

# Detecting, Tracking, and Exploiting Spectrum Opportunities in Unslotted Primary Systems

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**Abstract**—We consider the design of an spectrum overlay network where secondary users detect, track, and exploit spectrum opportunities in unslotted primary systems. We develop a decision-theoretic framework for the optimal joint design of the spectrum sensor and tracking and access strategies, and obtain structural solutions. Our results suggest the equivalence between the design of overlay networks in unslotted primary systems and that in slotted primary systems where channel occupancy by primary users can only change at known and fixed time instants. This equivalence points to the possibility of reducing the design of spectrum overlay in unslotted primary systems to that in slotted primary systems, a significantly simpler problem, and porting results available on the latter to the former.

## I. INTRODUCTION

We consider the design of an ad hoc overlay network where secondary users detect, track, and exploit spectrum opportunities in unslotted primary systems. Integrated in the design are the spectrum sensor at the physical layer for opportunity detection, the sensing strategy at the MAC layer for channel selection in order to track the rapidly varying opportunities, and the access strategy that decides whether to transmit in order to maximize the throughput of the secondary user without causing unacceptable interference to primary users.

The spectrum of interest consists of multiple channels. The occupancy of each channel by primary users is modelled as a continuous-time Markov chain, which has been shown to match well with the spectrum usage in wireless LAN [1]. The secondary network adopts a slotted transmission structure. At the beginning of each slot, a secondary user decides which channel to sense and potentially transmit over. The objective is to track spectrum opportunities that vary in both time and frequency. After a channel is chosen, the user senses this channel for a period of time to assess whether it is suitable for transmission. In other words, *based on the sensing measurements collected at the beginning of the slot, the user detects whether the channel will be idle for the remaining of the slot*. Since primary users use unslotted transmission structure, the state (idle or busy) of the channel can change at any time. The detection outcome is thus subject to errors even when the channel state during the sensing period is perfectly known. The next decision the user needs to make

is whether to transmit based on the imperfect detection outcome. The objective is to minimize missed opportunities while limiting the probability of colliding with primary users. The optimal solution requires an integrated design of these three components: the spectrum sensor for opportunity detection, the sensing strategy for opportunity tracking, and the access strategy for opportunity exploitation.

**Related Work** Opportunistic spectrum access (OSA) in slotted primary systems has been addressed in [2]–[4], where the channel occupancy is modelled as a discrete-time Markov chain and a decision-theoretic framework based on the theory of *constrained* Partially Observable Markov Decision Process (POMDP) is developed. Within this framework, a separation principle has been established in [3] for the optimal joint design of OSA in the presence of sensing errors. It shows that the design of the sensing strategy for opportunity tracking can be decoupled from that of the spectrum sensor and the access strategy, leading to *closed-form* solutions for the spectrum sensor and the access strategy and *unconstrained* design of the sensing strategy.

Based on a continuous-time Markovian model of channel occupancy, the design of slotted secondary systems in unslotted primary systems is addressed in [5]. The focus there is the access strategy; a round robin scheme is used for channel selection. Furthermore, access decisions are based on the channel state at the beginning of each slot, which is assumed to be perfectly known; opportunity detection is not part of the design. In this paper, we take the viewpoint that access decisions should be based on the channel state during the *transmission* period rather than the *sensing* period of each slot. Opportunity detection — infer the state of the channel during the transmission period from measurements taken in the sensing period — is thus a crucial issue, and the operating characteristics of the spectrum sensor need to be chosen carefully.

An overview of challenges and recent developments in OSA can be found in [6].

**Contributions** In this paper, we develop a decision-theoretic framework for the joint design of spectrum sensor and sensing and access strategies of overlay systems in unslotted primary systems. Even though the underlying primary systems are modelled as *continuous-time* Markov chains, we show that the joint design can be formulated as a *discrete-time* constrained POMDP. *The key observation is that the difference between*

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unslotted and slotted primary systems—that transmissions of primary users can start and end at arbitrary time instants—simply contributes to sensing errors. This suggests that the design of overlay networks in unslotted primary systems is equivalent to that in slotted primary systems with imperfect sensing, which has been addressed in [3], [4]. The significance of this result is that the design of spectrum overlay in unslotted primary systems can be reduced to the counterpart in slotted primary systems, a fundamentally simpler problem, and results available on the latter can be ported to the former problem.

## II. NETWORK MODEL

Consider a spectrum consisting of  $N$  channels, each with bandwidth  $B_n$  ( $n = 1, \dots, N$ ). The occupancies of these  $N$  channels by primary users are modelled as independent continuous-time Markov processes with two states:  $S_n(t) = 0$  (busy) and  $S_n(t) = 1$  (idle). More specifically, for channel  $n$ , the sojourn times in the busy and idle states are exponentially distributed with rates  $\mu_n$  and  $\lambda_n$ , respectively.

We consider an ad hoc overlay network where secondary users independently search for and access spectrum opportunities in these  $N$  channels. This overlay network adopts a slotted transmission structure with slot length  $L$ . In each slot, a secondary user chooses one of the  $N$  channels to sense and decides whether to transmit over the chosen channel based on the sensing outcome. Consider a slot starting at time  $t$ . The beginning of each slot is used for spectrum sensing which takes  $L_s$  seconds. The remaining time  $[t + L_s, t + L]$  of the slot is used for transmission if the user chooses to. If the channel remains idle for the whole period of  $[t + L_s, t + L]$ , the transmission is successful. Otherwise, a collision with primary users occurs. The receiver acknowledges each successful transmission at the end of the slot. Note that even if the channel is idle during the sensing period  $[t, t + L_s]$ , it may become busy in any segment of the transmission period  $[t + L_s, t + L]$ , resulting in a collision. For convenience, we use  $k$  ( $k = 1, \dots, T$ ) as the slot index, *i.e.*, slot  $k$  starts at  $t_k \triangleq (k - 1)L$  and ends at  $kL$ .

Our goal is to develop an optimal OSA strategy for the secondary user, which sequentially determines which channel in the spectrum to sense, how to design the spectrum sensor, and whether to access based on the sensing outcome. We focus on an individual selfish user. The design objective is to maximize the throughput of this user during a desired period of  $T$  slots under the constraint that the probability of collision  $P_n(k)$  perceived by the primary users in any channel  $n$  and slot  $t$  is capped below a pre-determined threshold  $\zeta$ , *i.e.*,

$$P_n(k) \triangleq \Pr\{\Phi_n(k) = 1 \mid O_n(k) = 0\} \leq \zeta, \quad \forall n, k, \quad (1)$$

where  $\Phi_n(k) \in \{0$  (no access),  $1$  (access) $\}$  denotes the access decision of the user, and  $O_n(k)$  denotes opportunity defined as

$$O_n(k) = \begin{cases} 1, & S_n(t) = 1 \quad \forall t \in [t_k + L_s, t_k + L] \\ 0, & \text{otherwise} \end{cases}.$$

## III. A CONSTRAINED POMDP FRAMEWORK

In this section, we show that even though the channel occupancies are given by continuous-time Markov processes, the joint design of the spectrum sensor and sensing and access strategies can be formulated as a discrete-time POMDP.

### A. The Spectrum Sensor

Suppose that channel  $n$  is chosen in slot  $k$ . The objective of the spectrum sensor is to decide, based on the measurements taken in  $[t_k, t_k + L_s]$ , whether channel  $n$  is an opportunity for transmission, *i.e.*, idle during  $[t_k + L_s, t_k + L]$ . The spectrum sensor thus performs a binary hypothesis test:

$$\mathcal{H}_0 : O_n(k) = 1 \text{ (idle)} \quad \text{vs.} \quad \mathcal{H}_1 : O_n(k) = 0 \text{ (busy)}. \quad (2)$$

Let  $\hat{O}_n(k) \in \{0$  (busy),  $1$  (idle) $\}$  denote the sensing outcome (*i.e.*, the result of the binary hypothesis test). The performance of the spectrum sensor is characterized by the probability of false alarm (PFA)  $\epsilon_n(k)$  and the probability of miss detection  $\delta_n(k)$ :

$$\epsilon_n(k) \triangleq \Pr\{\hat{O}_n(k) = 0 \mid O_n(k) = 1\}, \quad (3a)$$

$$\delta_n(k) \triangleq \Pr\{\hat{O}_n(k) = 1 \mid O_n(k) = 0\}. \quad (3b)$$

For a given PFA  $\epsilon_n(k)$ , the largest achievable probability of detection (PD), denoted as  $P_{D,\max}^{(n)}(\epsilon_n(k))$ , can be attained by the optimal NP detector with the constraint that the PFA is no larger than  $\epsilon_n(k)$  or an optimal Bayesian detector with a suitable set of risks [7, Sec. 2.2.1]. All operating points  $(\epsilon, \delta)$  above  $P_{D,\max}^{(n)}$  are thus infeasible. The feasible set of operating points of the spectrum sensor is thus  $\{(\epsilon, \delta) : 0 \leq \epsilon \leq 1 - \delta \leq P_{D,\max}^{(n)}(\epsilon)\}$  as illustrated in Fig. 1. Note that every sensor operating point  $(\epsilon_n, \delta_n)$  below  $P_{D,\max}^{(n)}$  lies on a line that connects two boundary points and hence can be achieved by randomizing between two optimal NP detectors with properly chosen constraints on the PFA [7, Sec. 2.2.2]. Therefore, the design of spectrum sensor is reduced to the choice of a desired feasible sensor operating point. Note that both the feasible set and the optimal operating point may vary from slot to slot.

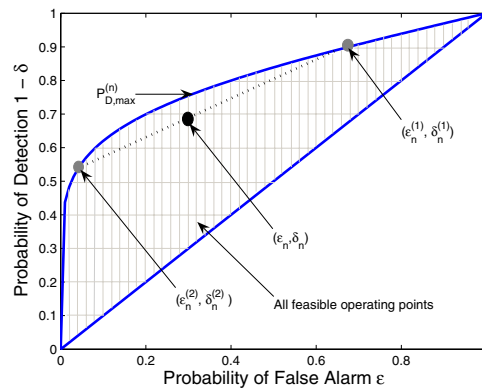


Fig. 1. Feasible set of sensor operating points  $(\epsilon_n, \delta_n)$ .

## B. The Sensing and Access Strategies

At the beginning of slot  $k$ , the secondary user first chooses a channel  $a(k) \in \{1, \dots, N\}$  to sense and a feasible sensor operating point  $(\epsilon_a(k), \delta_a(k))$ . It then determines whether to access  $\Phi_a(k) \in \{0 \text{ (no access)}, 1 \text{ (access)}\}$  by taking into account the sensing outcome  $\hat{O}_a(k) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$  provided by the spectrum sensor that is designed according to the chosen operating point  $(\epsilon_a(k), \delta_a(k))$ . At the end of this slot, the receiver acknowledges a successful transmission  $\Psi_a(k) \in \{0 \text{ (no ACK)}, 1 \text{ (ACK)}\}$ .

## C. A Constrained POMDP Formulation

We show here that the joint design of OSA can be formulated as a constrained POMDP defined as follows.

**State Space** The underlying system state is given by the state of each channel at the beginning of each slot. Let  $S_n(k) \triangleq S_n(t) |_{t=(k-1)L}$ . The system state in slot  $k$  is thus  $\mathbf{S}(k) = [S_1(k), \dots, S_N(k)] \in \{0, 1\}^N$ . It is straightforward that  $\{S_n(k)\}$  is a discrete-time Markov chain with transition matrix  $\mathbf{P}_n \triangleq \{\Pr[S_n(k+1) = j | S_n(k) = i]\}$  given by  $\mathbf{P}_n = \exp(\mathbf{Q}_n L)$ , where

$$\mathbf{Q}_n = \begin{pmatrix} -\mu_n & \mu_n \\ \lambda_n & -\lambda_n \end{pmatrix}.$$

is the transition rate matrix of the continuous-time Markov process that models the occupancy of channel  $n$ .

**Action Space** The action in each slot consists of three parts: a sensing decision  $a(k)$ , a spectrum sensor design  $(\epsilon_a(k), \delta_a(k))$ , and an access decision  $\Phi_a(k) \in \{0, 1\}$ .

**Observation Space** Optimal channel selection for opportunity tracking relies on the exploitation of the entire observation history of the user. To ensure synchronous hopping in the spectrum without introducing extra control message exchange, the user and its desired receiver must use *common observations* for channel selection. Since sensing errors may cause different sensing outcomes at the transmitter and the receiver, the acknowledgement  $\Psi_a(k)$  is the common observation.

**Reward** A natural definition of the reward is the number of bits that can be delivered by the user. Given sensing action  $a(k)$  and access action  $\Phi_a(k)$ , the immediate reward  $R_{\Psi_a(k)}$  can be defined as

$$R_{\Psi_a(k)} = \Psi_a(k) B_a L_{tx} = O_a(k) \Phi_a(k) B_a L_{tx}, \quad (4)$$

where  $L_{tx} = L - L_s$  is the transmission time in each slot.

**Belief Vector** Due to partial spectrum monitoring and sensing errors, a secondary user cannot directly observe the system state  $\mathbf{S}(k)$ . It can, however, infer the state from its decision and observation history. The statistical information on the system state provided by the entire decision and observation history can be encapsulated in a belief vector  $\Omega(k) \triangleq [\omega_1(k), \dots, \omega_N(k)]$ , where  $\omega_n(k)$  denotes the conditional probability (given the decision and observation history) that  $S_n(k) = 1$ . Note that we have used the channel independence to reduce the dimension of the belief vector from  $2^N$  to  $N$  [2].

**Policy** A joint design of OSA is given by policies of the above POMDP. Specifically, a sensing policy  $\pi_s$  specifies a sequence of functions (one for each slot), each mapping a belief vector  $\Omega(k)$  at the beginning of slot  $k$  to a channel  $a(k)$  to be sensed in slot  $k$ . Similarly, a sensor operating policy  $\pi_\delta$  specifies, in each slot  $k$ , a spectrum sensor design  $(\epsilon_a(k), \delta_a(k))$  based on the current belief vector  $\Omega(k)$  and the chosen channel  $a(k)$ . An access policy  $\pi_c$  specifies an access decision  $\Phi_a(k) \in \{0, 1\}$  in each slot  $k$  based on the current belief vector  $\Omega(k)$  and the sensing outcome  $\hat{O}_a(k)$ .

The above defined policies are deterministic. For unconstrained POMDPs, there always exist deterministic optimal policies. For constrained POMDPs, however, we may need to resort to randomized policies to achieve optimality. For randomized policies, we design the probability distribution of the action to be taken, rather than a specific deterministic action. Due to the uncountable space of probability distributions, randomized policies are usually computationally prohibitive.

**Objective and Constraint** We aim to develop the optimal joint design of OSA  $\{\pi_\delta^*, \pi_s^*, \pi_c^*\}$  that maximizes the expected total number of bits that can be delivered by the user in  $T$  slots under the collision constraint given in (1):

$$\begin{aligned} \{\pi_\delta^*, \pi_s^*, \pi_c^*\} &= \arg \max_{\pi_\delta, \pi_s, \pi_c} \mathbb{E}_{\{\pi_\delta, \pi_s, \pi_c\}} \left[ \sum_{k=1}^T R_{\Psi_a(k)} \middle| \Omega(1) \right] \\ \text{s.t. } P_a(k) &= \Pr\{\Phi_a(k) = 1 | O_a(k) = 0\} \leq \zeta, \quad \forall a, k, \end{aligned} \quad (5)$$

where  $\Omega(1)$  is the initial belief vector, which can be set to the stationary distribution of the underlying Markov process if no information on the initial system state is available.

## IV. THE OPTIMAL JOINT DESIGN

The first step to solving (5) is to express the objective and the constraint explicitly as functions of the actions. We establish first the optimality of deterministic sensing and sensor operating policies, which significantly simplifies the action space.

### A. The Optimality of deterministic policies

**Theorem 1:** For the optimal joint design of OSA given by (5), there exist deterministic optimal sensing and sensor operating policies.

*Proof:* omitted due to space limit. ■

As a result of Theorem 1, the user needs to choose, in each slot, a channel  $a$  to sense, a feasible sensor operating point  $(\epsilon_a, \delta_a)$ , and a pair of transmission probabilities  $(f_a(0), f_a(1))$ , where

$$f_a(\theta) \triangleq \Pr\{\Phi_a = 1 | \hat{O}_a = \theta\}$$

is the probability of accessing channel  $a$  given sensing outcome  $\hat{O}_a = \theta \in \{0, 1\}$ .

## B. The Objective Function

Let  $V_k(\Omega(k))$  be the value function, which represents the maximum expected reward that can be obtained starting from slot  $k$  given belief vector  $\Omega(k)$  at the beginning of slot  $k$ . Given that the user takes action  $A = \{a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))\}$  and observes acknowledgement  $\Psi_a = \psi$ , the reward that can be accumulated starting from slot  $t$  consists of two parts: the immediate reward  $R_{\Psi_a} = \psi B_a L_{tx}$  and the maximum expected future reward  $V_{k+1}(\Omega(k+1))$ , where  $\Omega(k+1) \triangleq \mathcal{T}(\Omega(k) | A, \psi)$  represents the updated belief vector after incorporating the action  $A$  and the acknowledgement  $\psi$  in slot  $k$ . We thus have the following optimality equation

$$V_k(\Omega(k)) = \max_A \sum_{s=0}^1 (s\omega_a + (1-s)(1-\omega_a)) \sum_{\psi=0}^1 U_{s,\psi}(A) [\psi B_a L_{tx} + V_{k+1}(\mathcal{T}(\Omega(k) | A, \psi))], \quad (6a)$$

$$V_T(\Omega(T)) = \max_A \sum_{s=0}^1 (s\omega_a + (1-s)(1-\omega_a)) U_{s,1}(A) B_a L_{tx}, \quad (6b)$$

where  $U_{s,\psi}(A) \triangleq \Pr\{\Psi_a = \psi | S_a = s\}$  is the conditional distribution of the acknowledgement given the current state  $S_a$  of channel  $a$  and action  $A$ . Since  $\Psi_a = O_a \Phi_a$ , we have

$$\begin{aligned} U_{s,0}(A) &= 1 - U_{s,1}(A) \\ U_{s,1}(A) &= \Pr[\Phi_a = 1, O_a = 1 | S_a = s] \\ &= \Pr[\Phi_a = 1 | O_a = 1, S_a = s] \Pr[O_a = 1 | S_a = s]. \end{aligned}$$

Due to space limit, we omit the detailed derivation of  $U_{s,1}(A)$ .

The updated belief vector  $\Omega(k+1) = \mathcal{T}(\Omega(k) | A, k)$  can be obtained from Bayes' rule.

## C. The Collision Constraint

The collision probability  $P_a(k)$  is determined by the sensor operating point  $(\epsilon_a, \delta_a)$  and the transmission probabilities  $(f_a(0), f_a(1))$ .

$$\begin{aligned} P_a(k) &\triangleq \Pr\{\Phi_a(k) = 1 | O_a(k) = 0\} \\ &= (1 - \delta_a)f_a(0) + \delta_a f_a(1) \leq \zeta. \end{aligned} \quad (8)$$

In principle, by solving (6) recursively (starting from the last slot  $T$  using (6b)) under the constraint of (8), we can obtain the maximum overall throughput  $V_1(\Omega(1))$  of the secondary user and the corresponding policies  $\{\pi_s^*, \pi_\delta^*, \pi_c^*\}$ . However, (6) is generally intractable due to the uncountable action space.

## V. STRUCTURAL SOLUTIONS

Obtaining the optimal design by directly solving (6) recursively is unattractive, suffering from the curse of dimensionality and providing little insights into system design. We thus consider a two-step approach that decouples the design of the sensing policy  $\pi_s$  from that of the spectrum sensor  $\pi_\delta$  and the access policy  $\pi_c$ . In the first step, we choose

the sensor operating policy  $\pi_\delta$  and the access policy  $\pi_c$  to maximize the instantaneous throughput subject to the collision constraint. Specifically, for any chosen channel  $a$ , the optimal sensor operating point  $(\epsilon_a^*, \delta_a^*)$  and transmission probabilities  $(f_a^*(0), f_a^*(1))$  are given by

$$\begin{aligned} \{(\epsilon_a^*, \delta_a^*), (f_a^*(0), f_a^*(1))\} &= \arg \max_{\epsilon_a, \delta_a, f_a(0), f_a(1)} \mathbb{E} [R_{\Psi_a(k)} | \Omega(t)] \\ &= \arg \max_{\epsilon_a, \delta_a, f_a(0), f_a(1)} \epsilon_a f_a(0) + (1 - \epsilon_a) f_a(1) \end{aligned}$$

s.t.  $P_a(t) = (1 - \delta_a) f_a(0) + \delta_a f_a(1) \leq \zeta.$

As given in Theorem 2 below,  $(\epsilon_a^*, \delta_a^*)$  and  $(f_a^*(0), f_a^*(1))$  can be obtained in closed-form.

*Theorem 2: For any chosen channel  $a$  in any slot, the optimal sensor should adopt the optimal NP detector with constraint  $\delta_a^* = \zeta$ . Correspondingly, the optimal access policy is to trust the sensing outcome given by the spectrum sensor, i.e.,  $f_a^*(0) = 0$  and  $f_a^*(1) = 1$ .*

*Proof:* omitted due to space limit. ■

In the second step, using the optimal sensor operating and access policies  $\{\pi_\delta^*, \pi_c^*\}$  given by Theorem 2, we choose the sensing policy to maximize the overall throughput. Specifically, the optimal sensing policy  $\pi_s^*$  is given by

$$\pi_s^* = \arg \max_{\pi_s} \mathbb{E}_{\pi_s} \left[ \sum_{k=1}^T R_{\Psi_a(k)} \middle| \Omega(1) \right]. \quad (10)$$

Note that the design of the sensing strategy has been reduced from a constrained POMDP (5) to an unconstrained one with finite action space. This is because the sensor operating points and the transmission probabilities determined by (9) have ensured the collision constraint regardless of channel selections. Unconstrained POMDPs have been well-studied. The optimal sensing policy can thus be readily obtained by using computationally efficient solution procedures in the literature.

It has been shown in [3] that the above separate design is optimal for OSA in slotted primary systems. This result is referred to as the separation principle for the joint design of OSA. We are currently investigating whether this separation principle also holds for OSA in unslotted primary systems, i.e., whether the above solution achieves optimality.

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