Quickest Change Detection in Multiple On-Off Processes: Switching With Memory

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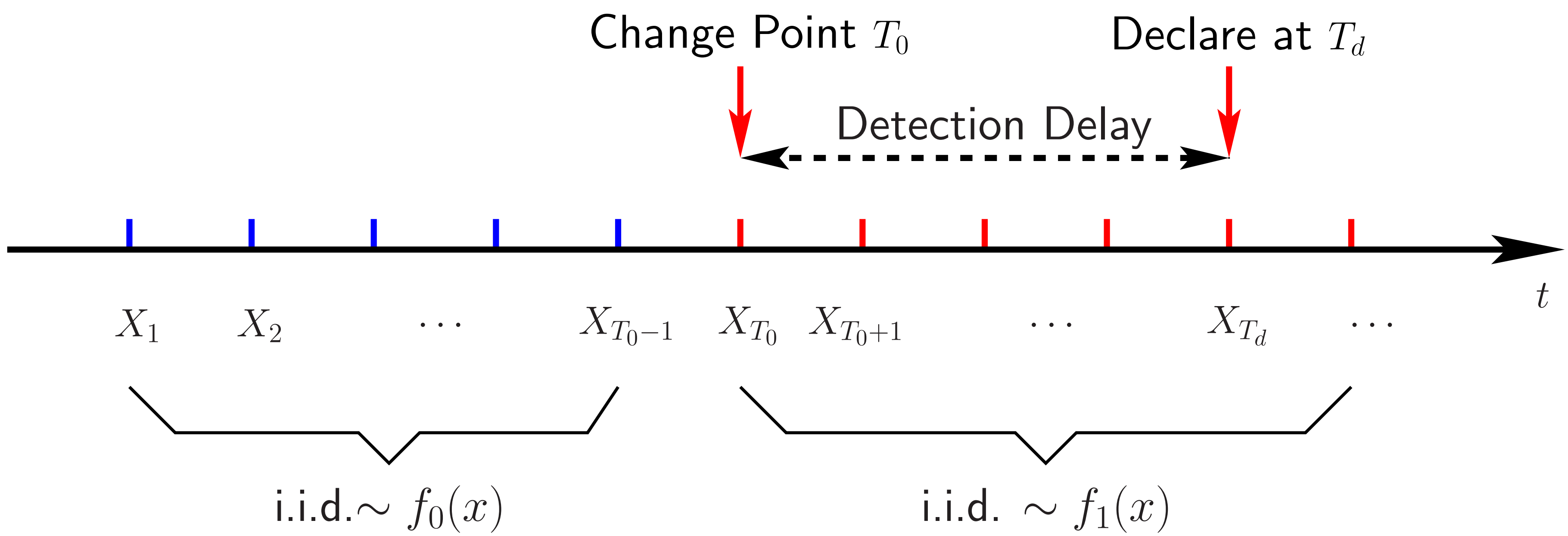
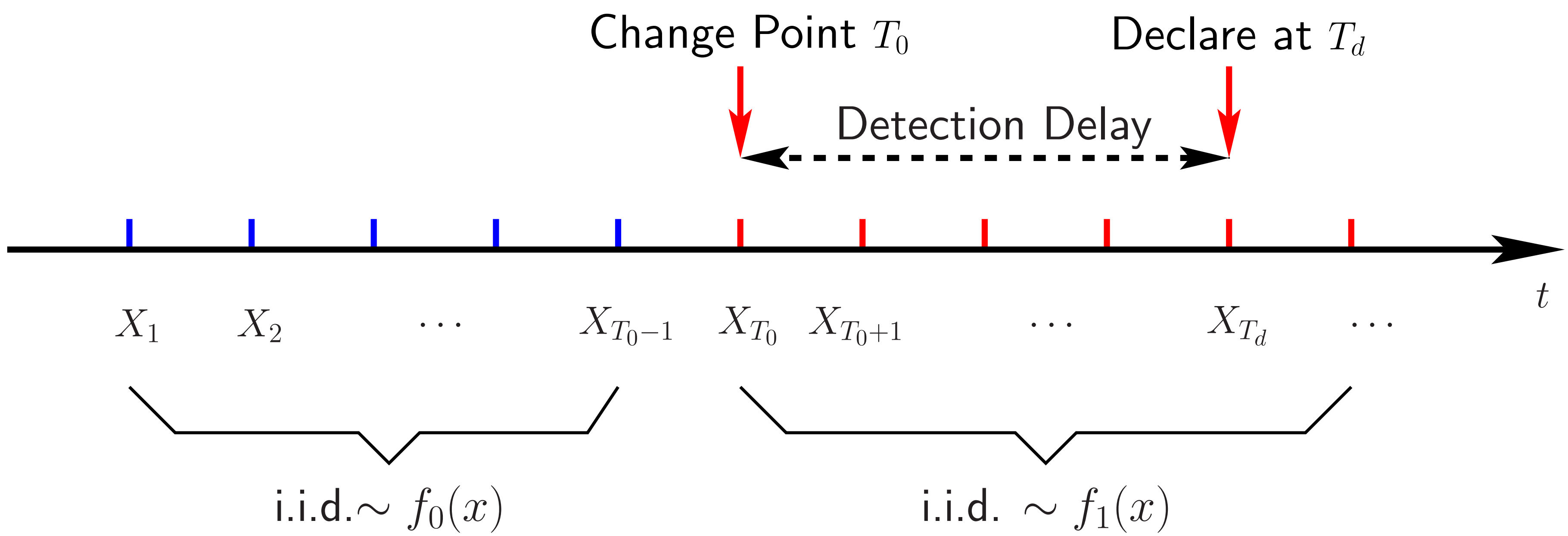
Quickest Change Detection



- **Quickest Detection:** min $\underbrace{\mathbb{E}[(T_d - T_0)^+]}_{\text{Detection Delay}}$ subject to $\underbrace{\Pr[T_d < T_0] \leq \zeta}_{\text{Reliability Constraint}}$

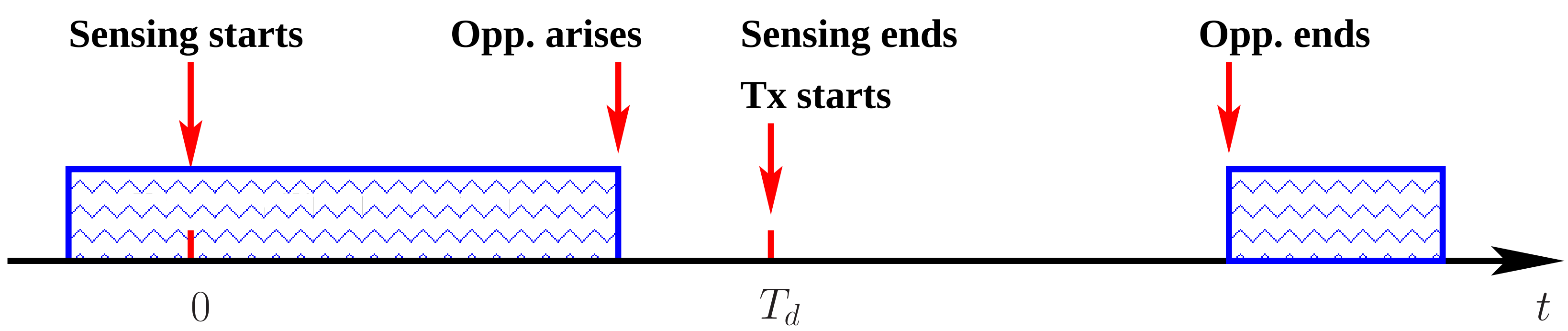
- **Tradeoff:** Detection delay vs. detection reliability.

Quickest Change Detection



- **Quickest Detection:** min $\underbrace{\mathbb{E}[(T_d - T_0)^+]}_{\text{Detection Delay}}$ subject to $\underbrace{\Pr[T_d < T_0] \leq \zeta}_{\text{Reliability Constraint}}$
  - Bayesian: Shiryayev'61, Borovkov'98, Tartakovsky&Veeravalli'05.
  - Minimax: CUSUM (Page'54, Lorden'71).

Application in Cognitive Radio



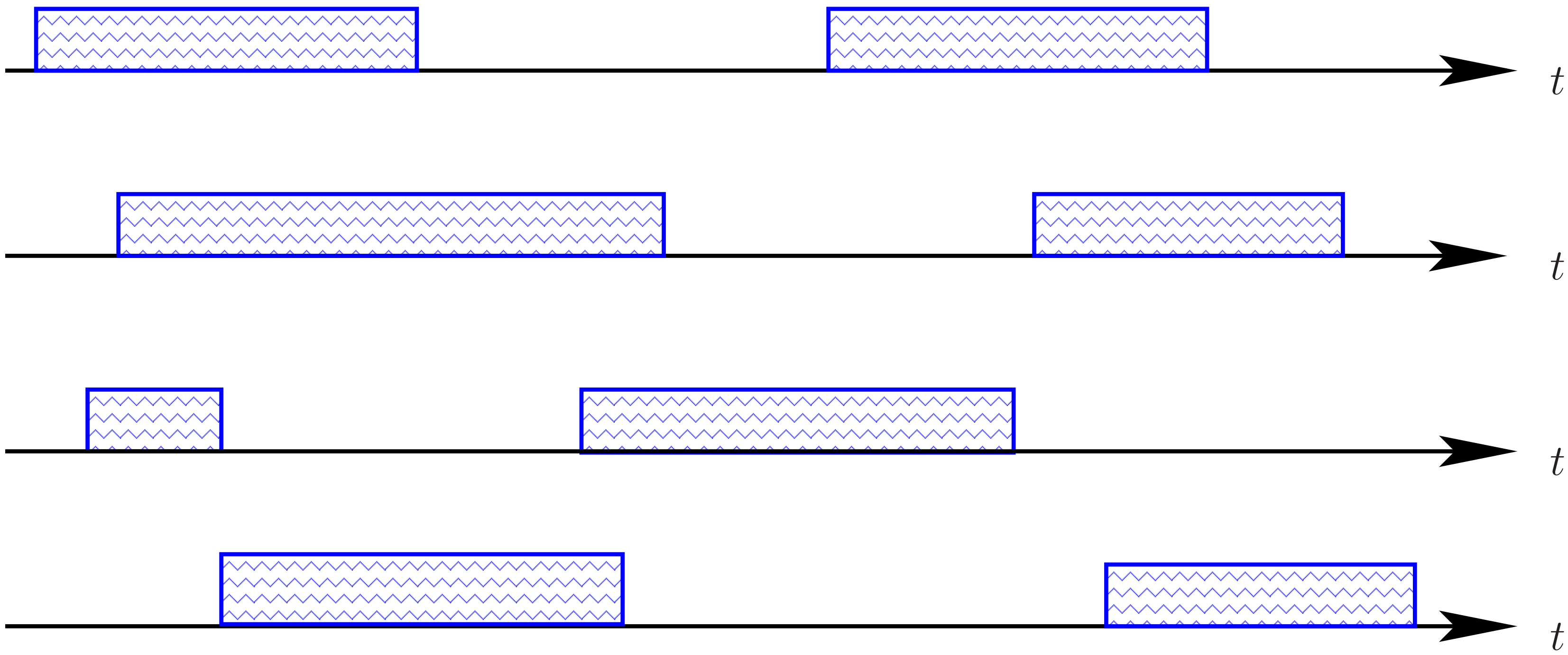
- Measurements: In busy states: i.i.d with distribution $f_0(x)$;

  In idle states: i.i.d with distribution $f_1(x)$.

- Stopping Time: At time $t = T_d$, the user declares an opportunity.

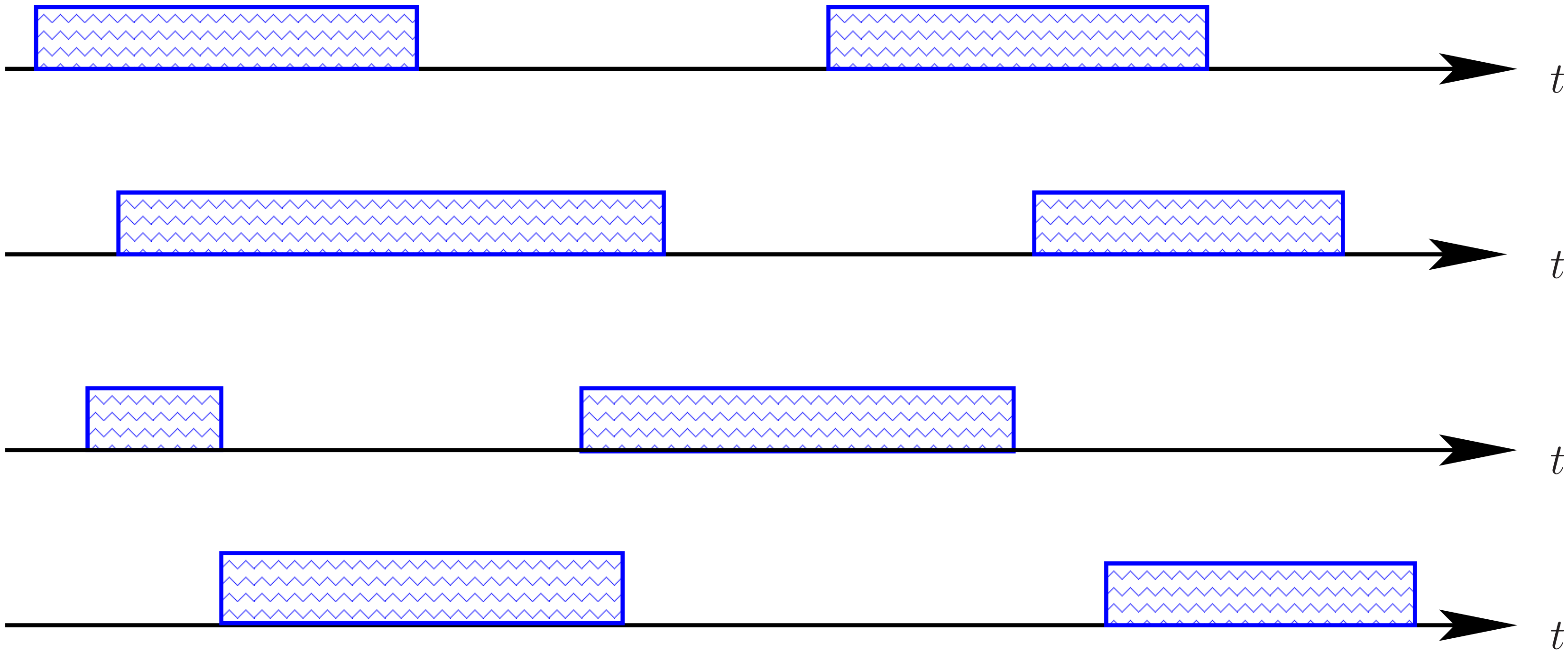
- Quickest Detection: min $\underbrace{\mathbb{E}[T_d]}_{\text{Detection Time}}$ subject to $\underbrace{\Pr[Z(T_d) = busy] \leq \zeta}_{\text{Interference Constraint}}$

Quickest Detection in Multiple On-Off Processes



- Two Fundamental Differences:
  - Channel occupancy is an on-off process with multiple change points.
  - There are multiple channels available.

Infinite Channel Case

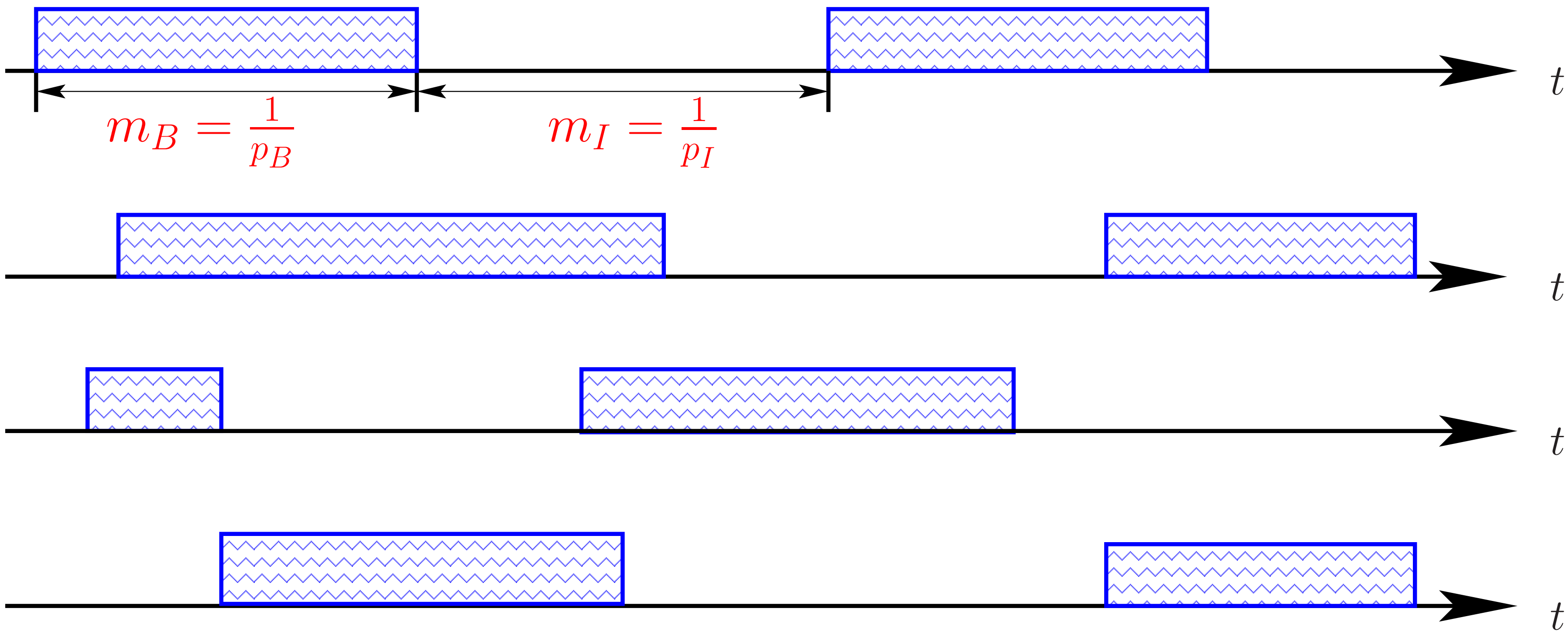


- Quickest Detection of Idle Periods in Multiple On-Off Processes:
  - Continue, switch, or declare?
- Tradeoffs:
  - Whether to declare: delay vs. reliability.
  - Whether to switch: loss of data vs. avoiding bad realizations.

Outline

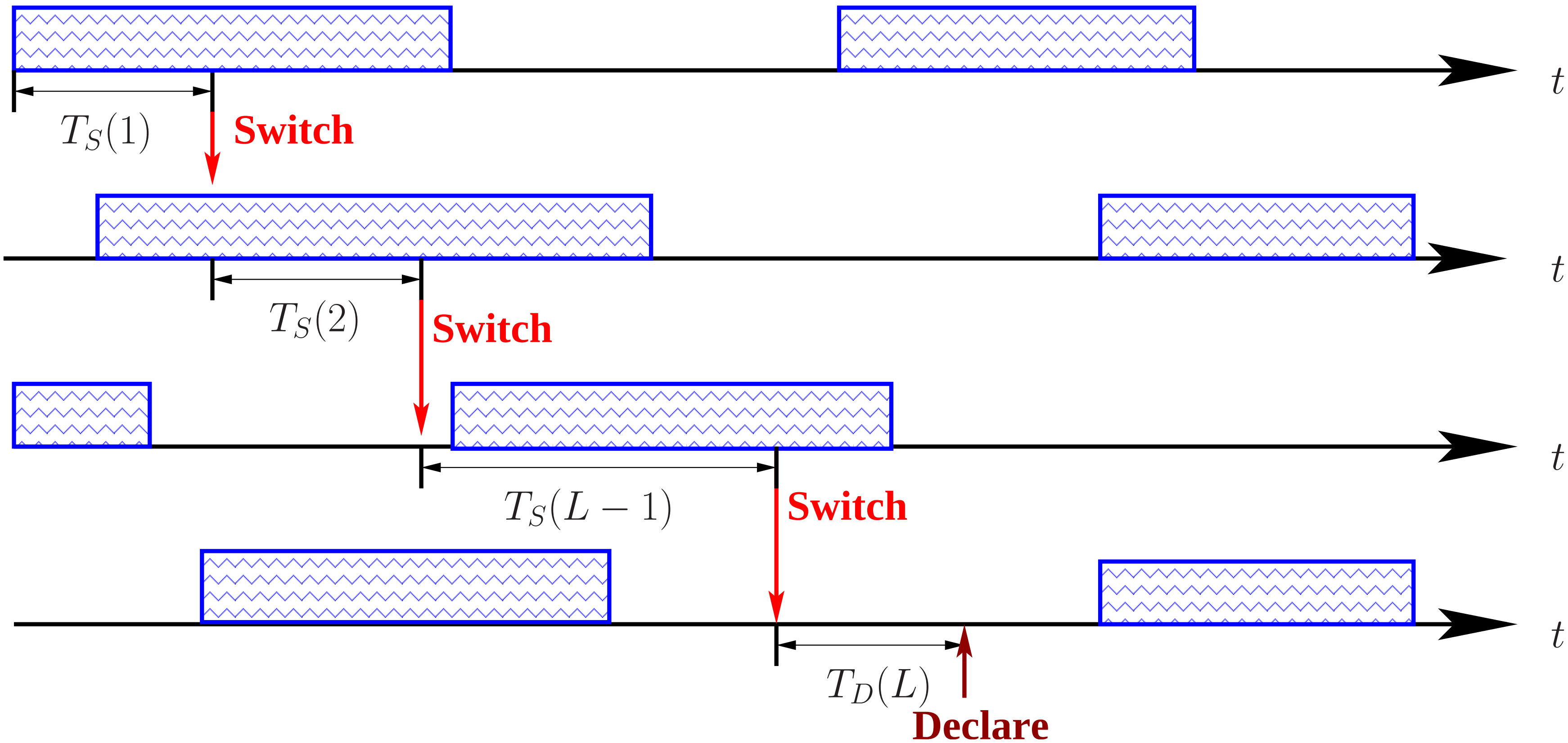
- Infinite channel case
  - A decision-theoretic formulation
  - The optimal detection rule: a threshold policy
- Finite channel case: switching with memory
  - Basic structure of the optimal policy
  - A low-complexity threshold policy
- Simulation examples
- Conclusion and work in progress

Infinite Channel Case



- Infinite number of independent homogeneous on-off processes.
- Busy period: geometrically distributed with mean $m_B = \frac{1}{p_B}$.
- Idle period: geometrically distributed with mean $m_I = \frac{1}{p_I}$.
- Fraction of idle time: $\lambda_0 = \frac{m_I}{m_I + m_B}$.

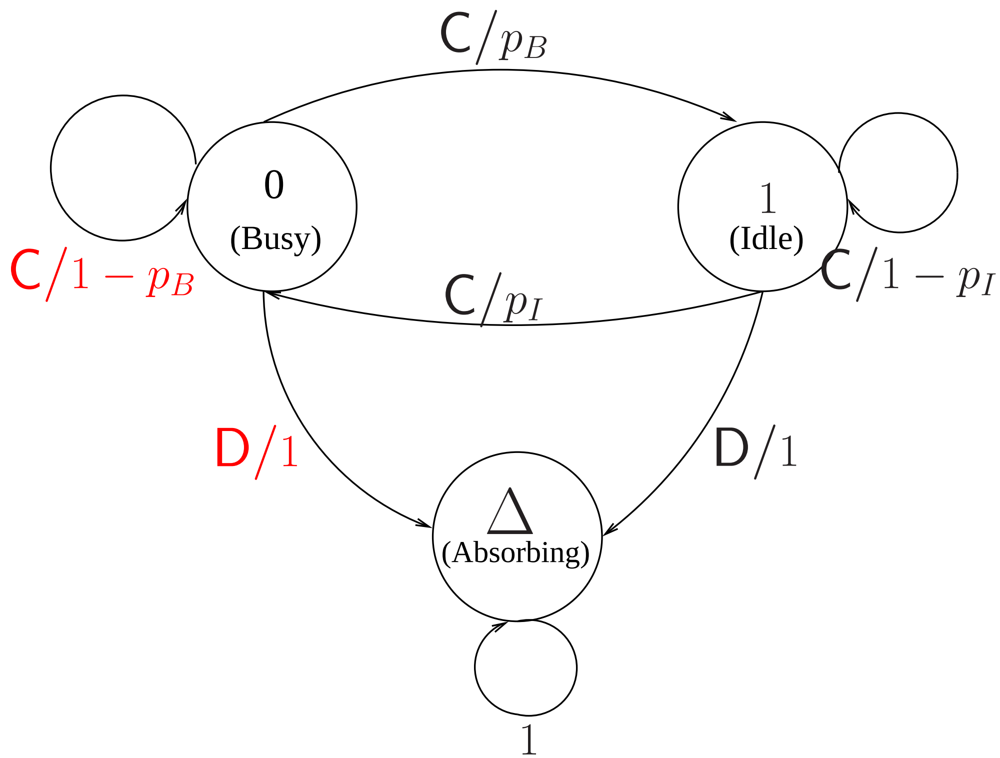
Infinite Channel Case



$$\underbrace{\min \mathbb{E}[\sum_{l=1}^{L-1} T_s(l) + T_d(L)]}_{\text{Detection Time}} \quad s.t. \quad \underbrace{\Pr[Z_L(\sum_{l=1}^{L-1} T_s(l) + T_d(L)) = \text{busy}] \leq \zeta}_{\text{Reliability Constraint}}$$

A POMDP Formulation

- State Space: 0 (busy), 1 (idle), $\triangle$ (absorbing state)
- Action Space: S (Switch), C (Continue), D(Declare)
- State Transition:



- Cost:
  - □ Switch or Continue: 1
  - □ Declare during a busy period: γ

A POMDP Formulation

- A Sufficient Statistic: the information state (belief)

$$\lambda_t = \Pr[Z_t = \text{idle} | X_1, X_2, \ldots, X_t]$$
$$\lambda_0 = \frac{m_I}{m_I + m_B}$$

- Update of the Information State

$$\lambda_t = \begin{cases} \mathcal{T}(\lambda_0|x) & a(t-1) = \mathsf{S},\ \ X_t = x \\ \mathcal{T}(\lambda_{t-1}|x) & a(t-1) = \mathsf{C},\ \ X_t = x \end{cases}.$$

- $\mathcal{T}(\lambda|x)$: updated information state based on the new measurement x.

$$\mathcal{T}(\lambda|x) \triangleq \frac{(\lambda\bar{p}_I + \bar{\lambda}p_B)f_1(x)}{(\lambda\bar{p}_I + \bar{\lambda}p_B)f_1(x) + (\lambda p_I + \bar{\lambda}\bar{p}_B)f_0(x)}.$$

A POMDP Formulation

- Channel switching and change detection policy π:

$$\lambda_t \in [0,1] \quad \Longrightarrow \quad a(t) \in \{S, C, D\}, \text{ for each time } t.$$

- Quickest change detection:

$$\pi^* = \arg\min_{\pi} \mathbb{E}_\pi[\sum_{t=0}^{\infty} \underbrace{R_{\pi(\lambda_t)}}_{\text{Cost}} \mid \lambda_0 = \frac{m_I}{m_B + m_I}],$$

Infinite Channel Case: Value Functions

- $V(\lambda_t)$: the minimum expected total cost-to-go when the current belief is λ_t.

$$V(\lambda_t) = \min\{ \underbrace{V_C(\lambda_t)}_{\text{Continue}} , \underbrace{V_S(\lambda_t)}_{\text{Switch}} , \underbrace{V_D(\lambda_t)}_{\text{Declare}} \}.$$

- $V_C(\lambda_t)$: the minimum expected total cost-to-go if continue at t.

$$V_C(\lambda_t) = 1 + \int_x \underbrace{P(x;\lambda_t)}_{\Pr[\text{ observe } x \text{ under } \lambda_t \text{ }]} V(\mathcal{T}(\lambda_t|x))\mathrm{d}x$$

- $V_S(\lambda_t)$: the minimum expected total cost-to-go if switch at t.

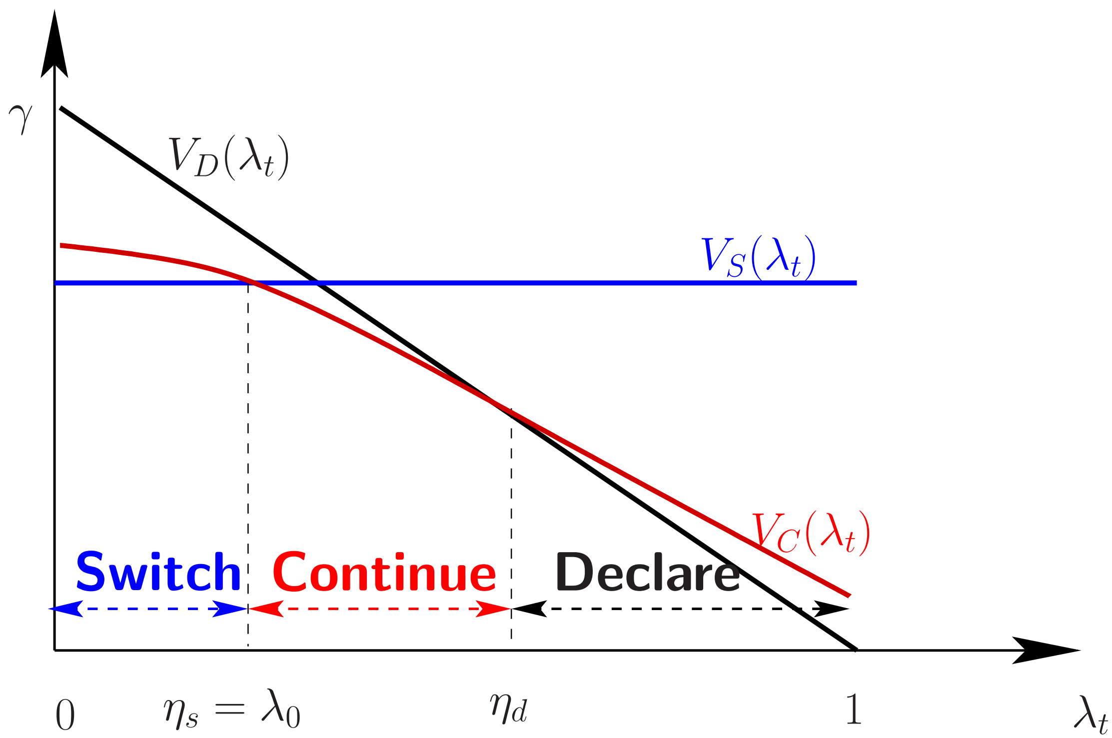
$$V_S(\lambda_t) = 1 + \int_x \underbrace{P(x;\lambda_0)}_{\Pr[\text{ observe } x \text{ under } \lambda_0 \text{ }]} V(\mathcal{T}(\lambda_0|x))\mathrm{d}x = V_C(\lambda_0)$$

- $V_D(\lambda_t)$: the minimum expected total cost-to-go if declare at t.

$$V_D(\lambda_t) = (1-\lambda_t)\gamma.$$

The Optimal Detection Rule: A Threshold Policy

$$V(\lambda_t) = \min\{ \underbrace{V_S(\lambda_t)}_{\text{Switch}}, \underbrace{V_C(\lambda_t)}_{\text{Continue}}, \underbrace{V_D(\lambda_t)}_{\text{Declare}} \}.$$

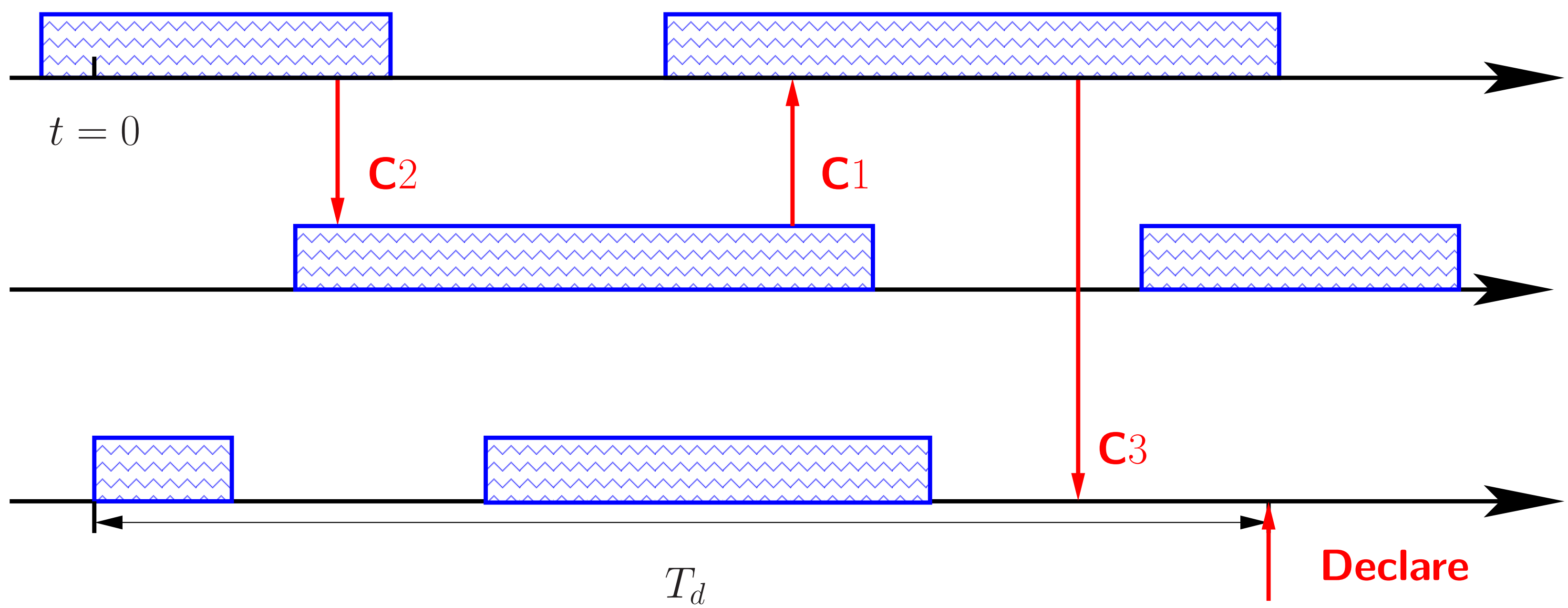


$$\pi^*_\infty(\lambda_t) = \begin{cases} \text{S}, & \lambda_t \in [0, \eta_s) \\ \text{C}, & \lambda_t \in [\eta_s, \eta_d) \\ \text{D}, & \lambda_t \in [\eta_d, 1] \end{cases}.$$

$$\eta_s = \lambda_0$$

$$\eta_d = 1 - \zeta$$

Finite Channel Case: Switching With Memory

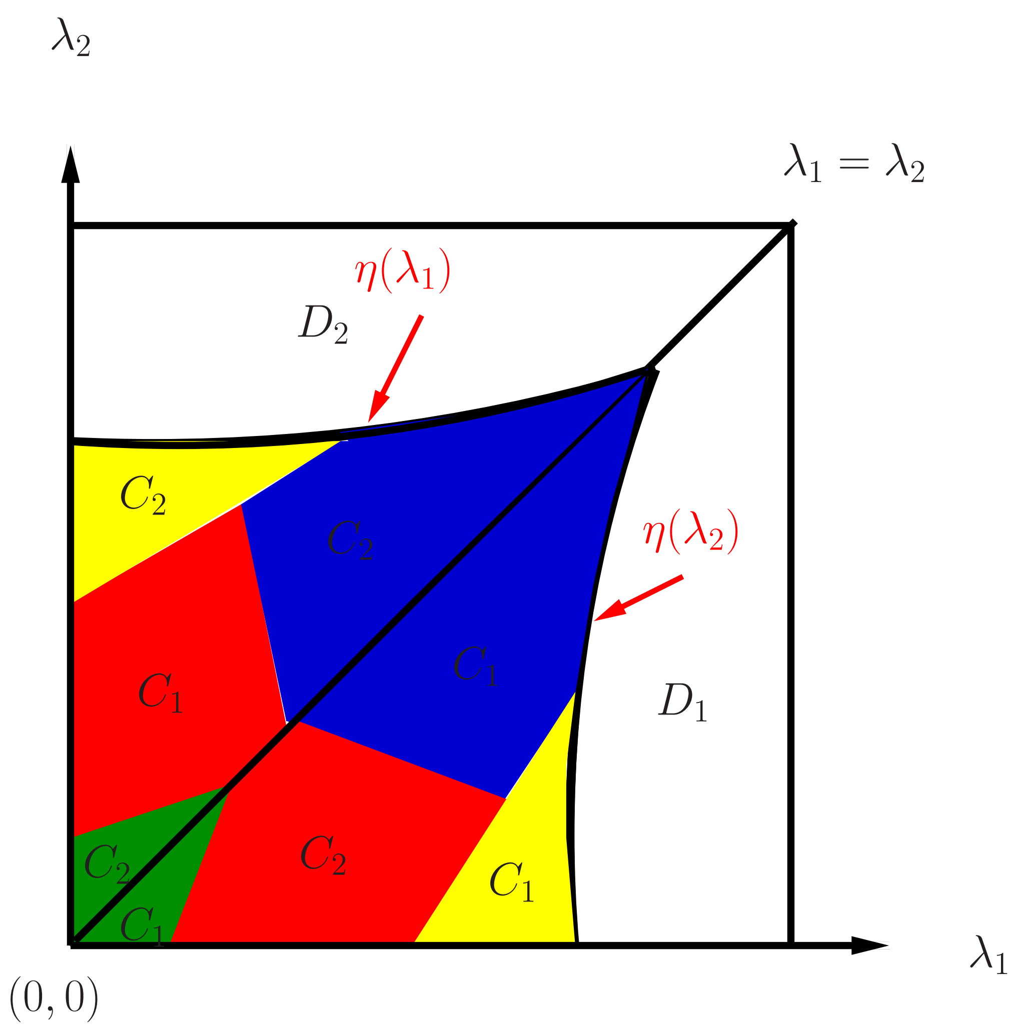


- Objective:

$$\min \underbrace{\mathbb{E}[T_d]}_{\text{Detection Time}} \quad s.t. \quad \underbrace{\Pr[Z(T_d) = \text{busy}] \leq \zeta}_{\text{Reliability Constraint}}$$

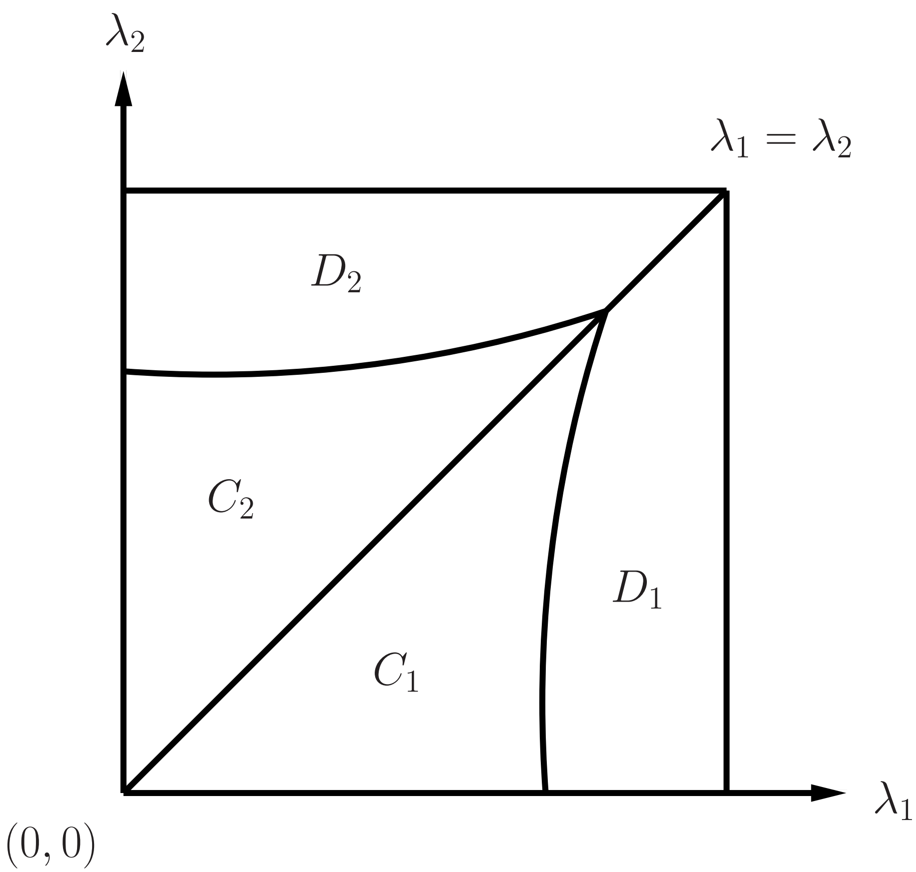
- Resulting POMDP:
  - $\underbrace{\lambda_t}_{1-D} \Longrightarrow \underbrace{\mathbf{\Lambda}(t) = [\lambda_1(t), \cdots, \lambda_N(t)]}_{N-D}$
  - $\{C, S, D\} \Longrightarrow \{C_1, \cdots, C_N, D_1, \cdots, D_N\}$

Basic Structure Of The Optimal Policy



- Always declare on channel $i = \arg\max_j \{\lambda_j\}$
- Declaring threshold η is monotonically increasing with λ
- $a^*(\lambda_1, \lambda_2) = C_1 \Leftrightarrow a^*(\lambda_2, \lambda_1) = C_2$

A Low-Complexity Threshold Policy



A threshold policy: $\hat{\pi}_N$:

- Continue on the channel with $\max \lambda$
- Declare on the channel with $\max \lambda$ when $\max \lambda \geq \eta(\mathbf{\Lambda}^{-i})$

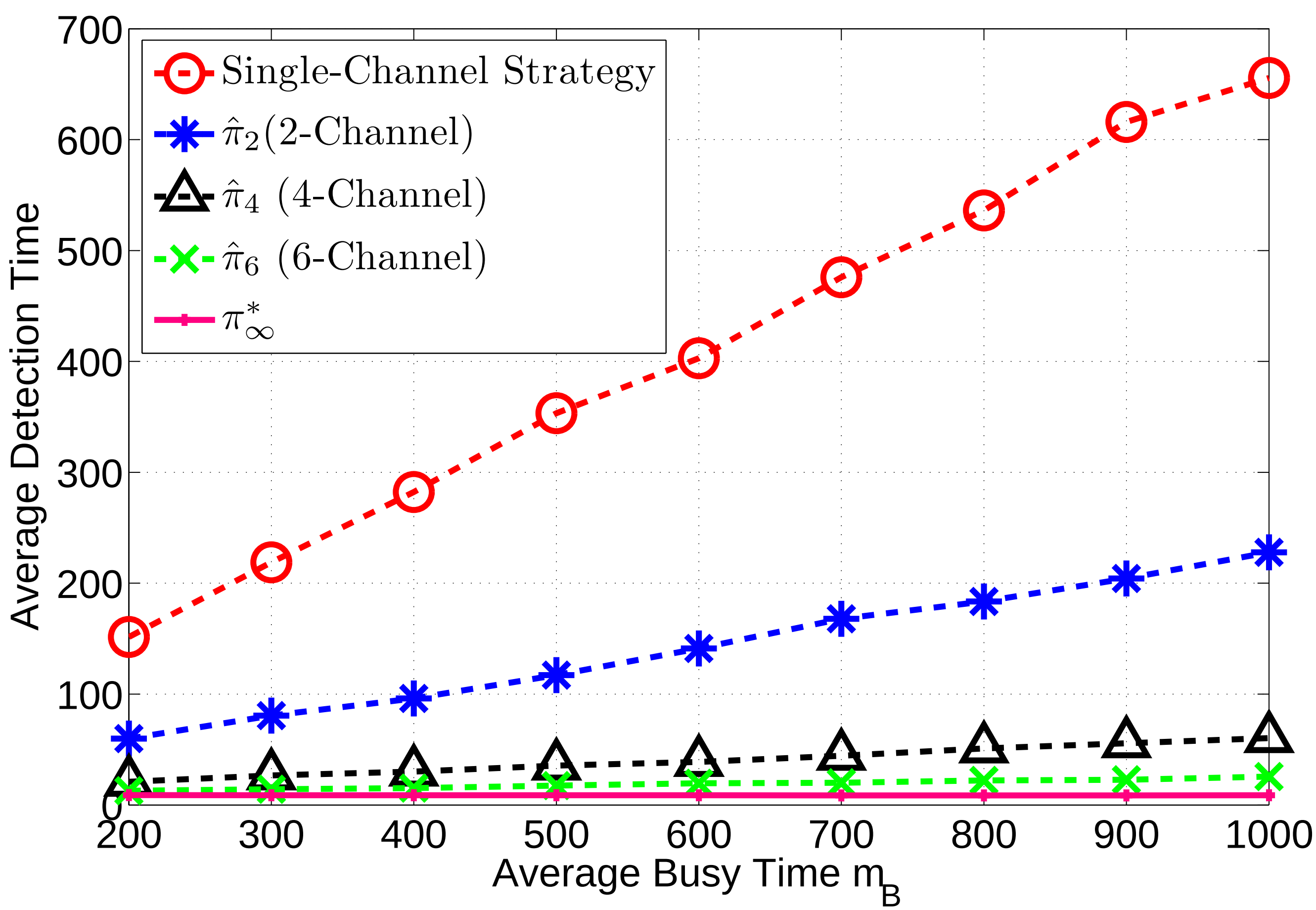
□ $\hat{\pi}_N \longrightarrow \pi^*_\infty$ as $N \to \infty$

Simulation Examples

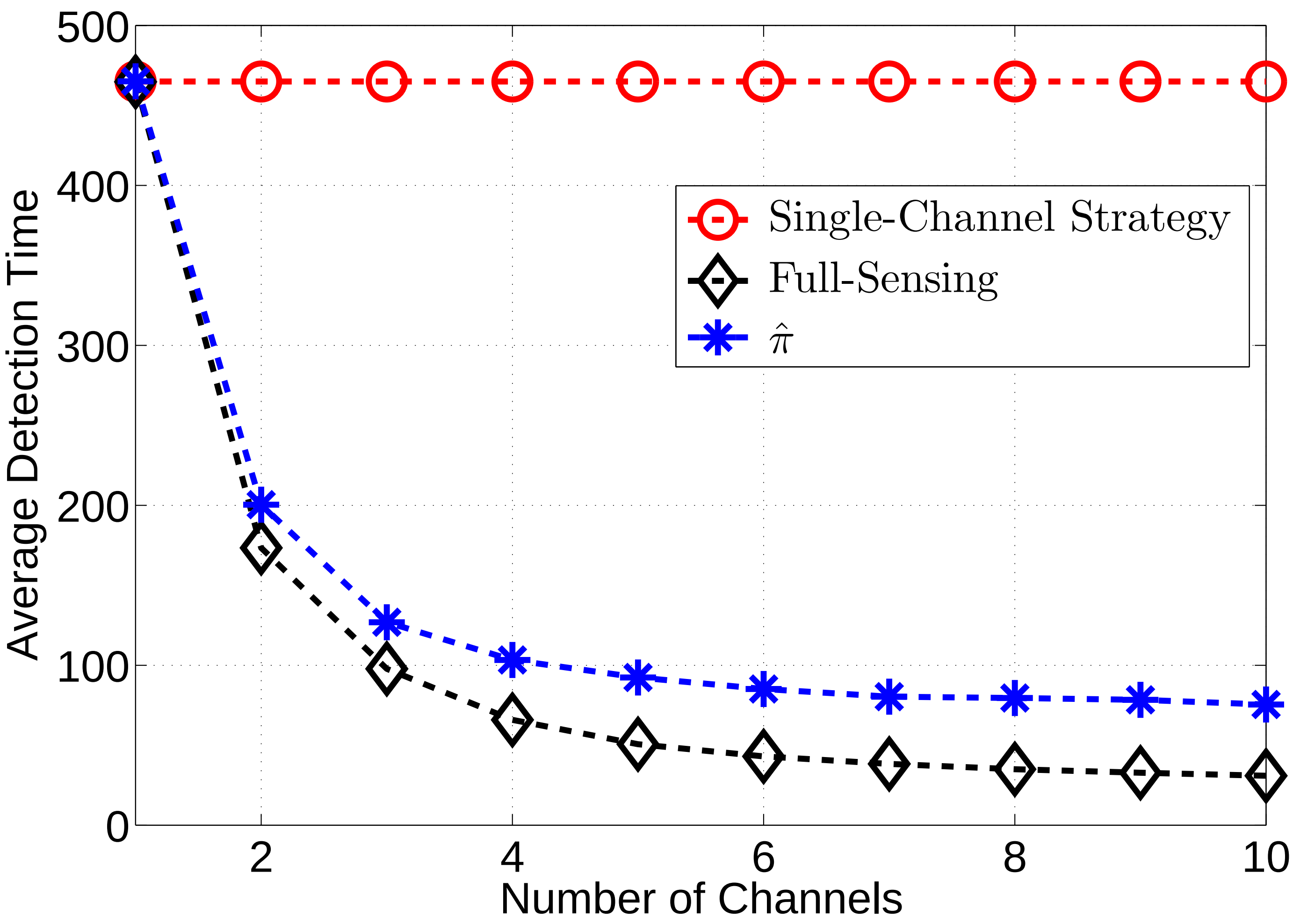
- $f_0(x),\ f_1(x)$: Gaussian with zero mean and different variances.
- $SNR = 10dB$.
- $\eta = 1 - \zeta$.

Simulation Example

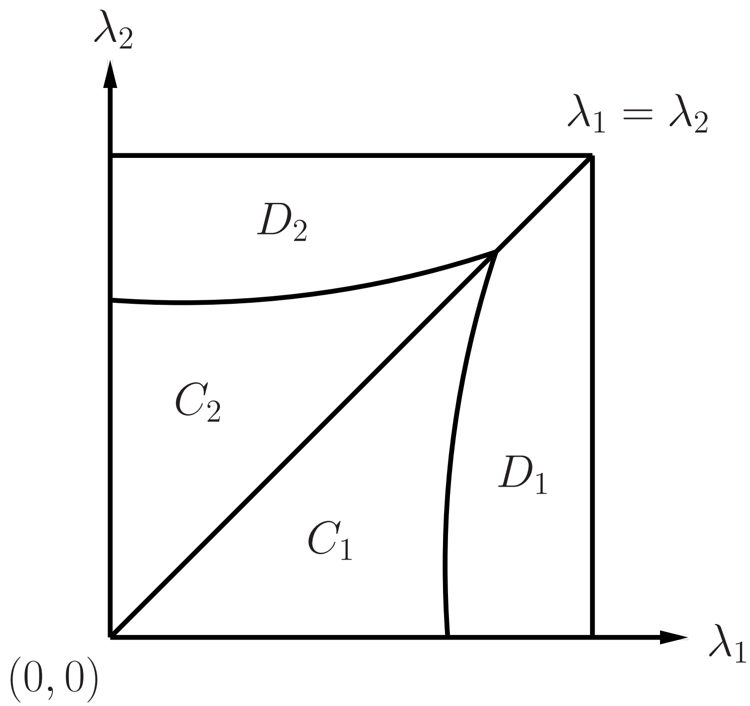
- Increase both m_B and m_I while keeping λ_0 fixed

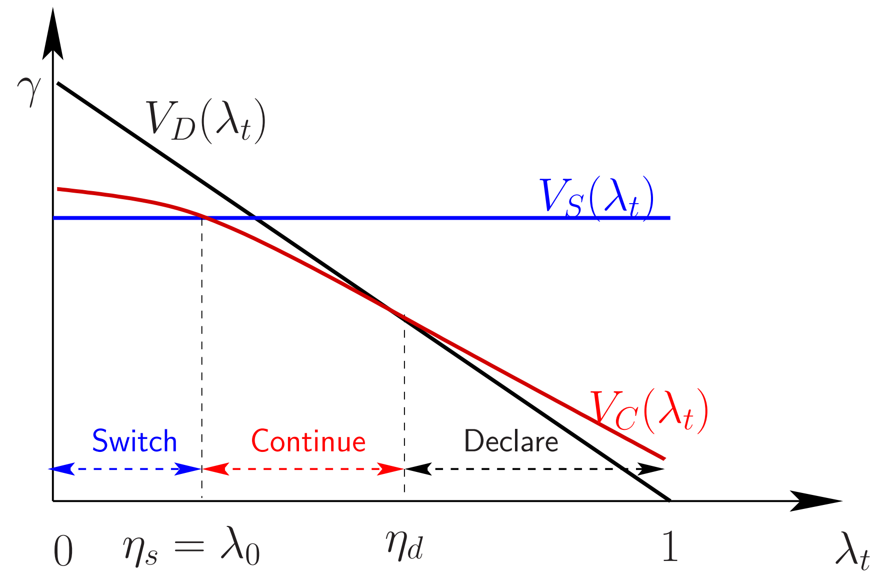


Simulation Example



Conclusion and Work in Progress





$$\hat{\pi}_N \xrightarrow{N \to \infty} \pi^*_\infty$$

Work in Progress:

- □ The optimality of the low complexity threshold policy $\hat{\pi}_N$.
- □ The asymptotic optimality of setting $\eta_d = 1 - \zeta$.