

# Low-Complexity Approaches to Spectrum Opportunity Tracking

*Qing Zhao\**, *Bhaskar Krishnamachari*<sup>◇</sup>, and *Keqin Liu\**

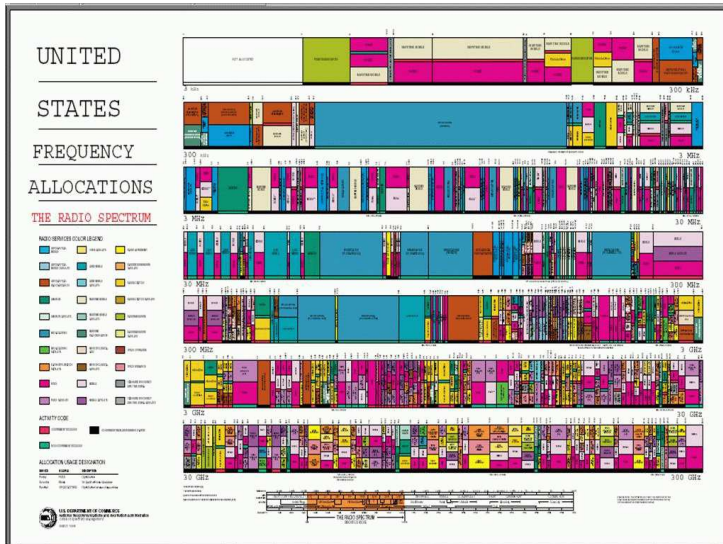
\*University of California at Davis

◇University of Southern California

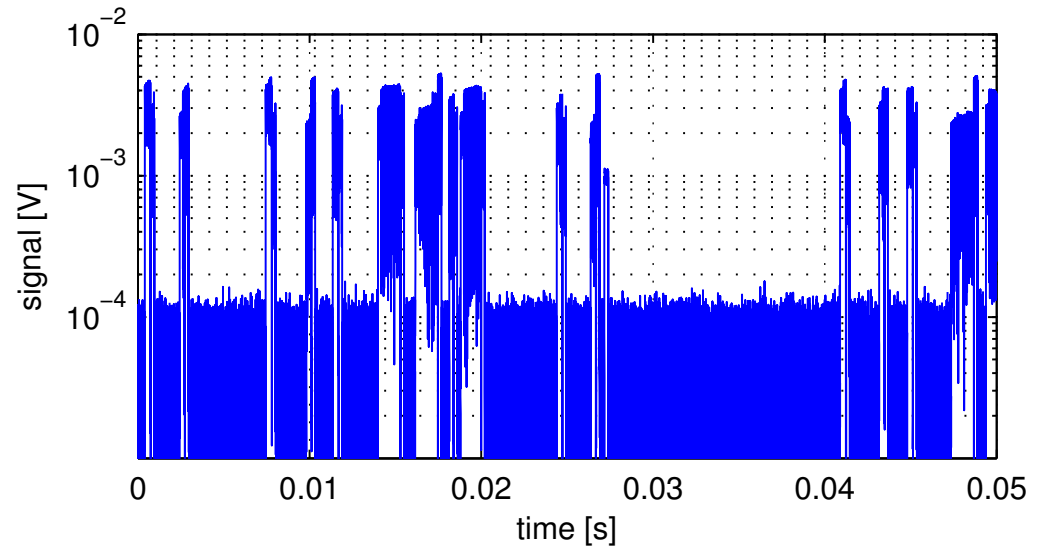
Supported by NSF and ARL-CTA.

# Opportunistic Spectrum Access

## Crowded Spectrum



## Pervasive Opportunities



## Three Basic Components:

- ▶ Spectrum Sensor: opportunity identification (PHY)
- ▶ Sensing policy: where to sense for opportunity tracking (MAC)
- ▶ Access policy: whether to tx given that sensing errors may occur (MAC)

**A joint PHY-MAC design is necessary for optimality.**

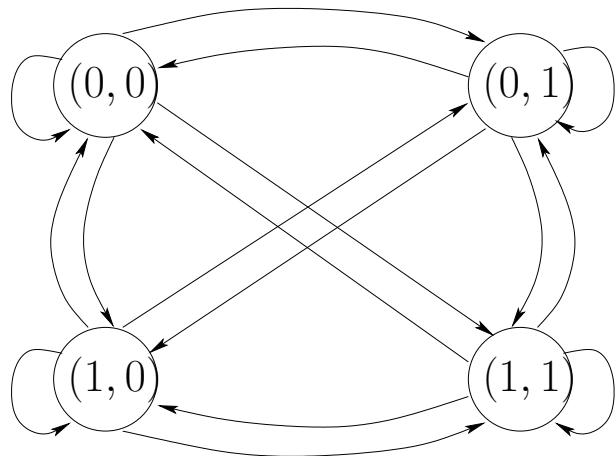
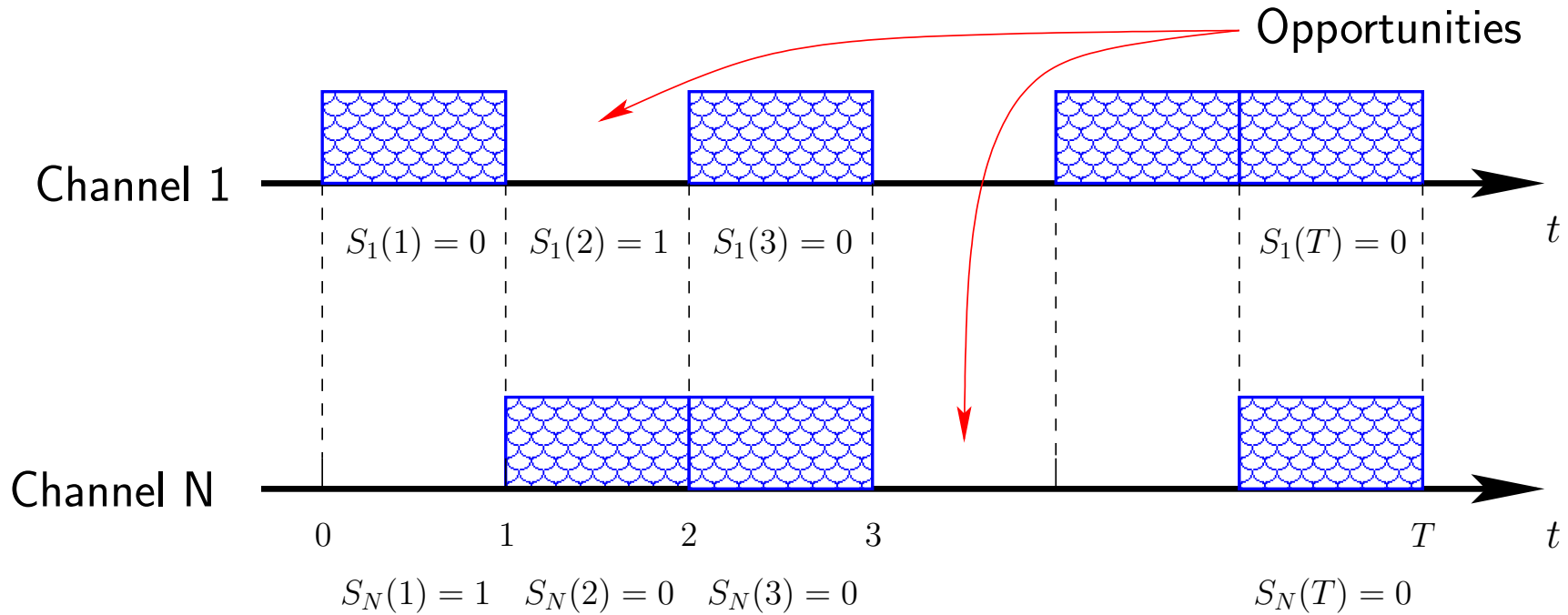
# Outline

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- ▶ Network model
- ▶ Joint design of spectrum sensor, sensing policy, and access policy
  - A **constrained** POMDP formulation
  - The separation principle:
    1. Spectrum sensor and access policy: **myopic** design in closed form
    2. Sensing policy: an **unconstrained** POMDP
- ▶ Low-complexity sensing policies for opportunity tracking
  - Structure and optimality of myopic sensing policy
  - Low-complexity index policies
- ▶ Conclusion

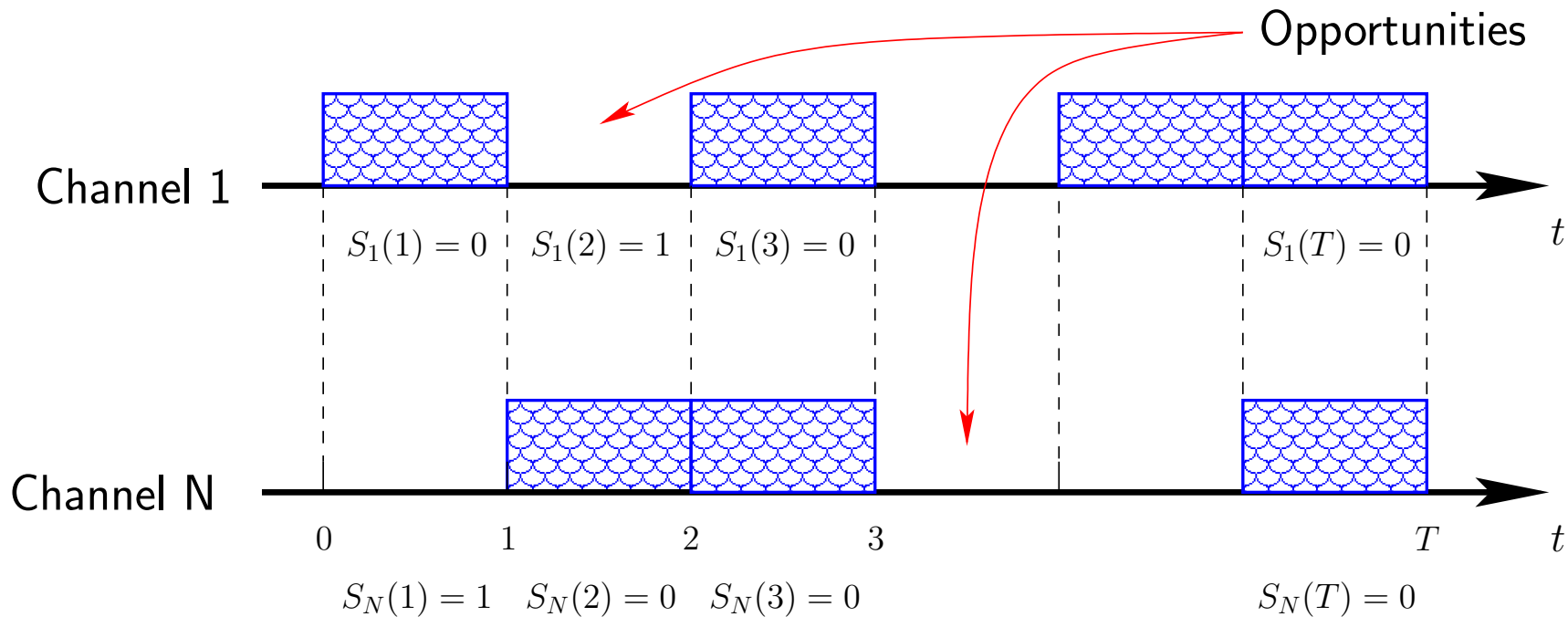
Y. Chen, Q. Zhao, and A. Swami, "Joint Design and Separation Principle for Opportunistic Spectrum Access in the Presence of Sensing Errors," submitted to *IEEE Transactions on Information Theory*, Feb. 2007; available at [http://arxiv.org/PS\\_cache/cs/pdf/0702/0702158.pdf](http://arxiv.org/PS_cache/cs/pdf/0702/0702158.pdf).

# Network Model



- ▶  $N$  channels, each with bandwidth  $B_i$ .
- ▶ A primary slotted network.
- ▶ Markovian spectrum usage ( $2^N$  states).
- ▶ Usage statistics unchanged for  $T$  slots.

## Network Model



- ▶ Secondary users search for opportunities independently.
- ▶ Can sense and access only one channel in each slot.
- ▶ A successful transmission is acknowledged at the end of the slot.

## Spectrum Sensor at PHY

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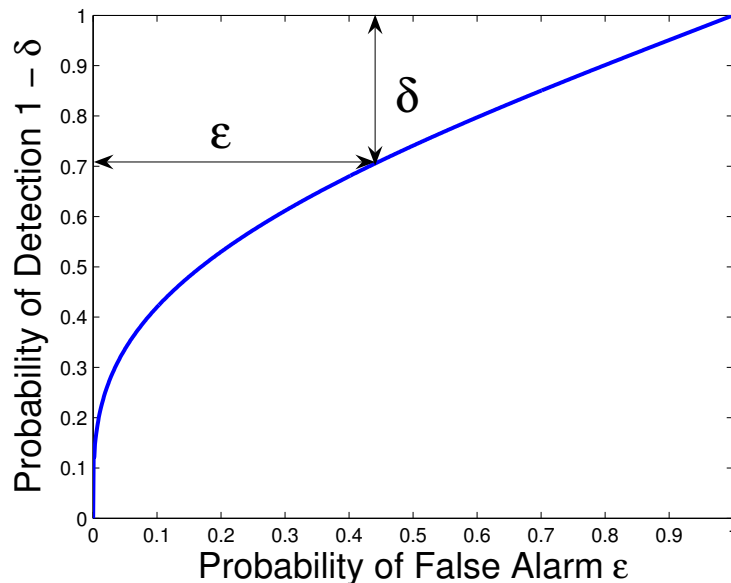
### Binary Hypotheses Test:

$$\mathcal{H}_0 \text{ (idle)} \quad \text{vs.} \quad \mathcal{H}_1 \text{ (busy)}$$

### Two Types of Sensing Errors:

- ▶ opportunity overlook:  $\mathcal{H}_0 \rightarrow \mathcal{H}_1$        $\epsilon \triangleq$  prob. of overlook
- ▶ opportunity misidentification:  $\mathcal{H}_1 \rightarrow \mathcal{H}_0$        $\delta \triangleq$  prob. of misidentification

### Receiver Operating Characteristics (ROC): $1 - \delta$ vs. $\epsilon$



## Spectrum Sensor at PHY

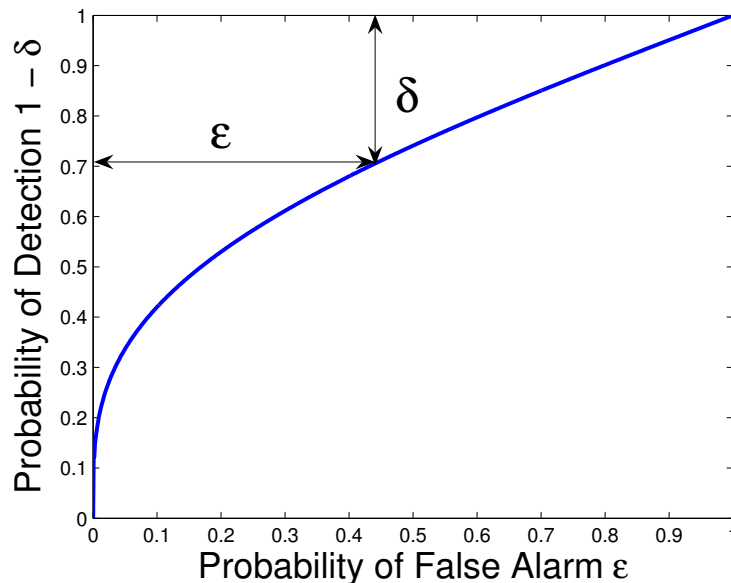
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### Two Types of Sensing Errors:

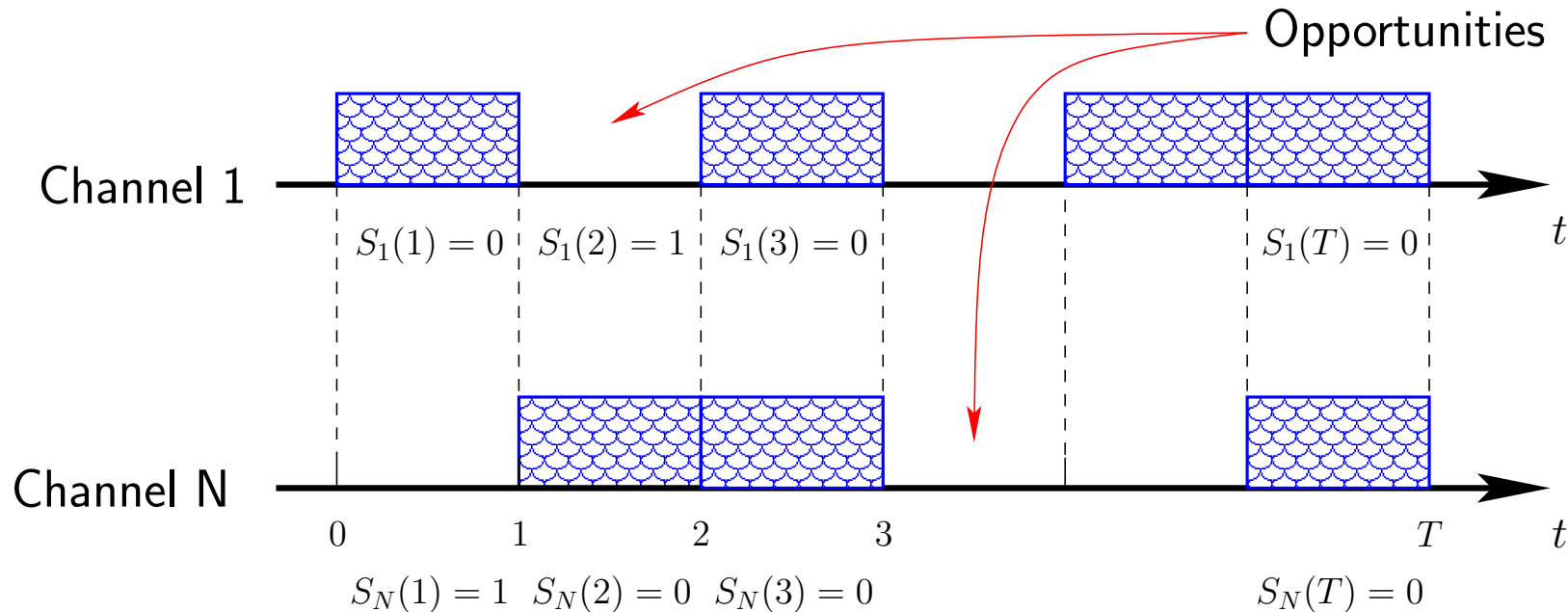
- ▶ opportunity overlook:  $\mathcal{H}_0 \rightarrow \mathcal{H}_1$        $\epsilon \triangleq$  prob. of overlook
- ▶ opportunity misidentification:  $\mathcal{H}_1 \rightarrow \mathcal{H}_0$        $\delta \triangleq$  prob. of misidentification

### Receiver Operating Characteristics (ROC): $1 - \delta$ vs. $\epsilon$



**Which point  $\delta$  to operate at?**  
**overlook vs. misidentification**

## Sensing and Access Policies



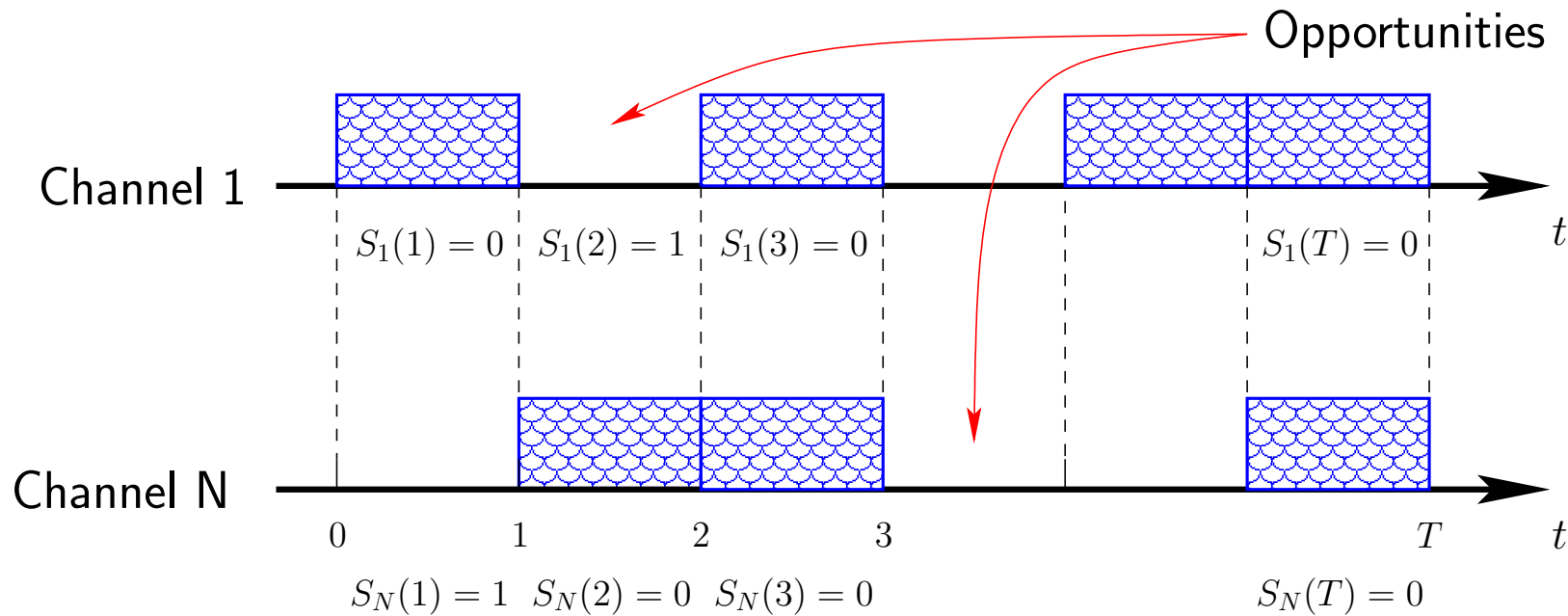
### Sensing Policy $\pi_s$

- Deterministic: choose the sensing action  $a$  in each slot
- Randomized: choose PMF of the sensing action in each slot

### Access Policy $\pi_c$

- Deterministic: whether to transmit based on sensing outcome
- Randomized: transmission probability based on sensing outcome

## Sensing and Access Policies



### Reward and Collision

- A reward  $R(t) = B_a$  is accrued when access an idle channel
- A collision with primary users occurs when access a busy channel

**Objective:** choose sensor operating policy  $\pi_\delta$ , sensing policy  $\pi_s$ , access policy  $\pi_c$

$$\max \mathbb{E}\left[\sum_{t=1}^T R(t)\right] \quad s.t. \quad \text{collision probability } P_c \leq \zeta$$

## A Constrained POMDP Formulation

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$$\{\pi_\delta^*, \pi_s^*, \pi_c^*\} = \arg \max \mathbb{E} \left[ \sum_{t=1}^T R(t) \right] \quad s.t. \quad \text{collision probability } P_c \leq \zeta$$

- ▶ A **constrained** Partially Observable Markov Decision Process (POMDP)
- ▶ Often requires randomized policy for optimality
- ▶ Analytically intractable
- ▶ Approximate numerical solutions provide little insight

# Separation Principle

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$$\{\pi_\delta^*, \pi_s^*, \pi_c^*\} = \arg \max \mathbb{E} \left[ \sum_{t=1}^T R(t) \right], \quad \text{subject to } P_c \leq \zeta$$

**Separation principle:**  $\pi_\delta$  and  $\pi_c$  can be decoupled from  $\pi_s$

- ▶ Choose sensor operating policy  $\pi_\delta$  and access policy  $\pi_c$  to maximize immediate reward  $R(t)$  under constraint  $P_c = \zeta$ .
- ▶ Choose sensing policy  $\pi_s$  to maximize total reward  $\mathbb{E} \left[ \sum_{t=1}^T R(t) \right]$ .

## Separation Principle

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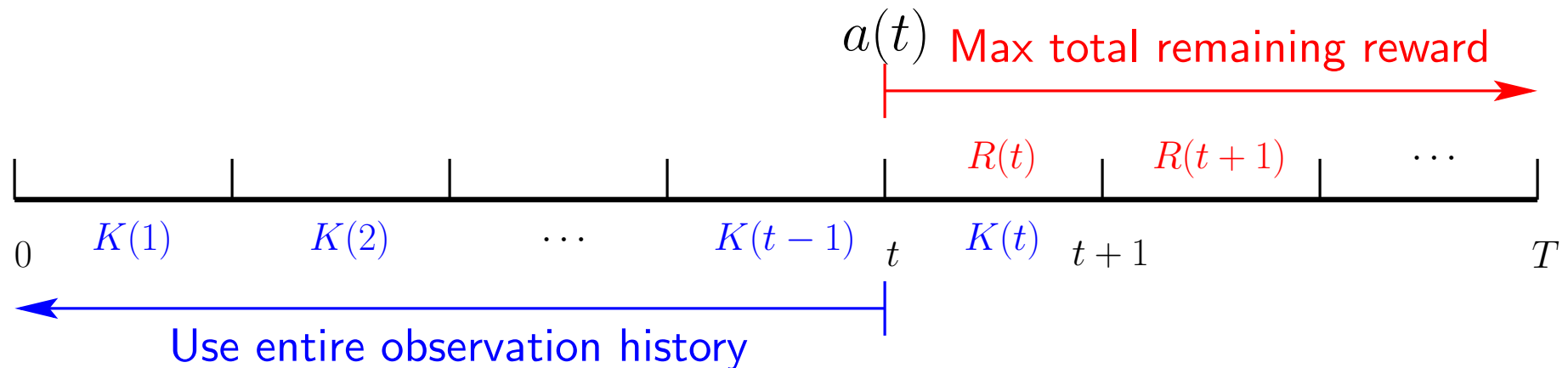
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**Separation principle:**  $\pi_\delta$  and  $\pi_c$  can be decoupled from  $\pi_s$

- ▶ Choose sensor operating policy  $\pi_\delta$  and access policy  $\pi_c$  to maximize immediate reward  $R(t)$  under constraint  $P_c = \zeta$ .
  - ⇒ Myopic is optimal: static optimization
  - ⇒ Deterministic policies  $\{\pi_\delta^*, \pi_c^*\}$  in closed form.
  - ⇒  $\pi_\delta^*$ :  $\delta^* = \zeta$ ;  $\pi_c^*$ : trust the sensor.
- ▶ Choose sensing policy  $\pi_s$  to maximize total reward  $\mathbb{E} \left[ \sum_{t=1}^T R(t) \right]$ .
  - ⇒ An unconstrained POMDP
  - ⇒ Optimality achieved with deterministic policies.

# The Optimal Sensing Policy for Opportunity Tracking

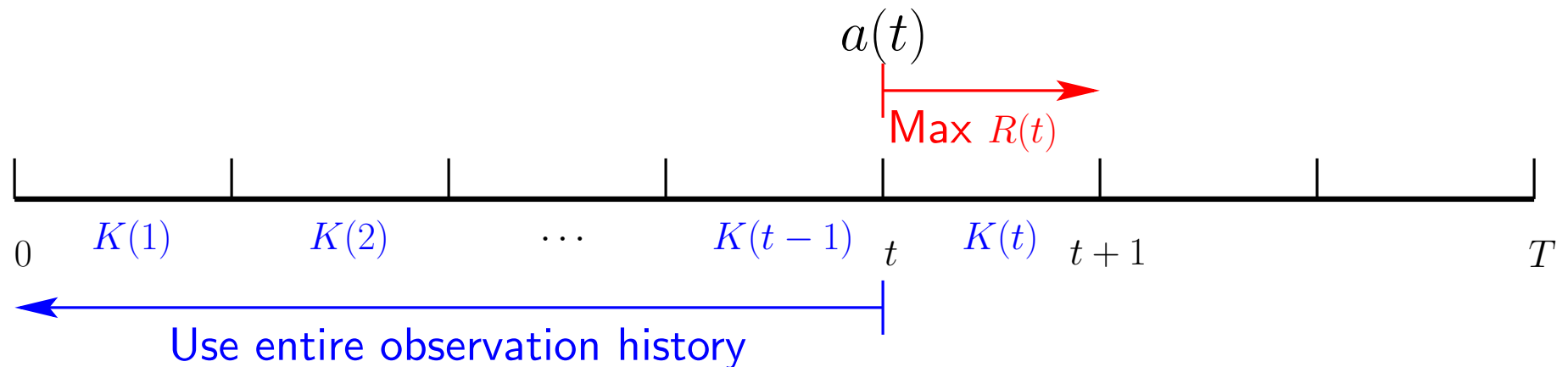
$$\pi_s^* = \arg \max \mathbb{E} \left[ \sum_{t=1}^T R(t) \right]$$



- ▶ Use common observations (ACK) to ensure Tx-Rx synchronous hopping.
- ▶ Computational complexity:  $\mathcal{O}(N^T)$
- ▶ Due to continuously growing observation history and foresighted planning

## The Myopic Sensing Policy

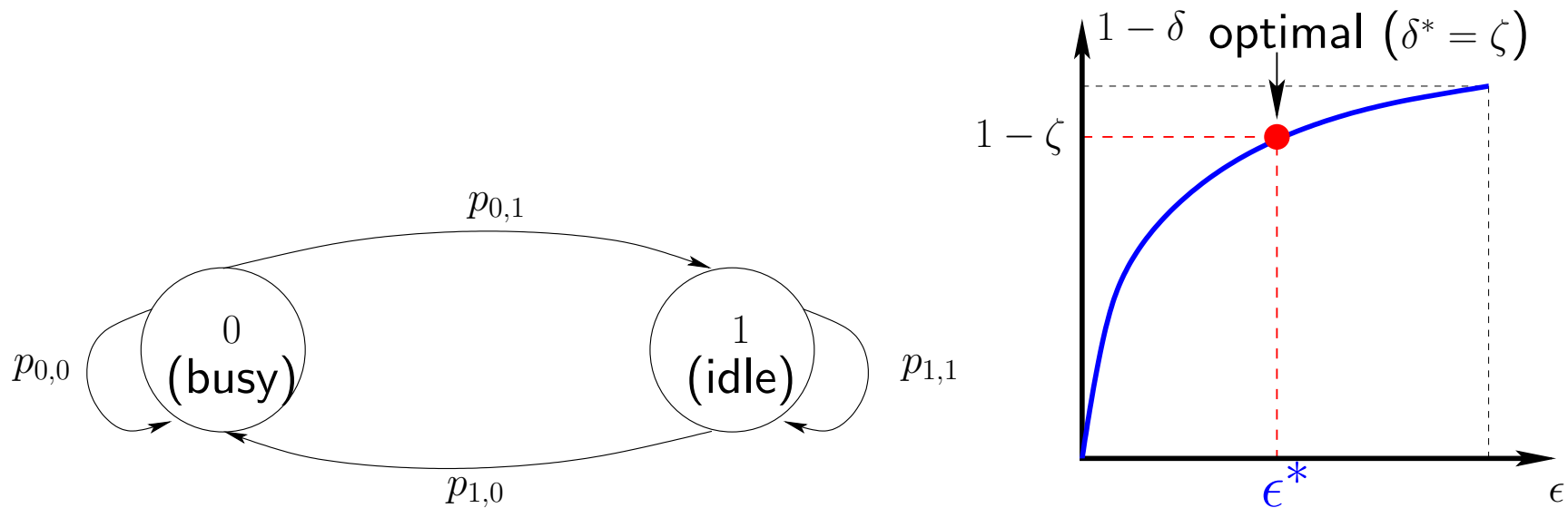
$$\pi_s^* = \arg \max \mathbb{E} \left[ \sum_{t=1}^T R(t) \right]$$



- Myopic policy: maximize immediate reward  $R(t)$

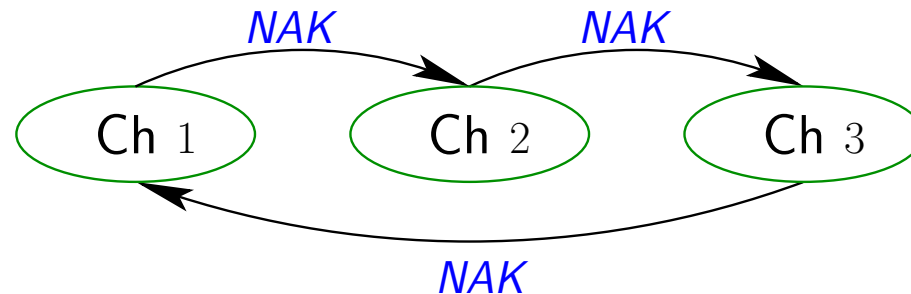
$$\begin{aligned} a(t) &= \arg \max_{a=1, \dots, N} R_a(t) \\ &= \arg \max_{a=1, \dots, N} \Pr[a \text{ is idle} \mid \underbrace{K(1), \dots, K(t-1)}_{\text{common observ.}}] B_a \end{aligned}$$

# Structure of Myopic Sensing for i.i.d. Markov Processes

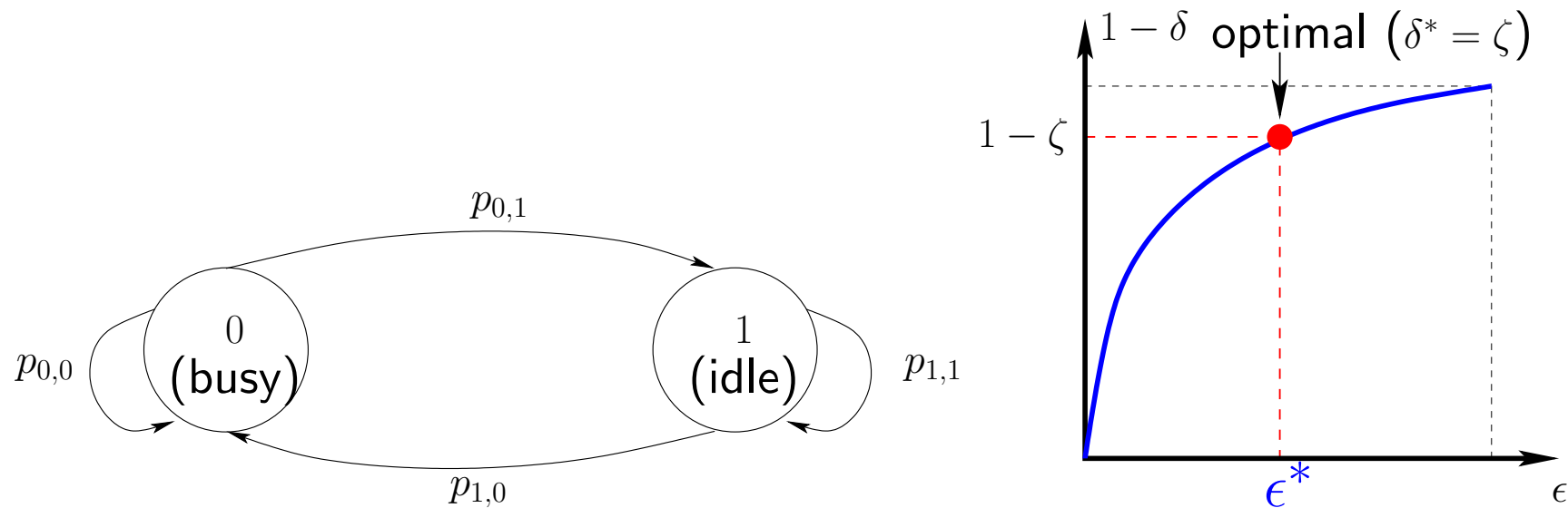


**The Structure of Myopic Sensing Policy:**  $p_{1,1} > p_{0,1}$  and  $\epsilon^* < \frac{p_{1,0}p_{0,1}}{p_{1,1}p_{0,0}}$

- ▶ Stay in the same channel after ACK and switch after NAK.
- ▶ Switch to the channel visited the longest time ago.
- ▶ A sufficient statistic: current acknowledgement.



## Structure of Myopic Sensing for i.i.d. Markov Processes

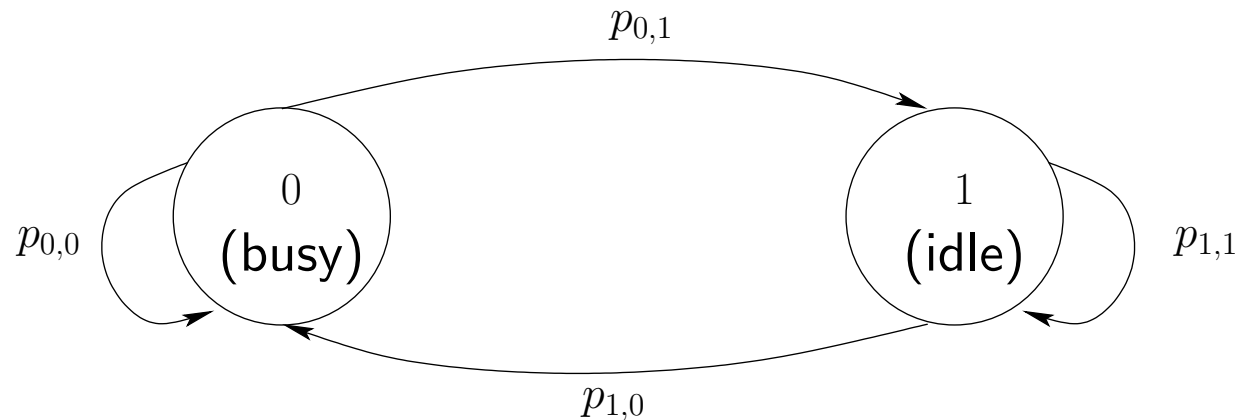


**The Structure of Myopic Sensing Policy:**  $p_{0,1} > p_{1,1}$  and  $\epsilon^* < \frac{p_{0,0}p_{1,1}}{p_{0,1}p_{1,0}}$

- ▶ Stay in the same channel after a NAK and switch after an ACK.
- ▶ Among channels visited an even number of slots ago, choose the most recent.
- ▶ If no such channels, choose the one visited the longest time ago.
- ▶ A sufficient statistic: current acknowledgement and last visit to each channel.

# Optimality of Myopic Sensing for i.i.d. Markov Processes

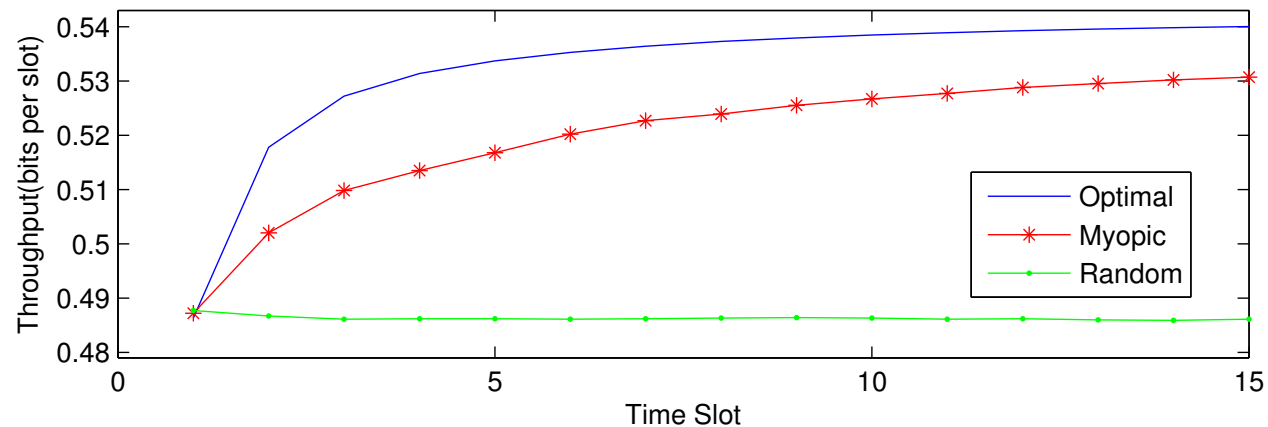
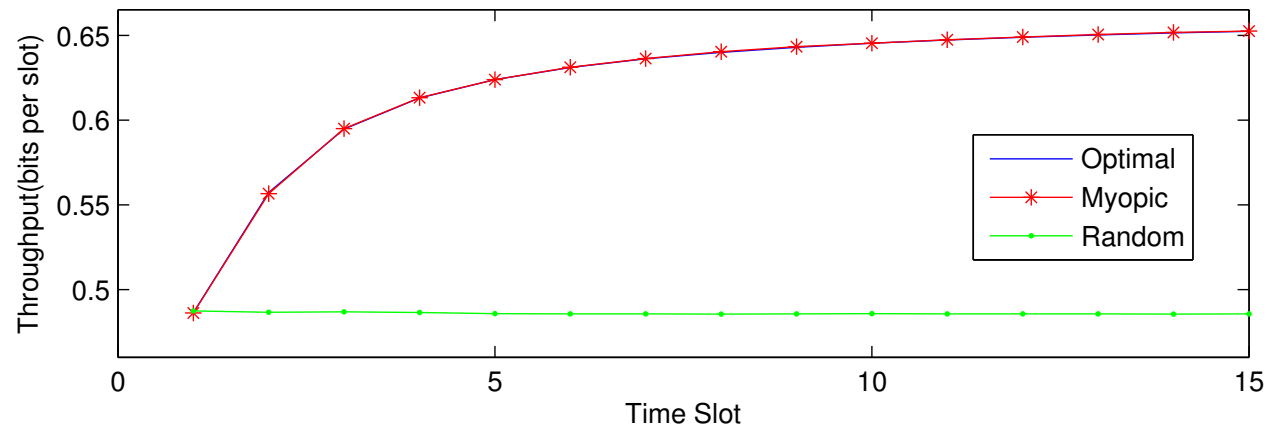
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## The Optimality of Myopic Sensing Policy

- ▶ Proven to be optimal for  $N = 2$
- ▶ Numerical results indicate its optimality for  $N > 2$ .

## A Numerical Example



- Cognitive: improved performance due to increasingly accurate state information drawn from accumulating observations.

## Conclusion

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### The Separation Principle:

- ▶ Spectrum sensor and access policy: **myopic** design in closed form
- ▶ Sensing policy: an **unconstrained** POMDP

### Low-complexity Sensing:

- ▶ Structure and optimality of myopic sensing for i.i.d. Markov channels
- ▶ Low-complexity index policies for non-i.i.d. channels

Y. Chen, Q. Zhao, and A. Swami, "Joint Design and Separation Principle for Opportunistic Spectrum Access in the Presence of Sensing Errors," submitted to *IEEE Transactions on Information Theory*, Feb. 2007; available at [http://arxiv.org/PS\\_cache/cs/pdf/0702/0702158.pdf](http://arxiv.org/PS_cache/cs/pdf/0702/0702158.pdf).