Connectivity and Quickest Opportunity Detection in Cognitive Radio Systems

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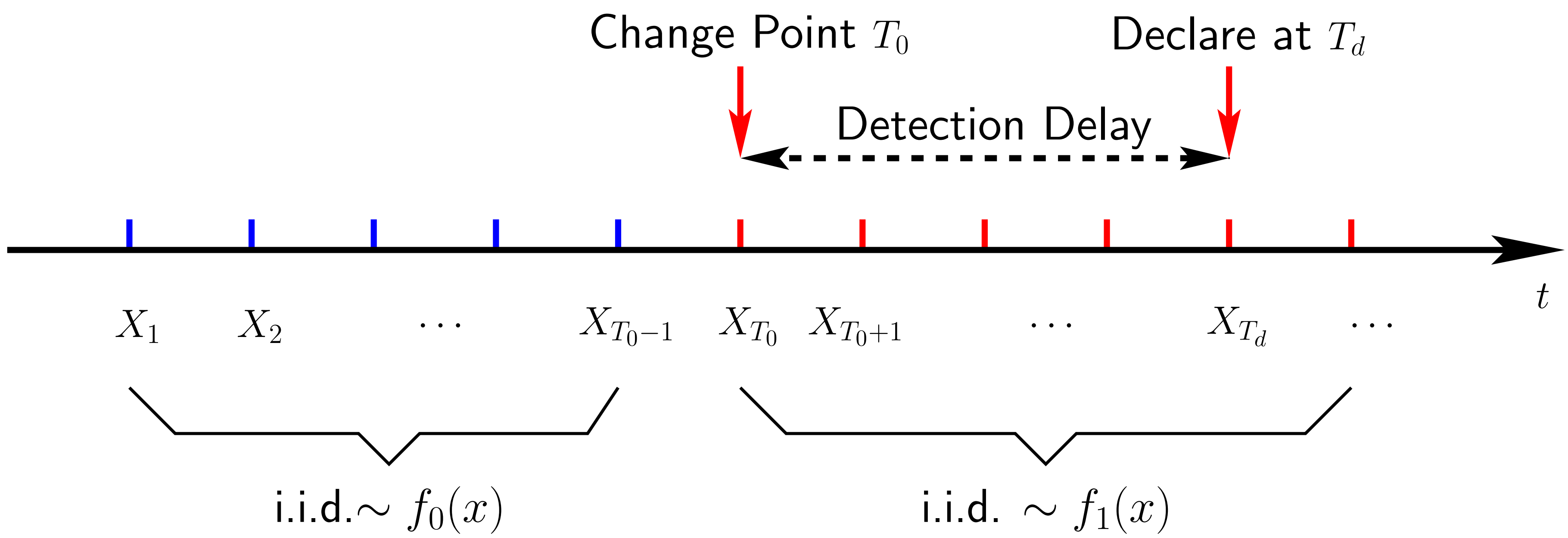
Quickest Detection of Spectrum Opportunity

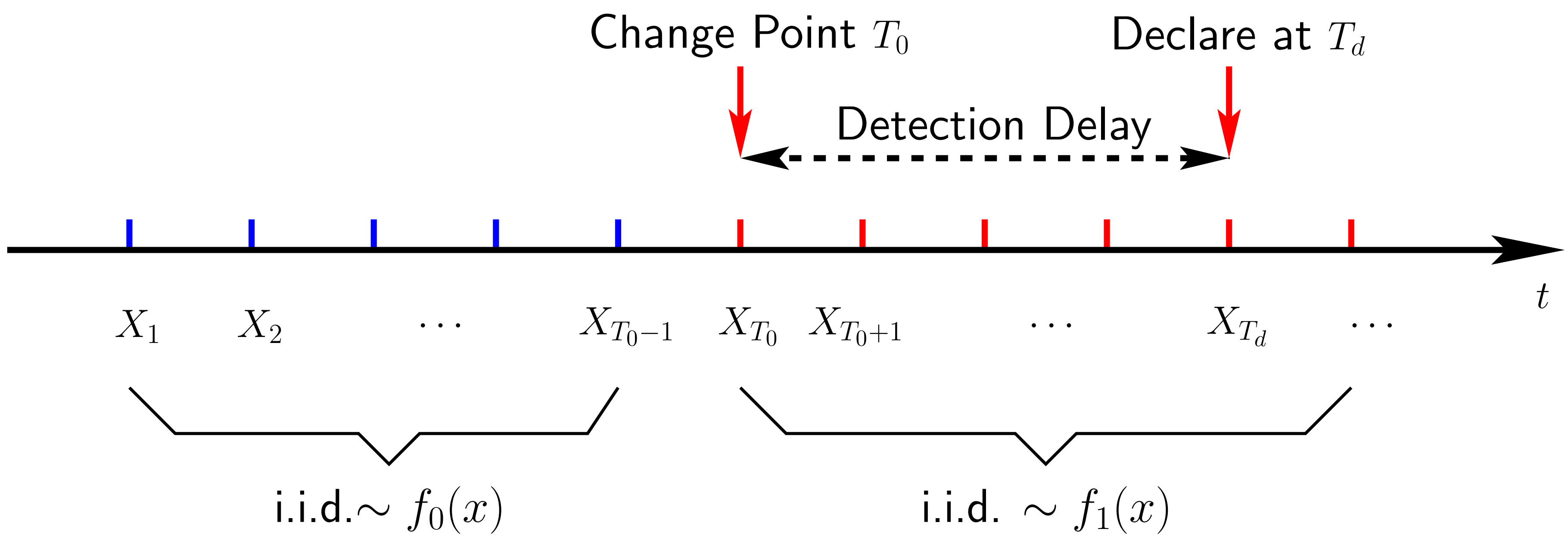
Quickest Change Detection



- **Quickest Detection:** min $\underbrace{\mathbb{E}[(T_d - T_0)^+]}_{\text{Detection Delay}}$ subject to $\underbrace{\Pr[T_d < T_0] \leq \zeta}_{\text{Reliability Constraint}}$

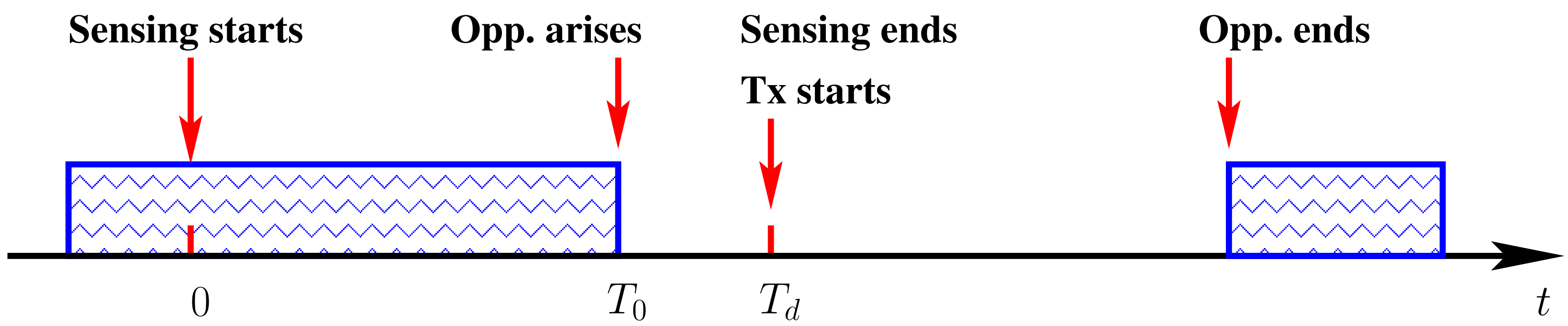
- **Tradeoff:** Detection delay vs. detection reliability.

Quickest Change Detection



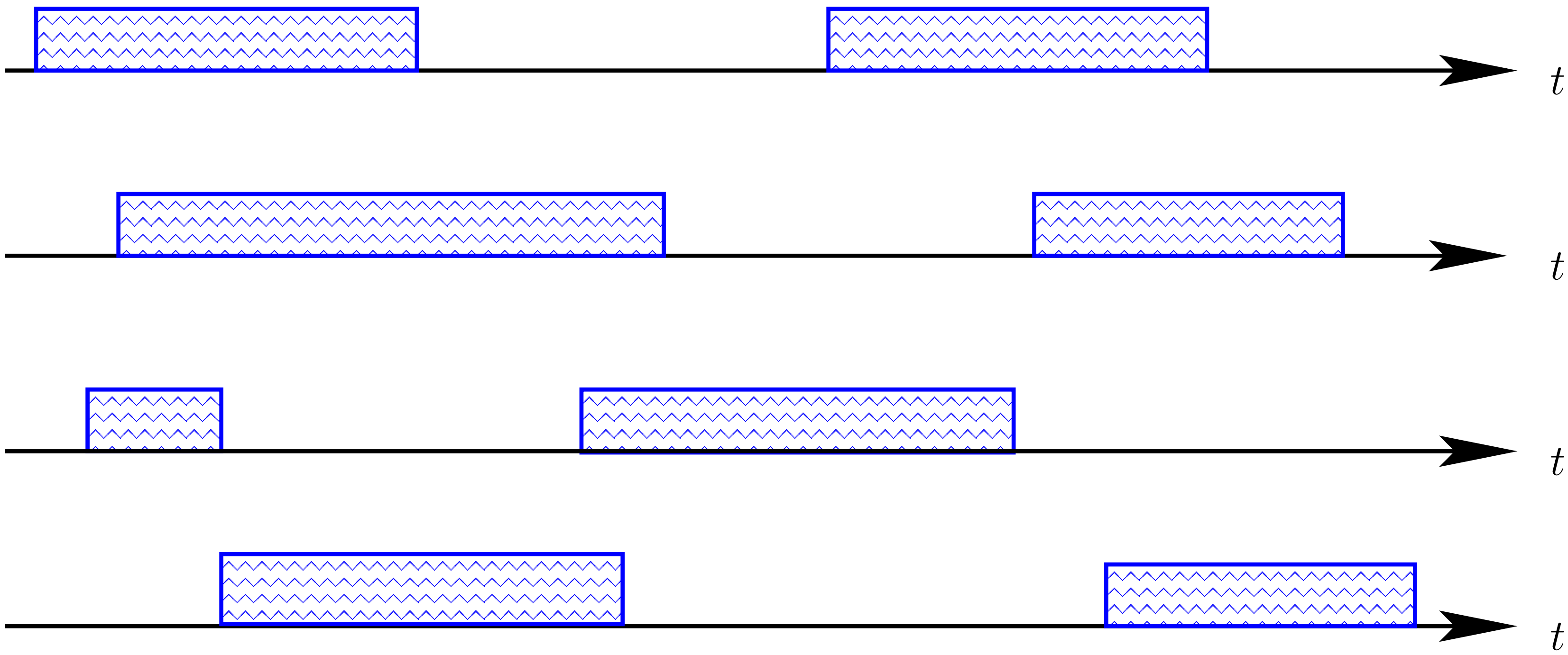
- **Quickest Detection:** min $\underbrace{\mathbb{E}[(T_d - T_0)^+]}_{\text{Detection Delay}}$ subject to $\underbrace{\Pr[T_d < T_0] \leq \zeta}_{\text{Reliability Constraint}}$
  - Bayesian: Shiryayev'61, Borovkov'98, Tartakovsky&Veeravalli'05.
  - Minimax: CUSUM (Page'54, Lorden'71).

Application in Cognitive Radio



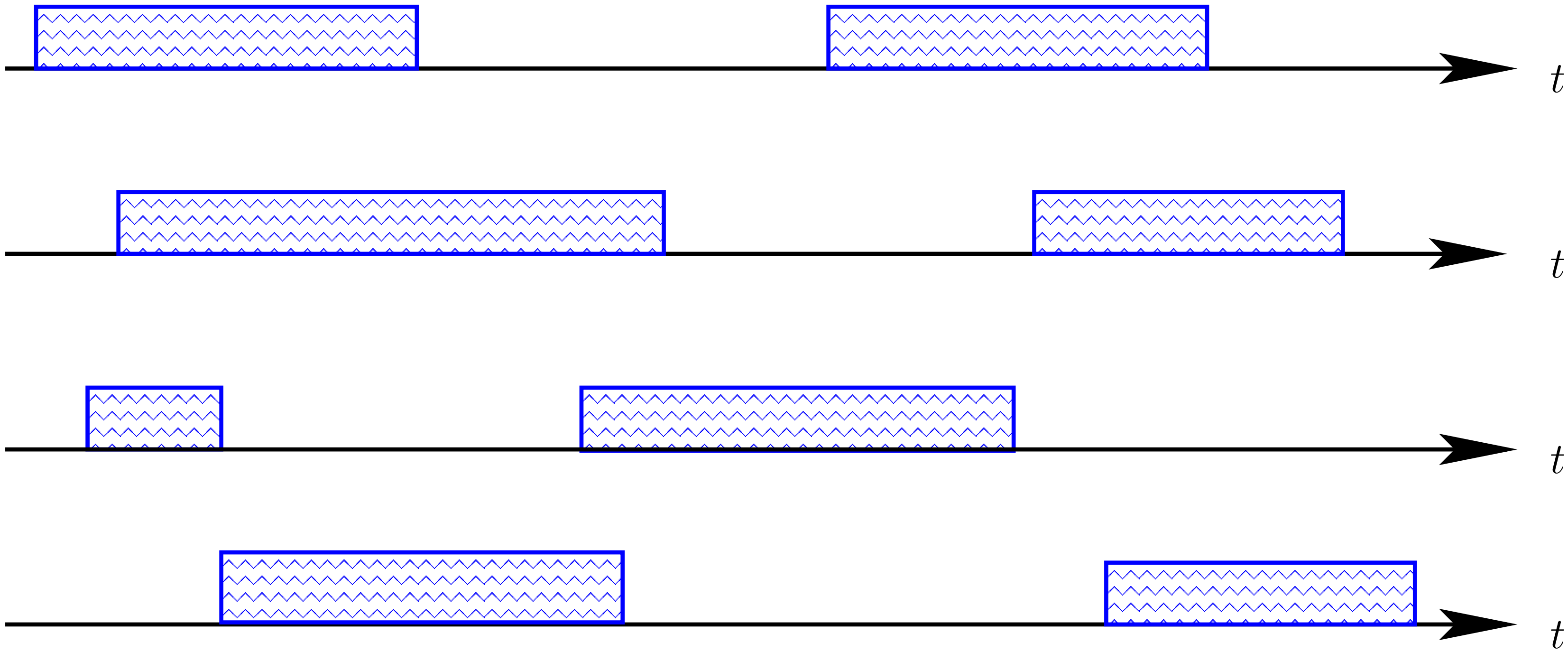
- **Measurements:** $\{X_1, X_2, \ldots, X_{T_0-1}\}$ are i.i.d with distribution $f_0(x)$;
  $\{X_{T_0}, X_{T_0+1}, \ldots\}$ are i.i.d with distribution $f_1(x)$.
- **Stopping Time:** At time $t = T_d$, the user declares an opportunity.
- **Quickest Detection:** min $\underbrace{\mathbb{E}[(T_d - T_0)^+]}_{\text{Detection Delay}}$ subject to $\underbrace{\Pr[T_d < T_0] \leq \zeta}_{\text{Interference Constraint}}$

Quickest Detection in Multiple On-Off Processes



- Two Fundamental Differences:
  - Channel occupancy is an on-off process with multiple change points.
  - There are multiple channels available.

Quickest Detection in Multiple On-Off Processes

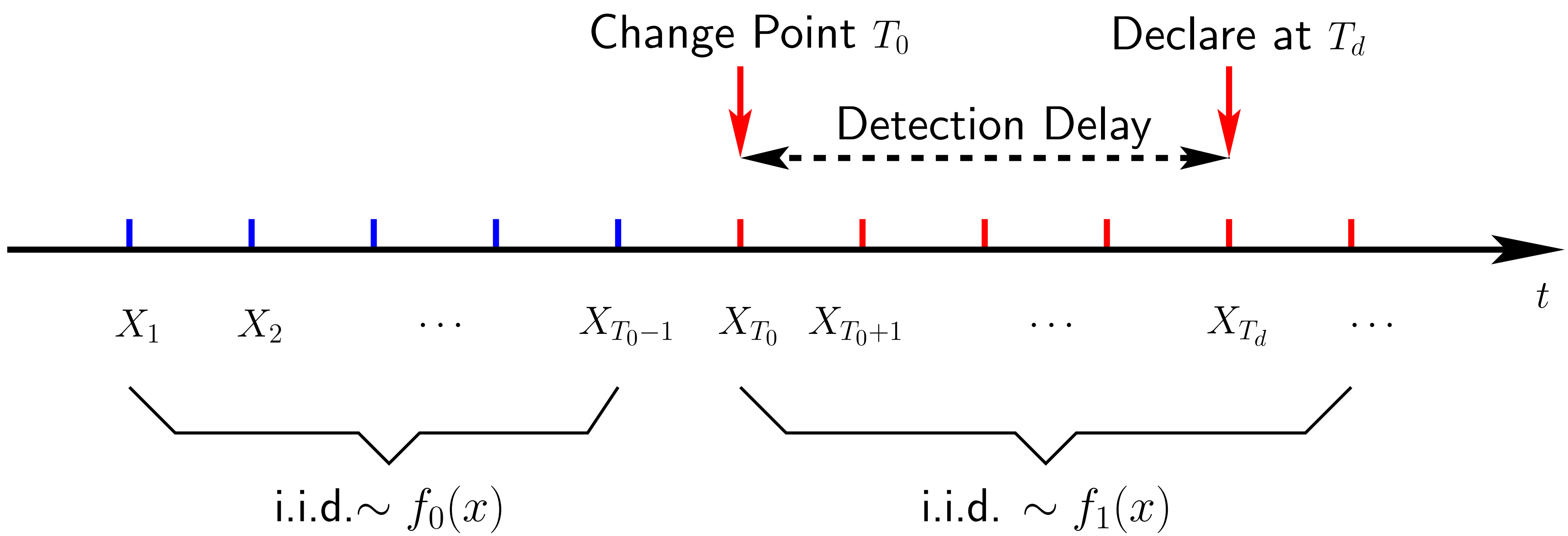


- Quickest Detection of Idle Periods in Multiple On-Off Processes:
  - Continue, switch, or declare?
- Tradeoffs:
  - Whether to declare: delay vs. reliability.
  - Whether to switch: loss of data vs. avoiding bad realizations.

Outline

- Quickest change detection in a single stochastic process
  - Shiryayev's algorithm
- Quickest detection in multiple on-off processes
  - A decision-theoretic formulation
  - The optimal detection rule: a threshold policy
- Simulation examples
- Conclusion and work in progress

Quickest Change Detection: Classic Bayesian Formulation

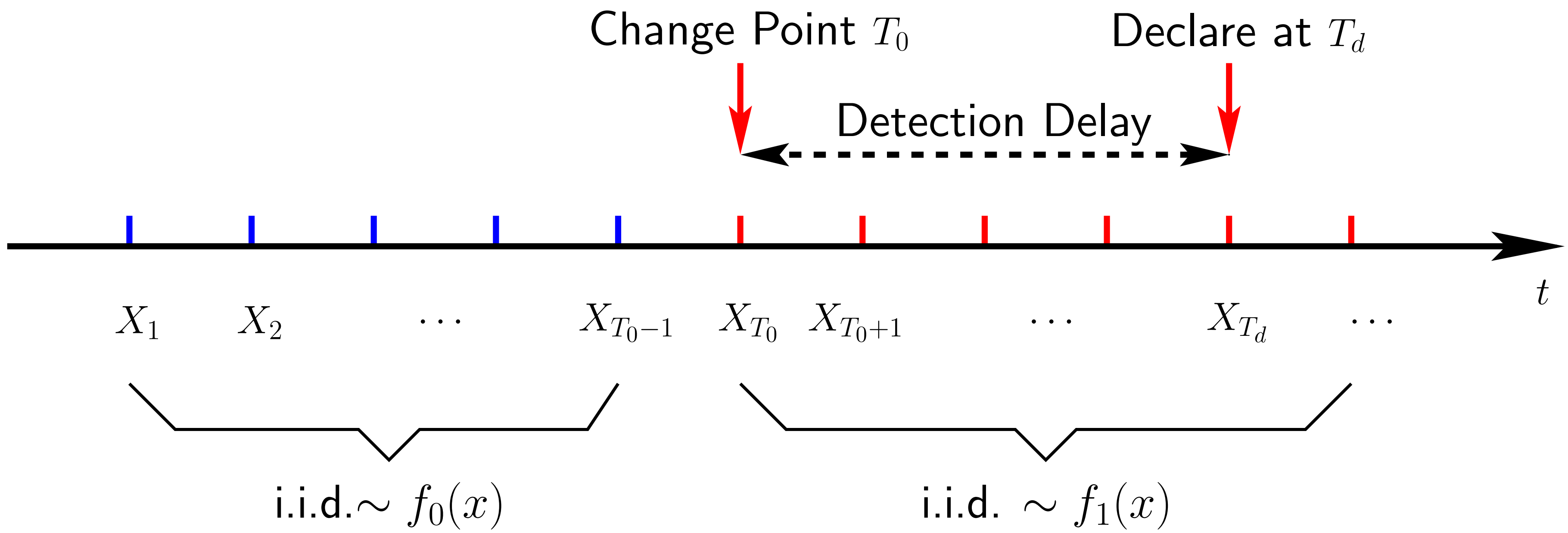


Bayesian Formulation:

- Priori distribution of change point T_0: geometric

$$\begin{aligned}\Pr[T_0 = 0] &= \lambda_0 \\ \Pr[T_0 = k] &= (1-\lambda_0)p(1-p)^{k-1}, \ \forall k > 0,\end{aligned}$$

Shiryayev's Algorithm



- A sufficient statistic: **a posterior probability** that change has occurred

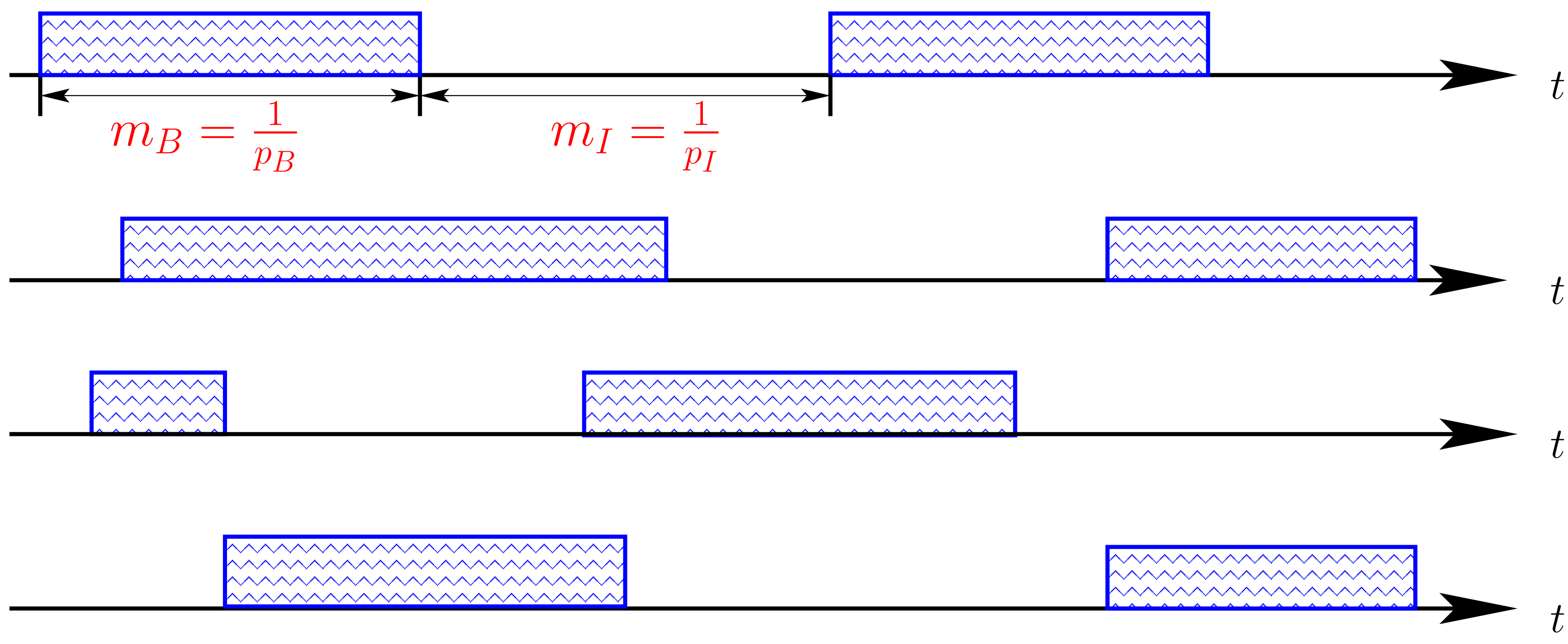
$$\lambda_t \triangleq \Pr[T_0 \leq t | X_1, X_2, \ldots, X_t].$$

- Shiryayev's detection rule:

$$T_d = \inf\{t : \lambda_t \geq \eta_d\}$$

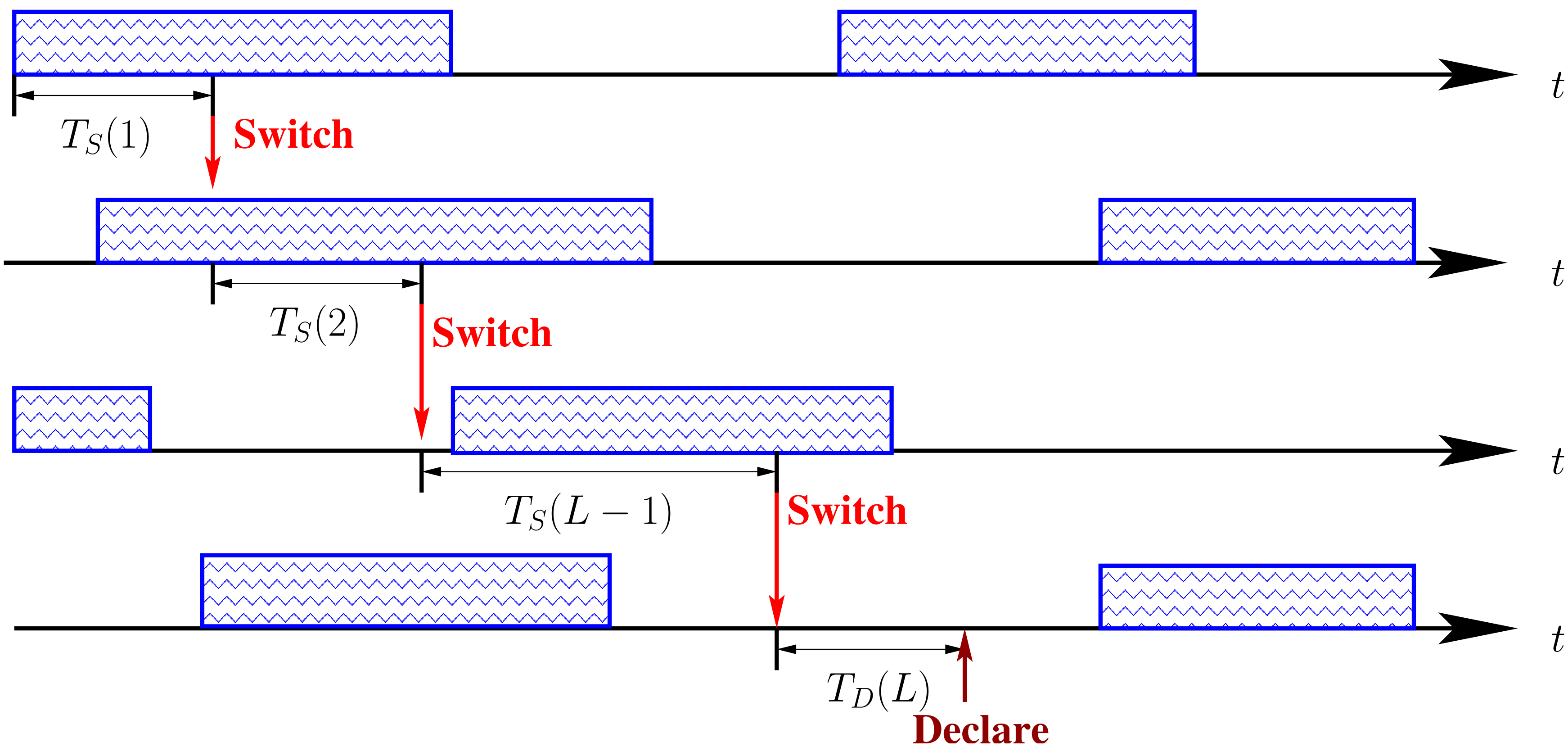
- Detection threshold η_d: determined by the reliability constraint ζ.
- Setting $\eta_d = 1 - \zeta$ is asymptotically optimal as $\zeta \rightarrow 0$.

Quickest Detection In Multiple On-Off Processes



- A number of independent homogeneous on-off processes.
- Busy period: geometrically distributed with mean $m_B = \frac{1}{p_B}$.
- Idle period: geometrically distributed with mean $m_I = \frac{1}{p_I}$.
- Fraction of idle time: $\lambda_0 = \frac{m_I}{m_I + m_B}$.

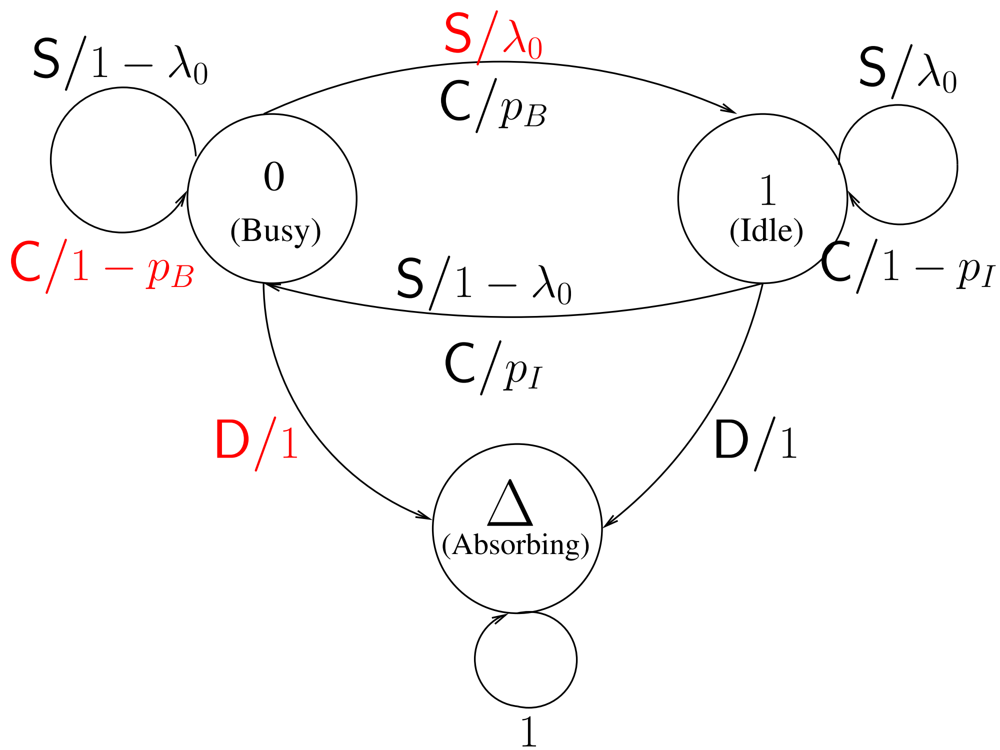
Quickest Detection In Multiple On-Off Processes



$$\underbrace{\min \mathbb{E}[\sum_{l=1}^{L-1} T_s(l) + T_d(L)]}_{\text{Detection Time}} \quad s.t. \quad \underbrace{\Pr[Z_L(\sum_{l=1}^{L-1} T_s(l) + T_d(L)) = \text{busy}] \leq \zeta}_{\text{Reliability Constraint}}$$

A POMDP Formulation

- State Space: 0 (busy), 1 (idle), Δ (absorbing state)
- Action Space: S (Switch), C (Continue), D(Declare)
- State Transition:



- Cost:
  - Switch or Continue: 1
  - Declare during a busy period: γ

A POMDP Formulation

- A Sufficient Statistic: the information state (belief)

$$\lambda_t = \Pr[Z_t = \textsf{idle} | X_1, X_2, \ldots, X_t]$$
$$\lambda_0 = \frac{m_I}{m_I + m_B}$$

- Update of the Information State

$$\lambda_t = \begin{cases} \mathcal{T}(\lambda_0 | x) & a(t-1) = \textsf{S}, \ X_t = x \\ \mathcal{T}(\lambda_{t-1} | x) & a(t-1) = \textsf{C}, \ X_t = x \end{cases}.$$

- $\mathcal{T}(\lambda|x)$: updated information state based on the new measurement x.

$$\mathcal{T}(\lambda|x) \triangleq \frac{(\lambda \bar{p}_I + \bar{\lambda} p_B) f_1(x)}{(\lambda \bar{p}_I + \bar{\lambda} p_B) f_1(x) + (\lambda p_I + \bar{\lambda} \bar{p}_B) f_0(x)}.$$

A POMDP Formulation

- Channel switching and change detection policy π:

$$\lambda_t \in [0,1] \quad \Longrightarrow \quad a(t) \in \{S, C, D\}, \text{ for each time } t.$$

- Quickest change detection:

$$\pi^* = \arg\min_{\pi} \mathbb{E}_\pi[\sum_{t=0}^{\infty} \underbrace{R_{\pi(\lambda_t)}}_{\text{Cost}} \mid \lambda_0 = \frac{m_I}{m_B + m_I}],$$

Quickest Change Detection: Value Functions

- $V(\lambda_t)$: the minimum expected total cost-to-go when the current belief is λ_t.

$$V(\lambda_t) = \min\{ \underbrace{V_C(\lambda_t)}_{\text{Continue}} , \underbrace{V_S(\lambda_t)}_{\text{Switch}} , \underbrace{V_D(\lambda_t)}_{\text{Declare}} \}.$$

- $V_C(\lambda_t)$: the minimum expected total cost-to-go if continue at t.

$$V_C(\lambda_t) = 1 + \int_x \underbrace{P(x;\lambda_t)}_{\Pr[\text{ observe } x \text{ under } \lambda_t \text{ }]} V(\mathcal{T}(\lambda_t|x))\mathrm{d}x$$

- $V_S(\lambda_t)$: the minimum expected total cost-to-go if switch at t.

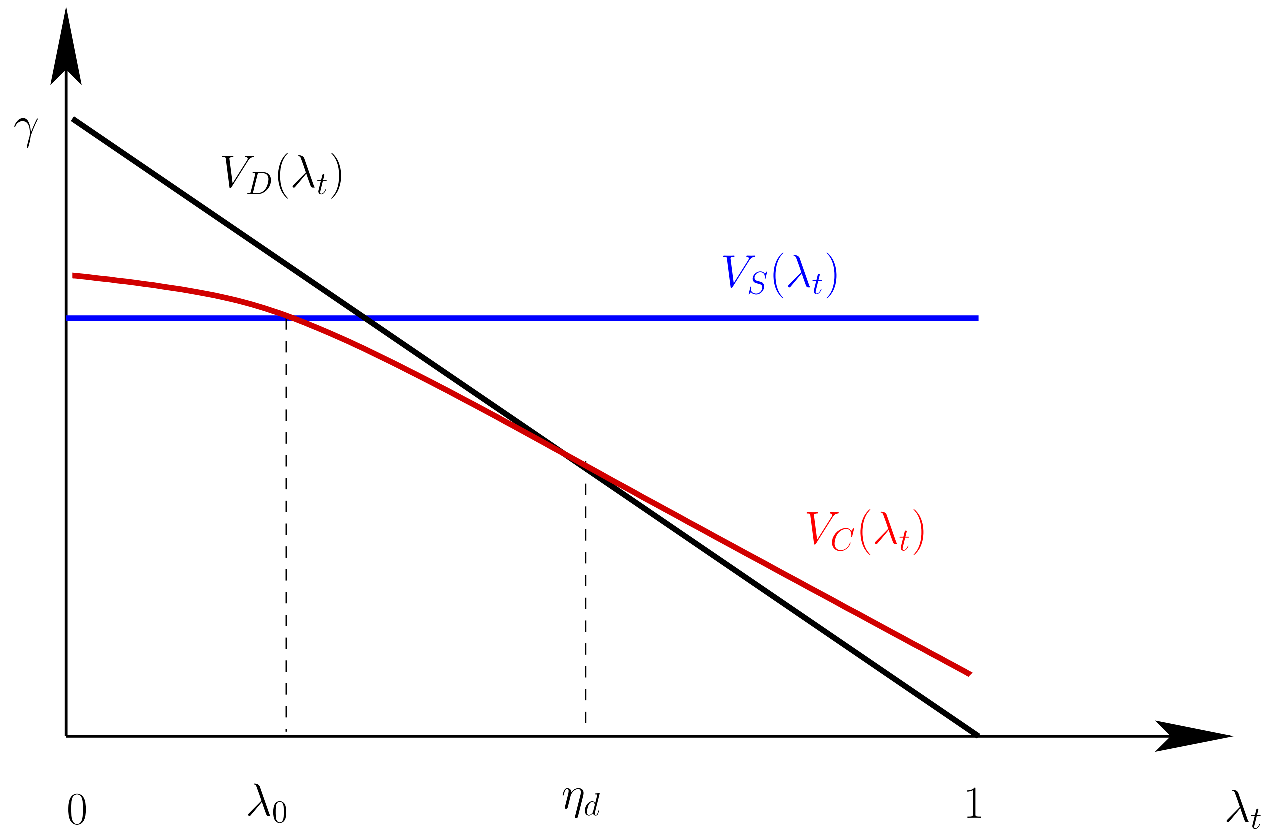
$$V_S(\lambda_t) = 1 + \int_x \underbrace{P(x;\lambda_0)}_{\Pr[\text{ observe } x \text{ under } \lambda_0 \text{ }]} V(\mathcal{T}(\lambda_0|x))\mathrm{d}x = V_C(\lambda_0)$$

- $V_D(\lambda_t)$: the minimum expected total cost-to-go if declare at t.

$$V_D(\lambda_t) = (1-\lambda_t)\gamma.$$

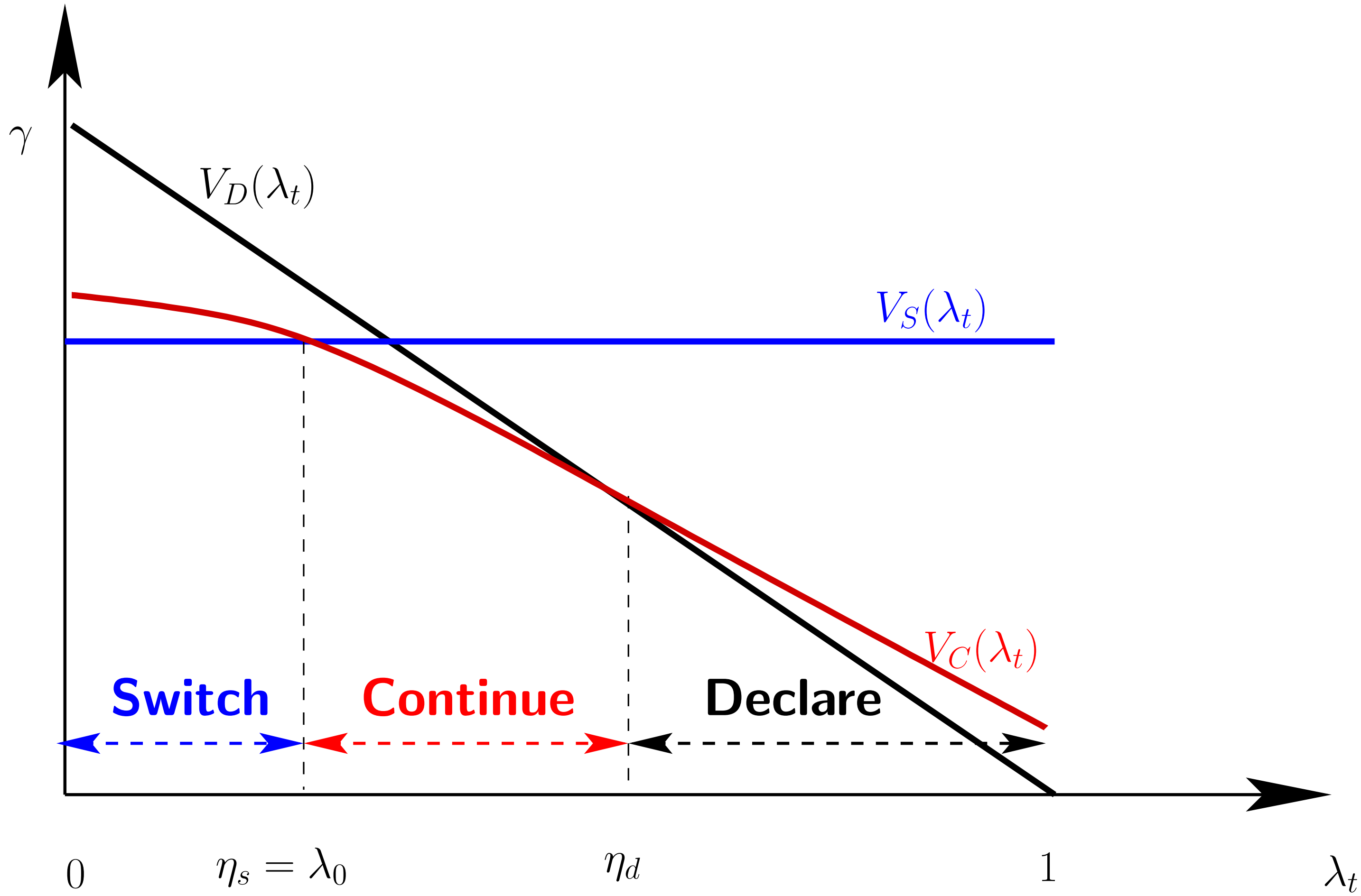
Quickest Change Detection: A Threshold Policiy

- $V_D(\lambda_t)$ is linear.
- $V_C(\lambda_t)$ is monotonically decreasing (if $\frac{1}{m_B} + \frac{1}{m_I} \leq 1$) and concave.
- $V_S(\lambda_t) = V_C(\lambda_0)$, where $\lambda_0 = \frac{m_I}{m_I + m_B}$.



Quickest Change Detection: A Threshold Policiy

$$V(\lambda_t) = \min\{ \underbrace{V_S(\lambda_t)}_{\text{Switch}}, \underbrace{V_C(\lambda_t)}_{\text{Continue}}, \underbrace{V_D(\lambda_t)}_{\text{Declare}} \}.$$

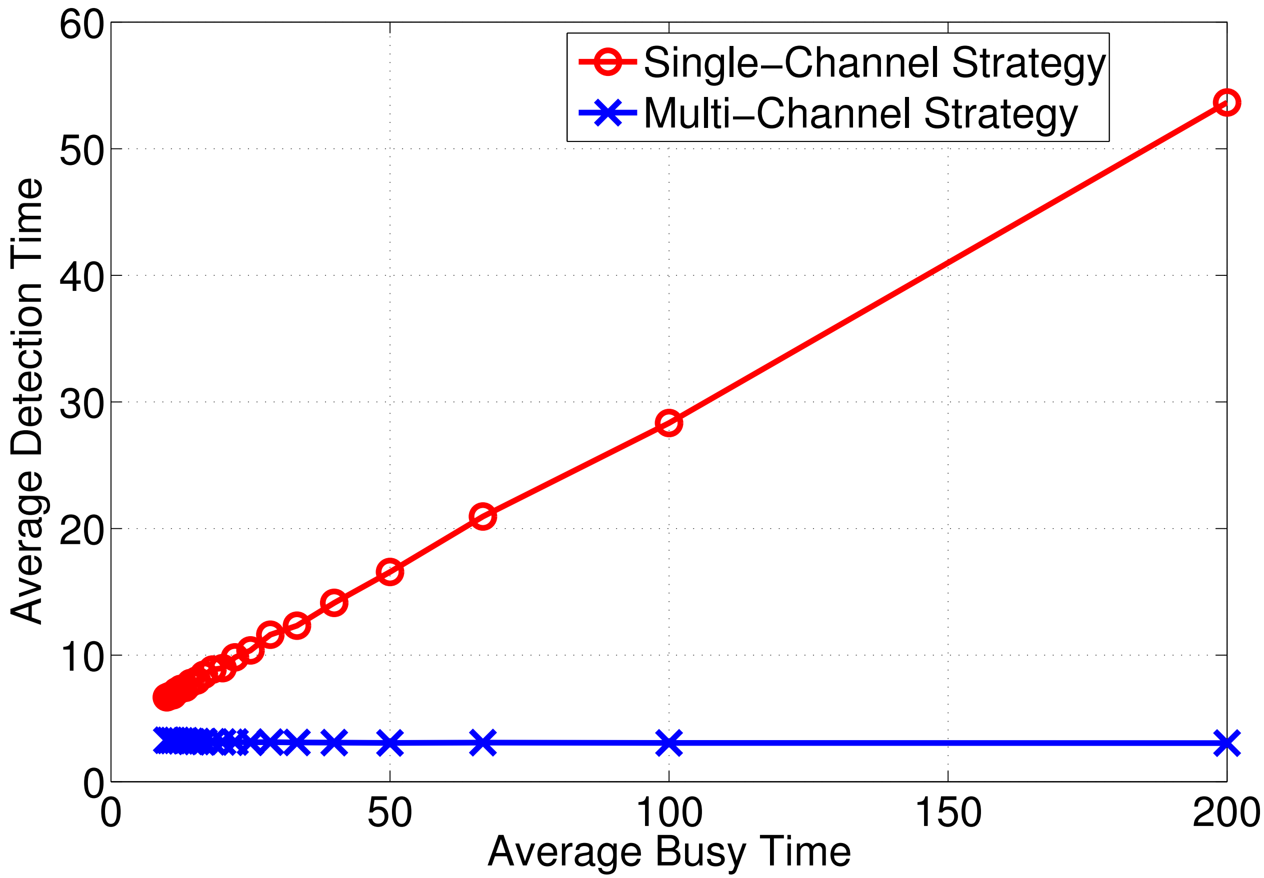


Simulation Examples

- $f_0(x),\ f_1(x)$: Gaussian with zero mean and different variances.
- $SNR = 10dB$.
- $\eta_d = 1 - \zeta$.

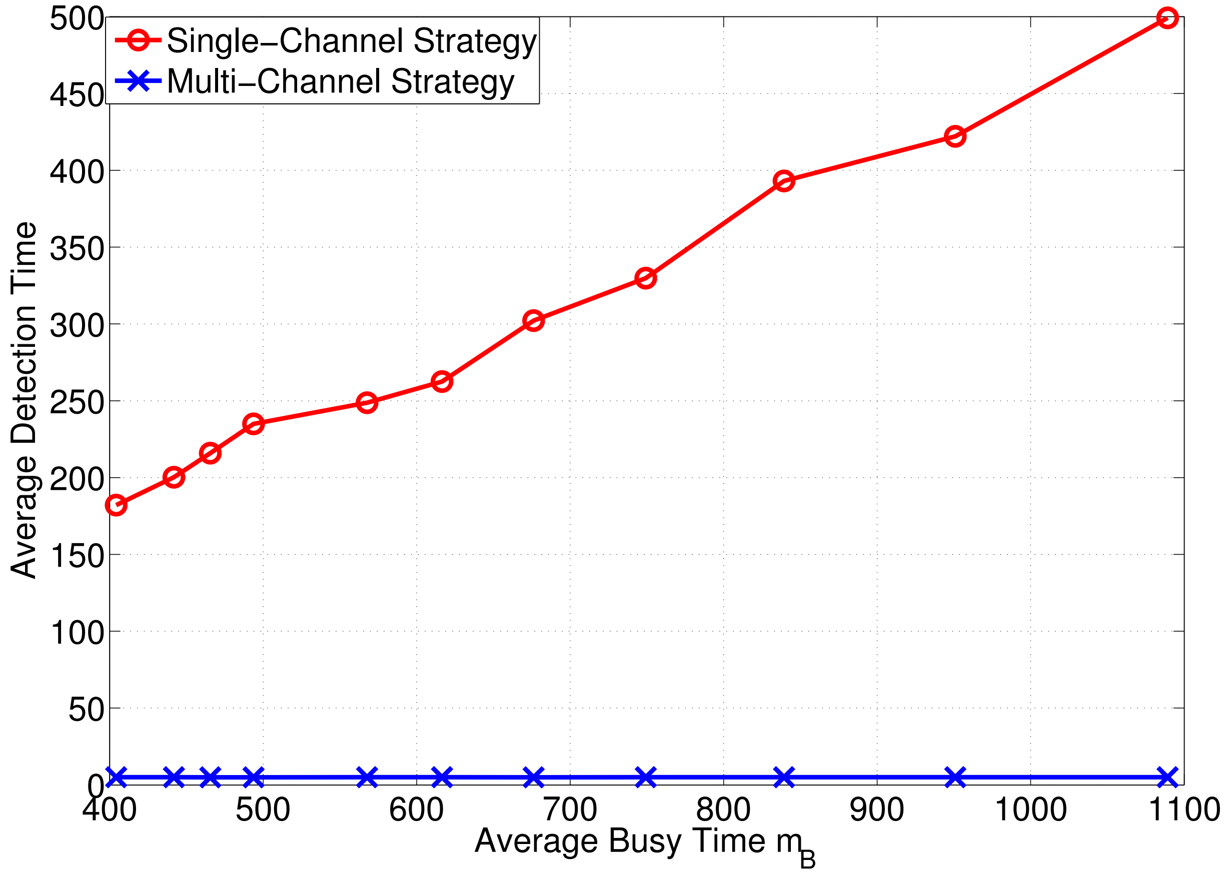
Simulation Example: Geometric Distribution

- Increase both m_B and m_I while keeping λ_0 fixed



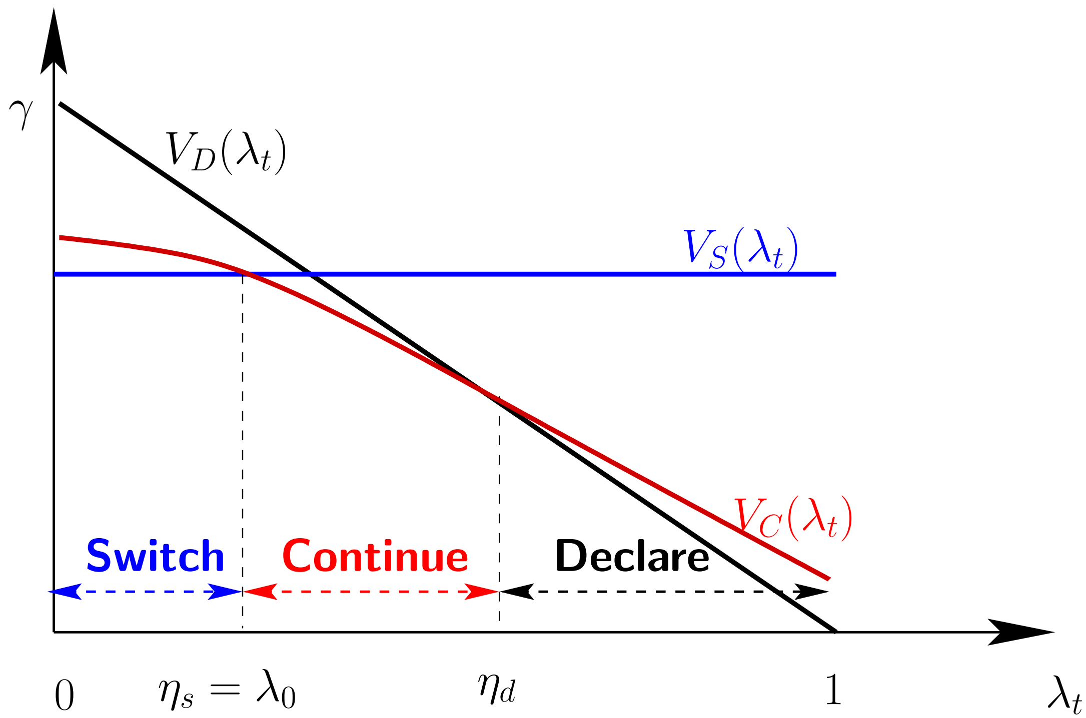
Simulation Example: Arbitrary Distributions

- Busy period: Pareto distribution with increasing tail index



Conclusion and Work in Progress

Quickest Detection in Multiple On-Off Processes:

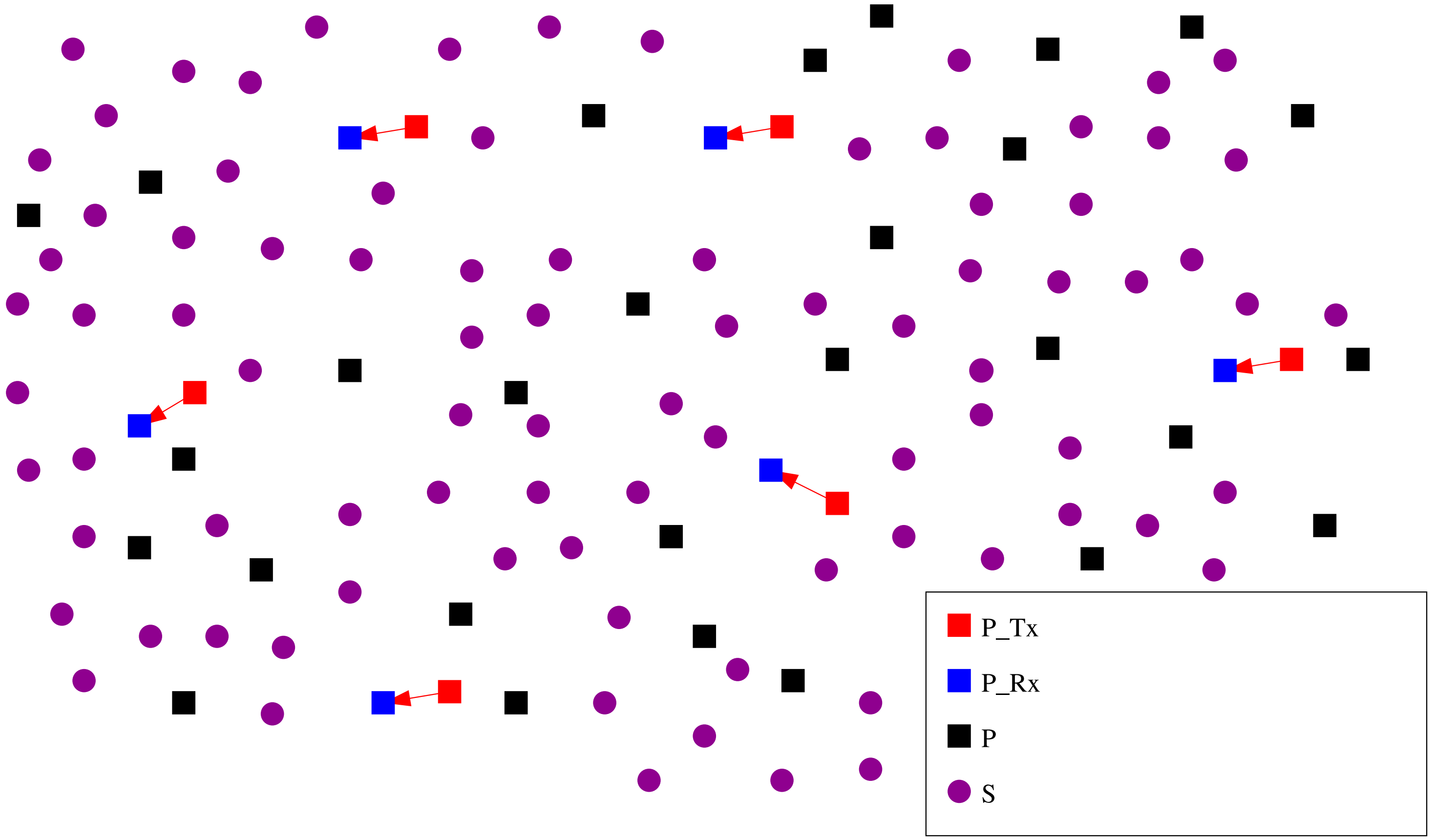


Work in Progress:

- Asymptotic optimality for arbitrary distributions and non-i.i.d. data.
- Minimax formulation.

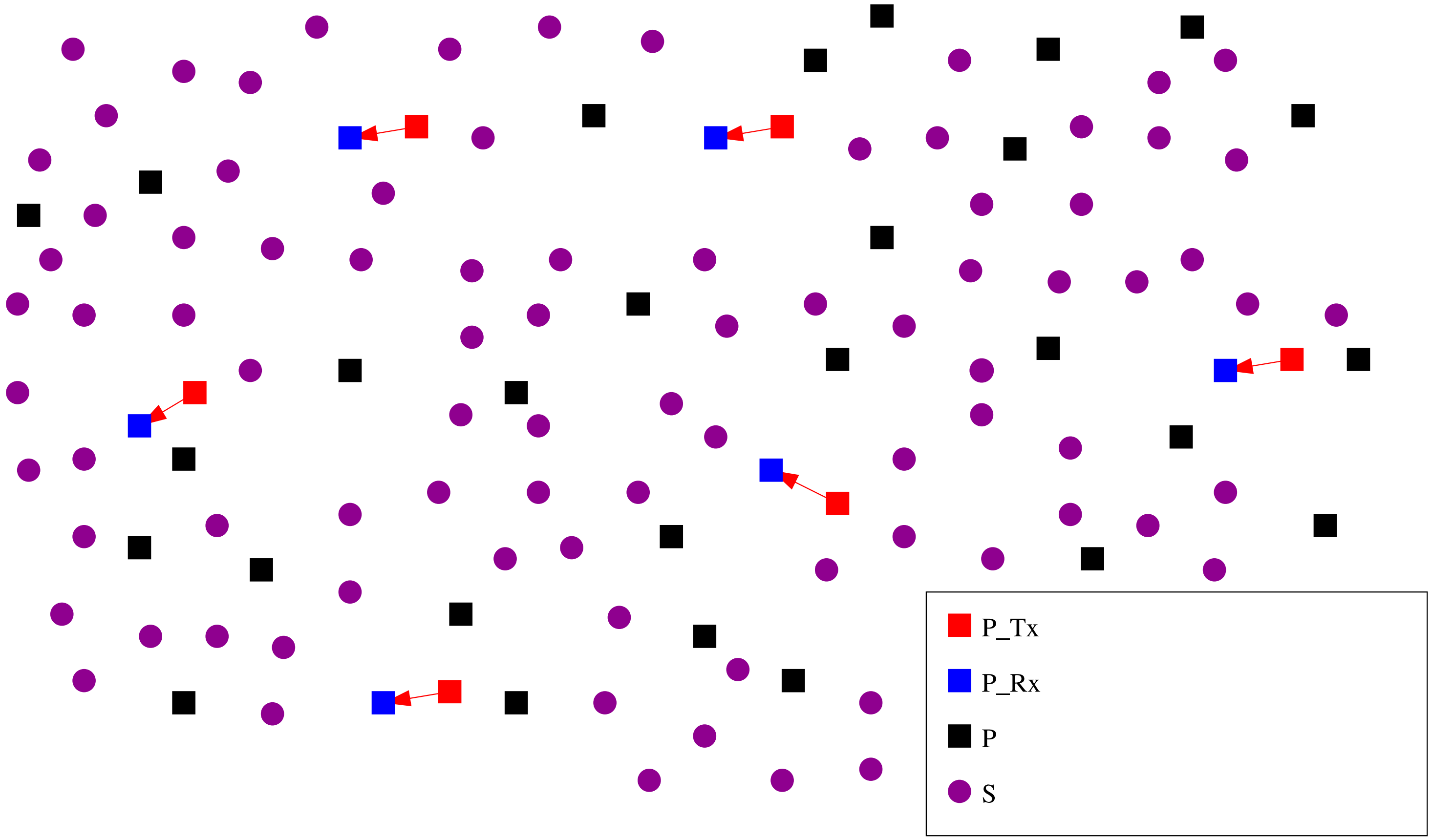
Connectivity

Connectivity: Poisson Primary + Poisson Secondary



Connectivity: the existence of an infinite connected component almost surely.

Connectivity

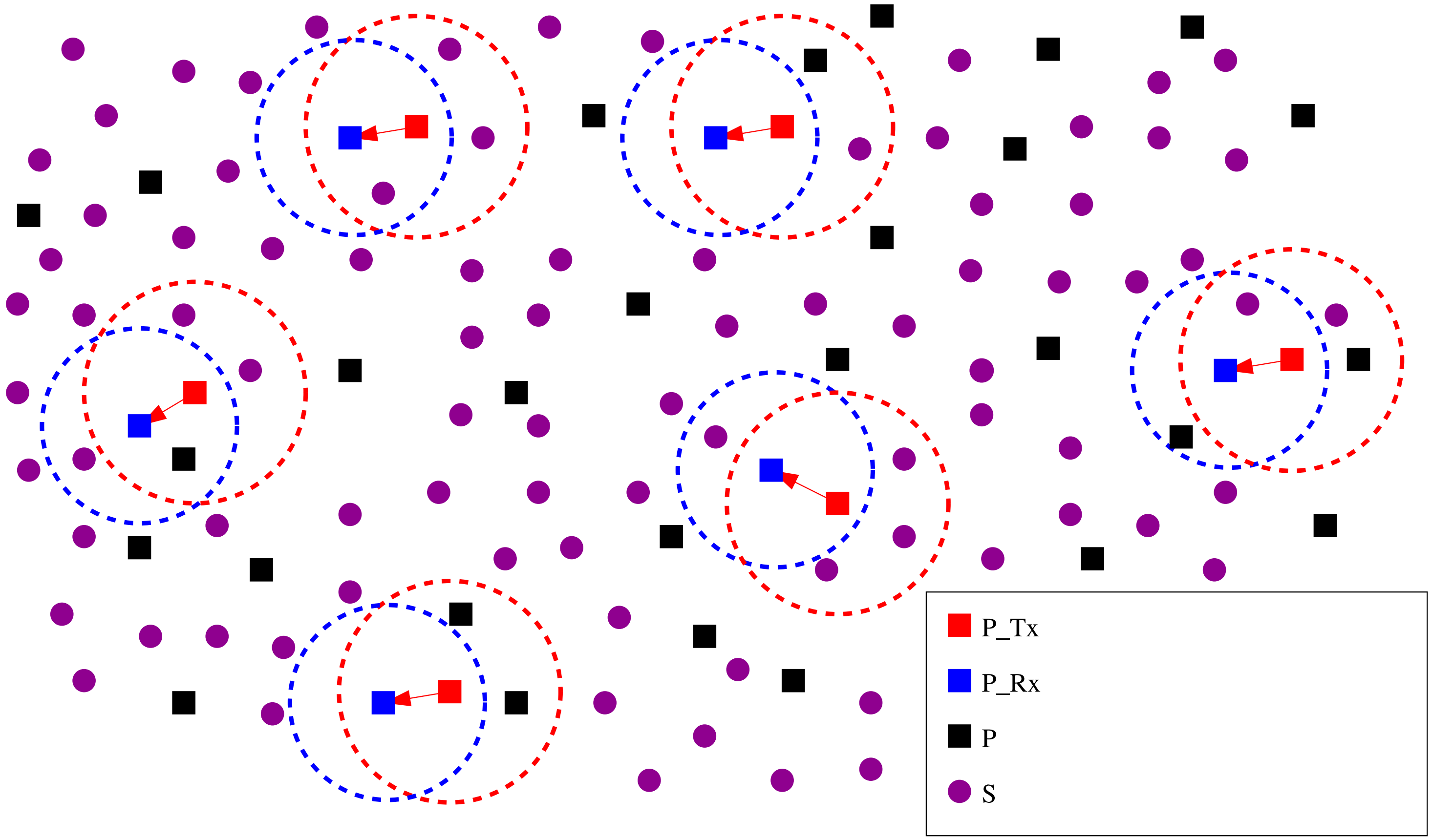


Connectivity: the existence of an infinite connected component almost surely.

Existence of A Link between Two Secondary Users:

- they are within Tx range;
- they see a bidirectional opportunity.

Who Sees An Opportunity Who Doesn't?



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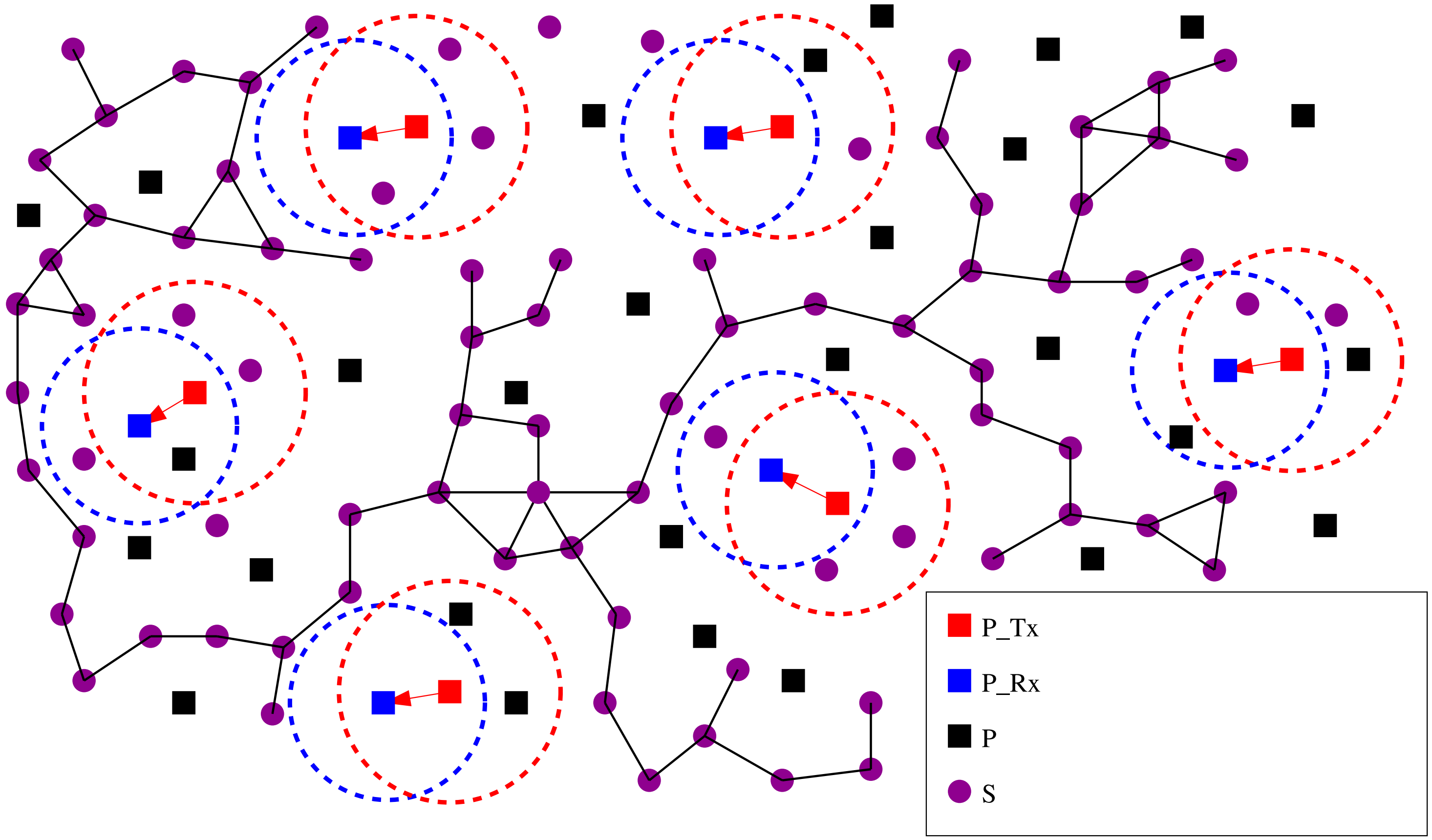

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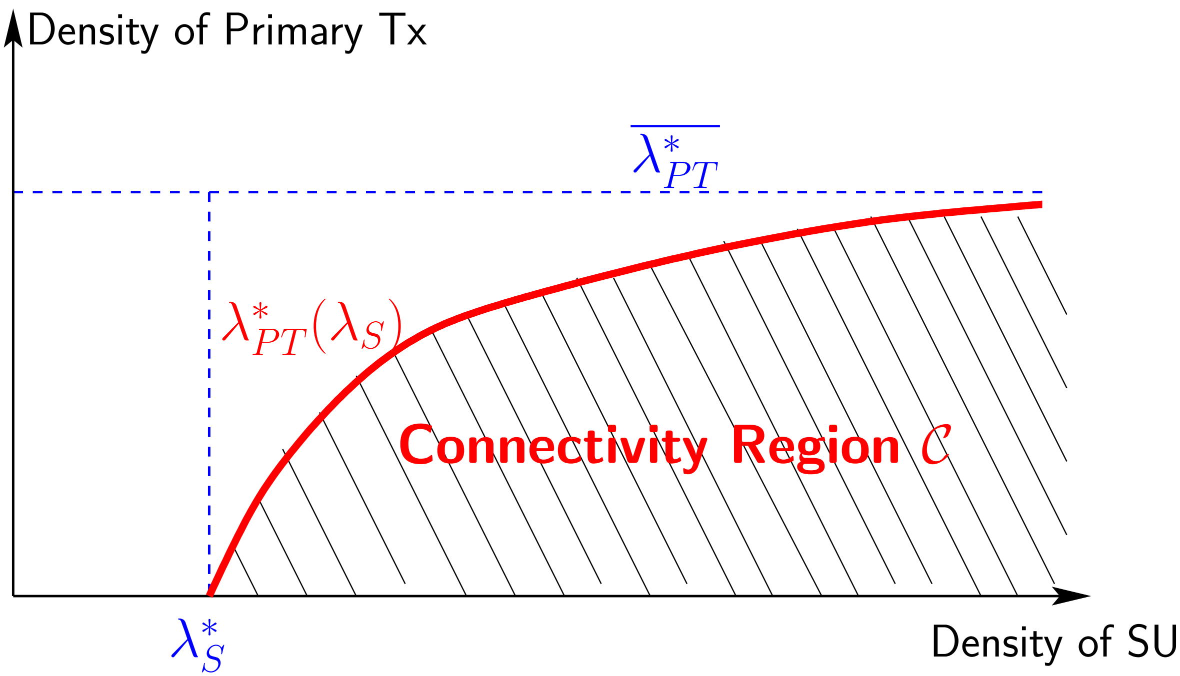
Connectivity Region



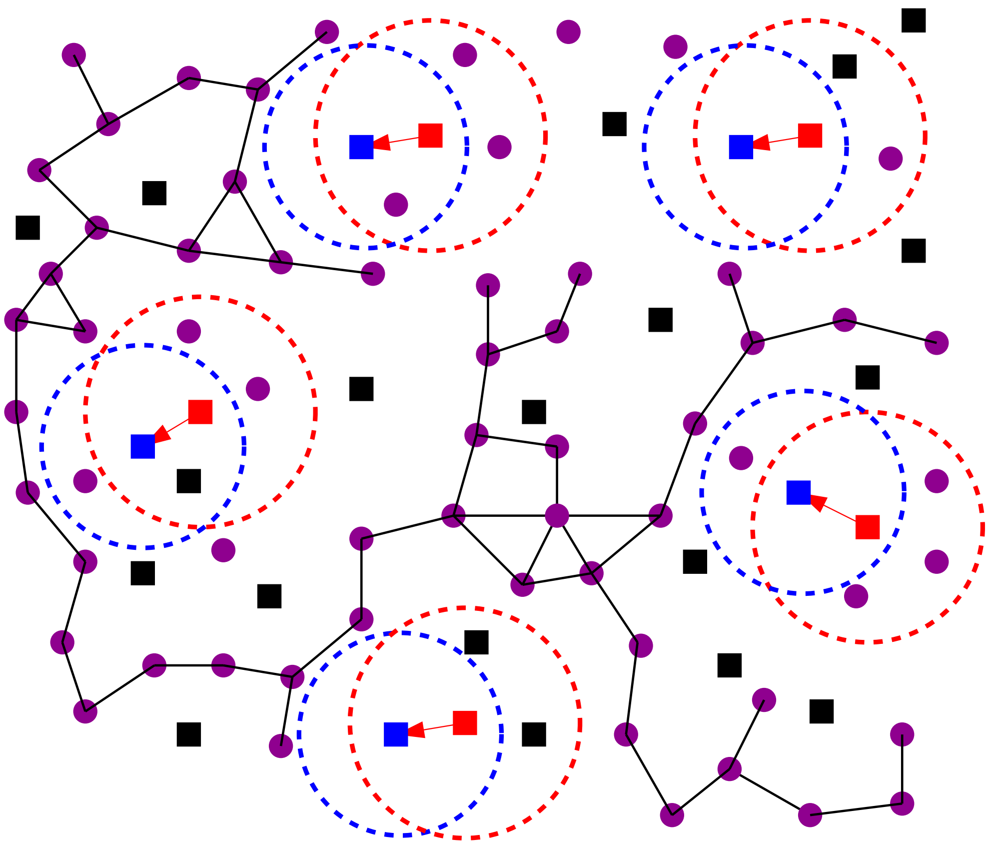
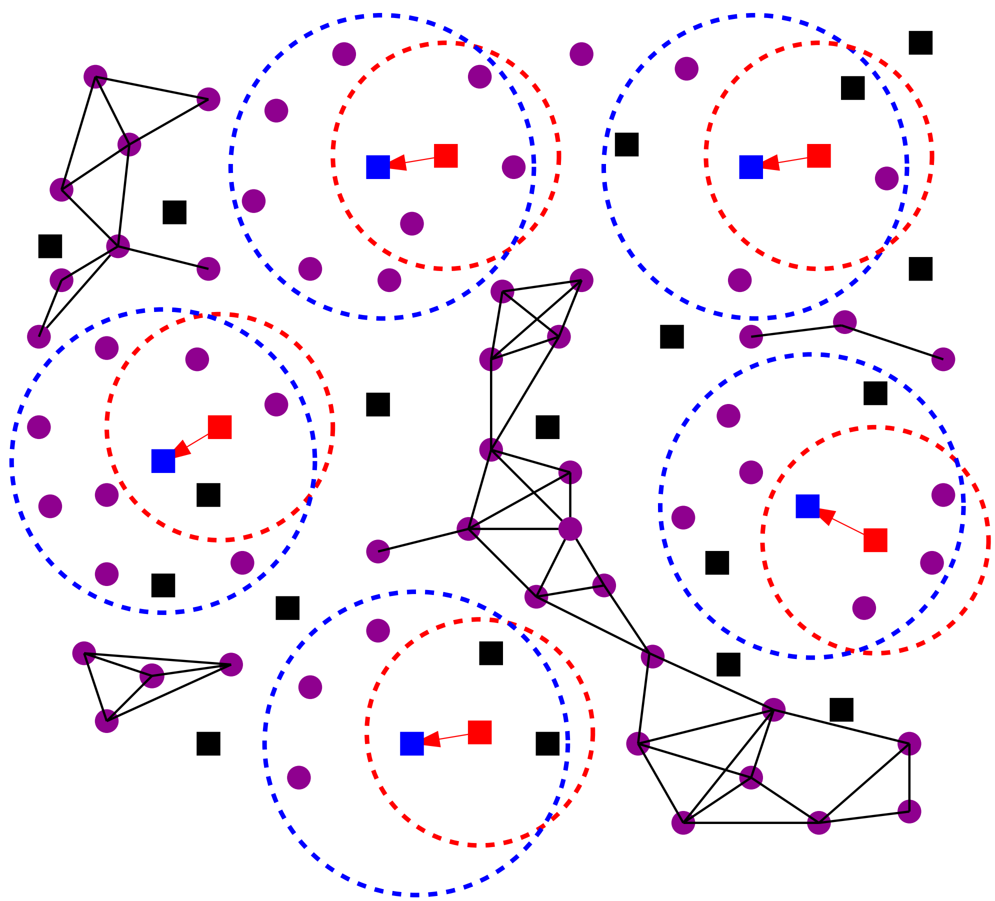
- $\forall(\lambda_S, \lambda_{PT}) \in \mathcal{C}$, there exists a *unique* infinite connected component.
- $\lambda^*_{PT}(\lambda_S)$ monotonically increases with λ_S.
- The critical density of secondary users: $\lambda^*_S = \lambda_c(r_{tx})$ (*CD of homogenous networks*).
- The critical density of primary Tx: $\overline{\lambda^*_{PT}} \leq \min\left\{\frac{1}{4\left(R_I^2 - r_p^2/4\right)}\lambda_c(1),\ \frac{1}{4\left(r_I^2 - r_p^2/4\right)}\lambda_c(1)\right\}$.

[0]W. Ren, Q. Zhao, D. Cheowtirakul, A. Swami, http://arxiv.org/abs/0903.1684

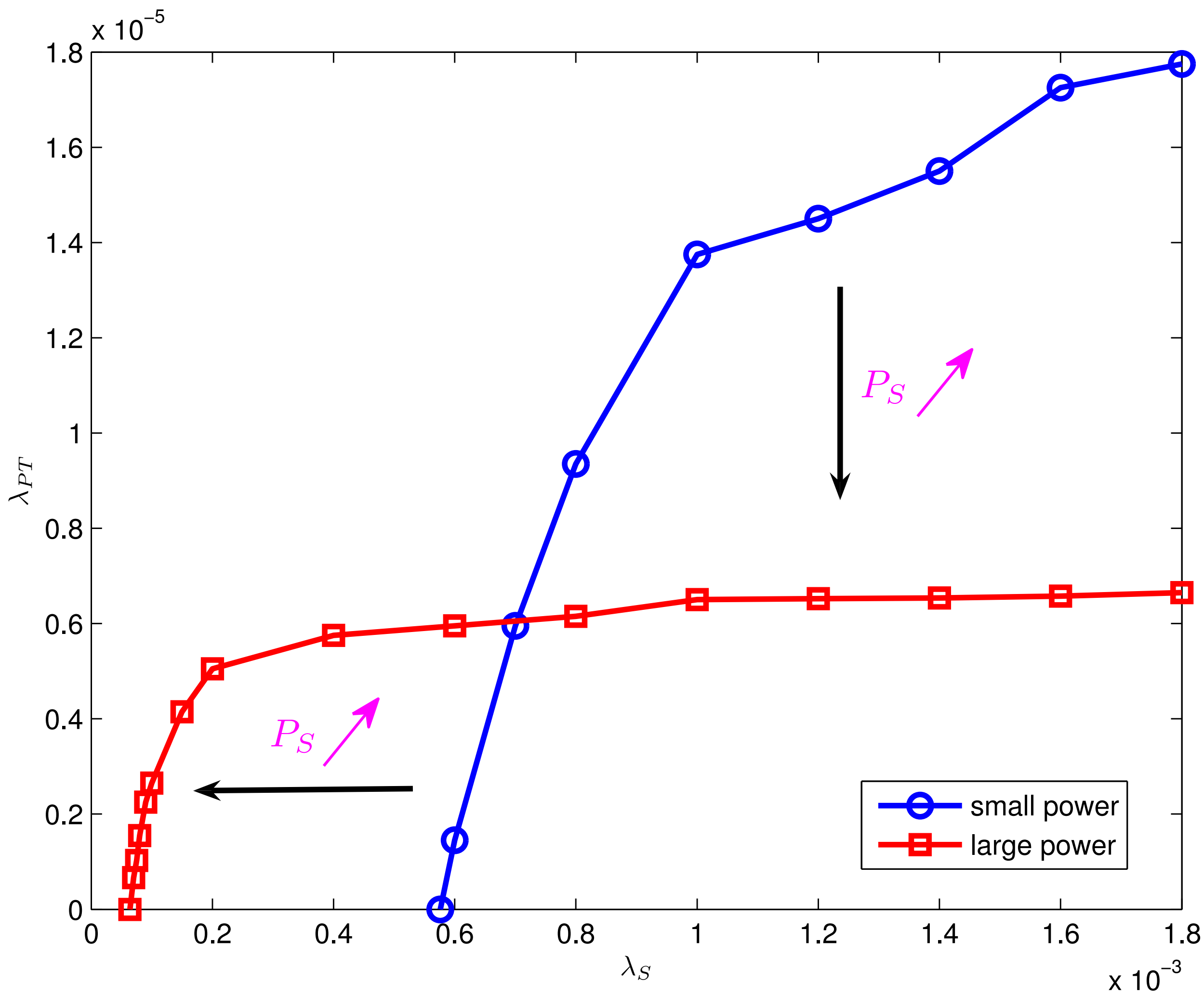
Proximity vs. Opportunity

Increasing P_S leads to more neighbors but fewer opportunities.

Small P_S *Large P_S*

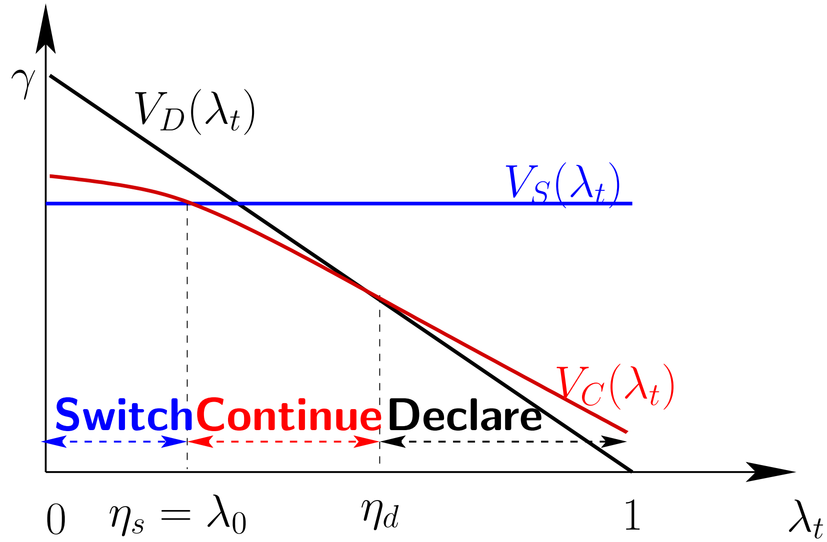



Proximity vs. Opportunity



Conclusion

Quickest Opportunity Detection:



Connectivity: