Separation Principle for Opportunistic Spectrum Access in Unslotted Primary Systems

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***Abstract*—We consider the design of opportunistic spectrum access (OSA) strategies that allow secondary users to search for and exploit spectrum opportunities in unslotted primary systems. We formulate the joint design of OSA as a constrained partially observable Markov decision process (POMDP). A separation principle for the joint design of OSA is established under certain conditions on the false alarm probability of the spectrum sensor. This result extends the separation principle for OSA in slotted primary systems to unslotted primary systems.**

***Index Terms*—Opportunistic Spectrum Access (OSA), Partially observable Markov decision process (POMDP).**

I. Introduction

A. Opportunistic Spectrum Access in Slotted Primary Systems

Opportunistic spectrum access (OSA) based on the cognitive radio technology addresses several critical challenges facing future generations of wireless systems, including radio spectrum scarcity, interference management, and coexistence and interoperability [1]. OSA in slotted primary systems has been addressed in [2]–[4], where the channel occupancy is modeled by a discrete-time Markov chain. A decision-theoretic framework based on the theory of *constrained Partially Observable Markov Decision Process* (POMDP) is developed in [2], [3]. This framework integrates the design of the spectrum sensor for opportunity detection, the sensing strategy for opportunity tracking in multiple channels, and the access strategy for transmission decisions based on imperfect sensing outcomes. It sets the system design of cognitive radio networks within a stochastic optimization framework that systematically tackles the tradeoffs between gaining access and limiting interference, learning the communication environment and exploiting dynamic spectrum opportunities.

While POMDPs often suffer from the curse of dimensionality, a separation principle has been shown to exist that leads to simple, robust, yet optimal design [3]. Specifically, the design of the spectrum sensor and the access strategy can be separated from that of the sensing strategy. Furthermore, the myopic policies are optimal for the design of the spectrum sensor and the access strategy, leading to simple closed-form solutions that do not require the knowledge of the underlying Markov model. It is further shown in [4] that when primary traffic is independent and homogeneous across channels, the optimal sensing strategy for opportunity tracking is again the myopic policy with a simple semi-universal structure that obviates the need to know the the precise knowledge of the underlying Markov model. These results reveal the existence of simple yet optimal solutions to OSA that are robust against model mismatch and variations.

B. Opportunistic Spectrum Access in Unslotted Primary Systems

The objective of this paper is to extend the separation principle to OSA in unslotted primary systems. The occupancy of each channel by primary users is modeled as a continuous-time Markov chain, which has been shown to match well with the spectrum usage in wireless LAN [5]. The secondary network adopts a slotted transmission structure. At the beginning of each slot, a secondary user decides which channel to sense and potentially transmit over. The problem appears to be significantly more complex than its counterpart in slotted primary systems due to the arbitrary starting and ending times of the primary transmissions and the half duplex mode of the secondary user that prevents it from sensing the channel during a transmission.

In our previous work [6], a certain equivalency between OSA in unslotted primary systems and that in slotted primary systems has been established. This equivalency points to the possibility of reducing the design of OSA in unslotted primary systems to that in slotted primary systems, a significantly simpler problem. Specifically, it is shown in [6] that even though the underlying primary systems are modeled as *continuous-time* Markov chains, the joint design of OSA fits into the *discrete-time* constrained POMDP framework developed in [2]–[4] for the slotted case. This result is based on the following two key observations: (i) Opportunity detection should be formulated as detecting the channel state during the *transmission* period of a secondary user's slot based on the measurements taken in the *sensing* period of the slot; (ii) under this formulation of opportunity detection, the difference between unslotted and slotted primary systems — that transmissions of primary users can start and end at arbitrary time

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instants — simply contributes to sensing errors.

Based on the constrained POMDP framework developed in [6] for OSA in unslotted primary systems, we show in this paper that the separation principle is preserved for the unslotted case under certain conditions. This result is not a straightforward outcome following [6] and the separation principle for the slotted case given in [3]. The main difficulty here is that the operating characteristics (probabilities of false alarm and miss detection) of the optimal spectrum sensor is time varying and dependent on the observation and decision history. This significantly enriches the design space and complicates the analysis of the optimal solution. In this paper, we show that when the variation of the false alarm probability with respect to the observation and decision history satisfies certain conditions, the separation principle is preserved; the same simple, robust, and optimal design of OSA can be achieved in unslotted primary systems.

II. Network Model

Consider a spectrum consisting of N channels, each with bandwidth B_n $(n = 1, \cdots, N)$. The occupancies of these N channels by primary users are modeled as independent continuous-time Markov processes with two states: $S_n(t) = 0$ (busy) and $S_n(t) = 1$ (idle). More specifically, for channel n, the sojourn times in the busy and idle states are exponentially distributed with rates μ_n and λ_n, respectively.

We consider an ad hoc overlay network where secondary users independently search for and access spectrum opportunities in these N channels. This overlay network adopts a slotted transmission structure with slot length L. In each slot, a secondary user chooses one of the N channels to sense and decides whether to transmit over the chosen channel based on the sensing outcome. Consider a slot starting at time t. The beginning of each slot is used for spectrum sensing which takes L_s seconds. The remaining time $[t + L_s, t + L)$ of the slot is used for transmission if the user chooses to. If the channel remains idle for the whole period of $[t+L_s, t+L)$, the transmission is successful. Otherwise, a collision with primary users occurs. The receiver acknowledges each successful transmission at the end of the slot. Note that even if the channel is idle during the sensing period $[t, t+L_s)$, it may become busy in any segment of the transmission period $[t+L_s, t+L)$, resulting in a collision. For convenience, we use k $(k = 1, \cdots, T)$ as the slot index, *i.e.*, slot k starts at $t_k \triangleq (k-1)L$ and ends at kL.

Our goal is to develop an optimal OSA strategy for the secondary user, which sequentially determines which channel in the spectrum to sense, how to design the spectrum sensor, and whether to access based on the sensing outcome. We focus on an individual selfish user. The design objective is to maximize the throughput of this user during a desired period of T slots under the constraint that the probability of collision $C_n(k)$ perceived by the primary users in any channel n and slot t is capped below a pre-determined threshold ζ, *i.e.*,

$$C_n(k) \triangleq \Pr\{\Phi_n(k) = 1 \,|\, O_n(k) = 0\} \leq \zeta, \quad \forall n, k, \qquad (1)$$

where $\Phi_n(k) \in \{0 \text{ (no access)}, 1 \text{ (access)}\}$ denotes the access decision of the user, and $O_n(k)$ denotes opportunity defined as

$$O_n(k) = \begin{cases} 1, & S_n(t) = 1 \ \forall t \in [t_k + L_s, t_k + L) \\ 0, & \text{otherwise} \end{cases}$$

III. A Constrained POMDP Framework

In this section, we show that even though the channel occupancies are given by continuous-time Markov processes, the joint design of the spectrum sensor and sensing and access strategies can be formulated as a discrete-time POMDP.

A. The Spectrum Sensor

Suppose that channel n is chosen in slot k. The objective of the spectrum sensor is to decide, based on the measurements taken in $[t_k, t_k + L_s)$, whether channel n is an opportunity for transmission, *i.e.*, idle during $[t_k + L_s, t_k + L)$. The spectrum sensor thus performs a binary hypothesis test:

$$\mathcal{H}_0 : O_n(k) = 1 \text{ (idle)} \quad \text{vs.} \quad \mathcal{H}_1 : O_n(k) = 0 \text{ (busy)}. \qquad (2)$$

Let $\hat{O}_n(k) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$ denote the sensing outcome (*i.e.*, the result of the binary hypothesis test). The performance of the spectrum sensor is characterized by the probability of false alarm (PFA) $\epsilon_n(k)$ and the probability of miss detection $\delta_n(k)$:

$$\epsilon_n(k) \triangleq \Pr\{\hat{O}_n(k) = 0 \,|\, O_n(k) = 1\}, \qquad (3a)$$

$$\delta_n(k) \triangleq \Pr\{\hat{O}_n(k) = 1 \,|\, O_n(k) = 0\}. \qquad (3b)$$

For a given PFA $\epsilon_n(k)$, the largest achievable probability of detection (PD), denoted as $P_{D,\max}^{(n)}(\epsilon_n(k))$, can be attained by the optimal NP detector with the constraint that the PFA is no larger than $\epsilon_n(k)$ or an optimal Bayesian detector with a suitable set of risks [7, Sec. 2.2.1]. All operating points (ϵ, δ) above $P_{D,\max}^{(n)}$ are thus infeasible. The feasible set of operating points of the spectrum sensor is thus $\{(\epsilon, \delta) : 0 \leq \epsilon \leq 1 - \delta \leq P_{D,\max}^{(n)}(\epsilon)\}$ as illustrated in Fig. 1. Note that every sensor operating point (ϵ_n, δ_n) below $P_{D,\max}^{(n)}$ lies on a line that connects two boundary points and hence can be achieved by randomizing between two optimal NP detectors with properly chosen constraints on the PFA [7, Sec. 2.2.2]. Therefore, the design of spectrum sensor is reduced to the choice of a desired feasible sensor operating point. Note that both the feasible set and the optimal operating point may vary from slot to slot.

B. The Sensing and Access Strategies

At the beginning of slot k, the secondary user first chooses a channel $a(k) \in \{1, \ldots, N\}$ to sense and a feasible sensor operating point $(\epsilon_a(k), \delta_a(k))$. It then determines whether to access $\Phi_a(k) \in \{0 \text{ (no access)}, 1 \text{ (access)}\}$ by taking into account the sensing outcome $\hat{O}_a(k) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$ provided by the spectrum sensor that is designed according to the chosen operating point $(\epsilon_a(k), \delta_a(k))$. At the end of this slot, the receiver acknowledges a successful transmission $\Psi_a(k) \in \{0 \text{ (no ACK)}, 1 \text{ (ACK)}\}$.

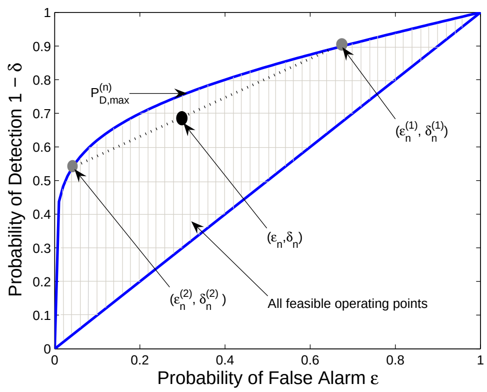

Fig. 1. Feasible set of sensor operating points (ϵ_n, δ_n).

C. A Constrained POMDP Formulation

We show here that the joint design of OSA can be formulated as a constrained POMDP defined as follows.

State Space The underlying system state is given by the state of each channel at the beginning of each slot. Let $S_n(k) \stackrel{\Delta}{=} S_n(t)\ |_{t=(k-1)L}$. The system state in slot k is thus $\mathbf{S}(k) = [S_1(k), \cdots, S_N(k)] \in \{0,1\}^N$. It is straightforward that $\{S_n(k)\}$ is a discrete-time Markov chain with transition matrix $\mathbf{P}^{(n)}(L) \stackrel{\Delta}{=} \Pr[S_n(k+1) = j \mid S_n(k) = i] = \exp(\mathbf{Q}_n L)$, where

$$\mathbf{Q}_n = \begin{pmatrix} -\mu_n & \mu_n \\ \lambda_n & -\lambda_n \end{pmatrix}.$$

is the transition rate matrix of the continuous-time Markov process that models the occupancy of channel n.

Lemma 1: The transition matrix for the discrete-time Markov chain $\{S_n(k)\}$, is given by the following expression:

$$\mathbf{P}^{(n)}(L) = \begin{pmatrix} 1 - \frac{1-\exp(-(\mu_n+\lambda_n)L)}{1+\frac{\lambda_n}{\mu_n}} & \frac{1-\exp(-(\mu_n+\lambda_n)L)}{1+\frac{\lambda_n}{\mu_n}} \\ \frac{\lambda_n}{\mu_n}\frac{1-\exp(-(\mu_n+\lambda_n)L)}{1+\frac{\lambda_n}{\mu_n}} & 1 - \frac{\lambda_n}{\mu_n}\frac{1-\exp(-(\mu_n+\lambda_n)L)}{1+\frac{\lambda_n}{\mu_n}} \end{pmatrix} \quad (4)$$

Proof: omitted due to space limit. ∎

Lemma 2: In un-slotted primary system, under any chosen slot length L, for each channel n:

$$P_{11}^{(n)}(L) > P_{01}^{(n)}(L) \quad (5)$$

Proof: Easy to check form *lemma* 1. ∎

These two lemmas lead to certain structures in the Value function which is used to prove separation principle.

Action Space The action in each slot consists of three parts: a sensing decision $a(k)$, a spectrum sensor design $(\epsilon_a(k), \delta_a(k)))$, and an access decision $\Phi_a(t) \in \{0,1\}$.

Observation Space Optimal channel selection for opportunity tracking relies on the exploitation of the entire observation history of the user. To ensure synchronous hopping in the spectrum without introducing extra control message exchange, the user and its desired receiver must use *common observations* for channel selection. Since sensing errors may cause different sensing outcomes at the transmitter and the receiver, the acknowledgement $\Psi_a(k)$ is the common observation.

Reward A natural definition of the reward is the number of bits that can be delivered by the user. Given sensing action $a(k)$ and access action $\Phi_a(k)$, the immediate reward $R_{\Psi_a(k)}$ can be defined as

$$R_{\Psi_a(k)} = \Psi_a(k) B_a L_{tx} = O_a(k)\Phi_a(k) B_a L_{tx}, \quad (6)$$

where $L_{tx} = L - L_s$ is the transmission time in each slot.

Belief Vector Due to partial spectrum monitoring and sensing errors, a secondary user cannot directly observe the system state $\mathbf{S}(k)$. It can, however, infer the state from its decision and observation history. The statistical information on the system state provided by the entire decision and observation history can be encapsulated in a belief vector $\Omega(k) \stackrel{\Delta}{=} [\omega_1(k), \cdots, \omega_N(k)]$, where $\omega_n(k)$ denotes the conditional probability (given the decision and observation history) that $S_n(k) = 1$. Note that we have used the channel independence to reduce the dimension of the belief vector from 2^N to N [2].

Policy A joint design of OSA is given by policies of the above POMDP. Specifically, a sensing policy π_s specifies a sequence of functions (one for each slot), each mapping a belief vector $\Omega(k)$ at the beginning of slot k to a channel $a(k)$ to be sensed in slot k. Similarly, a sensor operating policy π_δ specifies, in each slot k, a spectrum sensor design $(\epsilon_a(k), \delta_a(k))$ based on the current belief vector $\Omega(k)$ and the chosen channel $a(k)$. An access policy π_c specifies an access decision $\Phi_a(k) \in \{0,1\}$ in each slot k based on the current belief vector $\Omega(k)$ and the sensing outcome $\hat{O}_a(k)$.

The above defined policies are deterministic. For unconstrained POMDPs, there always exist deterministic optimal policies. For constrained POMDPs, however, we may need to resort to randomized policies to achieve optimality. For randomized policies, we design the probability distribution of the action to be taken, rather than a specific deterministic action. Due to the uncountable space of probability distributions, randomized policies are usually computationally prohibitive.

Objective and Constraint We aim to develop the optimal joint design of OSA $\{\pi_\delta^*, \pi_s^*, \pi_c^*\}$ that maximizes the expected total number of bits that can be delivered by the user in T slots under the collision constraint given in (1):

$$\{\pi_\delta^*, \pi_s^*, \pi_c^*\} = \arg\max_{\pi_\delta, \pi_s, \pi_c} \mathbb{E}_{\{\pi_\delta, \pi_s, \pi_c\}}\left[\sum_{k=1}^{T} R_{\Psi_a(k)} \middle| \Omega(1)\right]$$
$$\text{s.t. } C_a(k) = \Pr\{\Phi_a(k) = 1 \mid O_a(k) = 0\} \le \zeta, \qquad \forall a, k, \quad (7)$$

where $\Omega(1)$ is the initial belief vector, which can be set to the stationary distribution of the underlying Markov process if no information on the initial system state is available.

IV. The Optimal Joint Design

The first step to solving (7) is to express the objective and the constraint explicitly as functions of the actions. We establish first the optimality of deterministic sensing and sensor operating policies, which significantly simplifies the action space.

A. The Optimality of deterministic policies

Theorem 1: For the optimal joint design of OSA given by (7)*, there exist deterministic optimal sensing and sensor operating policies.*

Proof: omitted due to space limit. ■

As a result of Theorem 1, the user needs to choose, in each slot, a channel a to sense, a feasible sensor operating point (ϵ_a, δ_a), and a pair of transmission probabilities $(f_a(0), f_a(1))$, where

$$f_a(\theta) \triangleq \Pr\{\Phi_a = 1 \,|\, \hat{O}_a = \theta\}$$

is the probability of accessing channel a given sensing outcome $\hat{O}_a = \theta \in \{0, 1\}$.

B. The Objective Function

Let $V_k(\Omega(k))$ be the value function, which represents the maximum expected reward that can be obtained starting from slot k given belief vector $\Omega(k)$ at the beginning of slot k. Given that the user takes action $A = \{a, (\epsilon_a, \delta_a), (f_a(0), f_a(1))\}$ and observes acknowledgement $\Psi_a = \psi$, the reward that can be accumulated starting from slot t consists of two parts: the immediate reward $R_{\Psi_a} = \psi B_a L_{tx}$ and the maximum expected future reward $V_{k+1}(\Omega(k+1))$, where $\Omega(k+1) \triangleq \mathcal{T}(\Omega(k) \,|\, A, \psi)$ represents the updated belief vector after incorporating the action A and the acknowledgement ψ in slot k. We thus have the following optimality equation

$$V_k(\Omega(k)) = \max_A \sum_{s=0}^{1} (s\omega_a + (1-s)(1-\omega_a)) \sum_{\psi=0}^{1} U_{s,\psi}(A) \left[\psi B_a L_{tx} + V_{k+1}(\mathcal{T}(\Omega(k) \,|\, A, \psi))\right], \tag{8a}$$

$$V_T(\Omega(T)) = \max_A \sum_{s=0}^{1} (s\omega_a + (1-s)(1-\omega_a)) U_{s,1}(A) B_a L_{tx}, \tag{8b}$$

where $U_{s,\psi}(A) \triangleq \Pr\{\Psi_a = \psi \,|\, S_a = s\}$ is the conditional distribution of the acknowledgement given the current state S_a of channel a and action A. Since $\Psi_a = O_a \Phi_a$, we have

$$\begin{aligned} U_{s,0}(A) &= 1 - U_{s,1}(A) \\ U_{s,1}(A) &= \Pr[\Phi_a = 1, O_a = 1 \,|\, S_a = s] \\ &= \Pr[\Phi_a = 1 \,|\, O_a = 1, S_a = s] \Pr[O_a = 1 \,|\, S_a = s]. \end{aligned}$$

Due to space limit, we omit the detailed derivation of $U_{s,1}(A)$.

The updated belief vector $\Omega(k+1) = \mathcal{T}(\Omega(k) \,|\, A, k)$ can be obtained from Bayes' rule, more specifically, for the channels except the one that is chosen in slot k, the updated belief value is just simply updated by the discrete-time Markov chain:

$$\omega_n(k+1) = P_{11}^{(n)}(L)\omega_n(k) + P_{01}^{(n)}(L)(1 - \omega_n(k)) \tag{10}$$

Where $\mathbf{P}^{(n)}(L)$ is the transition matrix of the discrete-time Markov chain for the nth channel and $n \neq a$.

And for the channel that is chosen in slot k to sense we have:

Lemma 3: **Belief Update for the chosen channel**

If $\Psi_a = 1$:

$$\omega_a(k+1) = 1. \tag{11}$$

If $\Psi_a = 0$:

$$w_a(k+1) = \frac{\sum_{s=0}^{1}(s\omega_a + (1-s)(1-\omega_a))(P_{s,1}^{(a)}(L) - U_{s,1}(A))}{\sum_{s=0}^{1}(s\omega_a + (1-s)(1-\omega_a))U_{s,0}(A)}$$

Proof: Omitted due to space limit. ■

C. The Collision Constraint

The collision probability $C_a(k)$ is determined by the sensor operating point (ϵ_a, δ_a) and the transmission probabilities $(f_a(0), f_a(1))$.

$$\begin{aligned} C_a(k) &\triangleq \Pr\{\Phi_a(k) = 1 \,|\, O_a(k) = 0\} \\ &= (1 - \delta_a) f_a(0) + \delta_a f_a(1) \leq \zeta. \end{aligned} \tag{12}$$

In principle, by solving (8) recursively (starting from the last slot T using (8b)) under the constraint of (12), we can obtain the maximum overall throughput $V_1(\Omega(1))$ of the secondary user and the corresponding policies $\{\pi_s^*, \pi_\delta^*, \pi_c^*\}$. However, (8) is generally intractable due to the uncountable action space.

V. Separation Principle

In this section we prove that among policies with the same channel to sense, the one that maximizes the immediate reward, maximizes the value function, hence it is optimum to first choose sensor operating policy π_δ, and access policy π_c to maximize the immediate reward subject to the given constraint, then solving an uncostrained POMDP to obtain the optimum sensing policy π_s. But first let consider how the belief vector is updated in the un-slotted primary systems. Note that in the following we assume that the channels are i.i.d, so we will not have the superscript (n) and subscript n in the following.

Theorem 2: ***The Separation Principle for OSA in unslotted primary systems with single-channel sensing***

Under the following conditions on the variation of Probability of False Alarm (ϵ),

$$\frac{P_{11}(L_s) - P_{01}(L_s)}{1 - P_{01}(L_s)}\left(1 - \frac{1}{\exp(-\lambda L_{tx}) P_{11}(L_s)}\right) \leq \frac{\epsilon'}{1-\epsilon} \leq \frac{P_{11}(L_s) - P_{01}(L_s)}{P_{11}(L_s)} \tag{13}$$

$$2\{P_{11}(L_s) - P_{01}(L_s)\}\epsilon' \leq \min\{-\epsilon'' P_{11}(L_s), -\epsilon'' P_{01}(L_s)\} \quad (14)$$

$$\frac{\epsilon'}{1-\epsilon} + \frac{\min\{P_{11}(L_s)\epsilon'', P_{11}(L_s)\epsilon''\}}{(P_{11}(L_s) - P_{01}(L_s))(1-\epsilon)} \geq \frac{P_{11}(L) - P_{01}(L)}{1 - P_{11}(L)} \quad (15)$$

where ϵ' and ϵ'' are, respectively, the first and the second derivative of ϵ with respect to belief value, the joint design of OSA given in (8) can be carried out in two steps without losing optimality.

Step 1: choose the sensor operating policy π_δ and the access policy π_c to maximize the instantaneous throughput subject to the collision constraint. Specifically, for any chosen channel a, the optimal sensor operating point $(\epsilon_a^*, \delta_a^*)$ and transmission probabilities $(f_a^*(0), f_a^*(1))$ are given by

$$\begin{aligned}\{(\epsilon_a^*, \delta_a^*), (f_a^*(0), f_a^*(1))\} &= \arg\max \mathbb{E}\left[R_{\Psi_a(k)} \,\middle|\, \Omega(k)\right] \\ &= \arg\max \epsilon_a f_a(0) + (1-\epsilon_a) f_a(1) \\ \text{s.t.} \quad C_a(k) = (1-\delta_a) f_a(0) + \delta_a f_a(1) \leq \zeta.\end{aligned}$$

Step 2: Using the optimal sensor operating and access policies $\{\pi_\delta^*, \pi_c^*\}$ given by Theorem 2, we choose the sensing policy to maximize the overall throughput. Specifically, the optimal sensing policy π_s^* is given by

$$\pi_s^* = \arg\max_{\pi_s} \mathbb{E}_{\pi_s}\left[\sum_{k=1}^{T} R_{\Psi_a(k)} \middle| \Omega(1)\right]. \quad (17)$$

Proof: The proof is based on the convexity of value function $V_k(\Omega(k))$ in each direction (when only one component of belief vector varies) with respect to the belief vector $\Omega(k)$ under the given conditions, and the structure of belief update. We omit the proof here due to space limit. ■

Theorem 3: For any chosen channel a in any slot, the optimal sensor should adopt the optimal Neyman-Pearson (NP) detector with constraint $\delta_a^* = \zeta$. Correspondingly, the optimal access policy is to trust the sensing outcome given by the spectrum sensor, i.e., $f_a^*(0) = 0$ and $f_a^*(1) = 1$.

Proof: omitted due to space limit. ■

This theorem gives us a closed form expression for the optimal spectrum sensor and access policies.

Note that the design of the sensing strategy has been reduced from a constrained POMDP in (7) to an unconstrained one with finite action space. This is because the sensor operating points and the transmission probabilities determined by (16) have ensured the collision constraint regardless of channel selections. Unconstrained POMDPs have been well-studied. The optimal sensing policy can thus be readily obtained by using computationally efficient solution procedures in the literature.

VI. Simulation Results

In this section, we show the simulated *PFA* to see how it really changes vs belief and how restrictive the conditions for Separation Principle are.

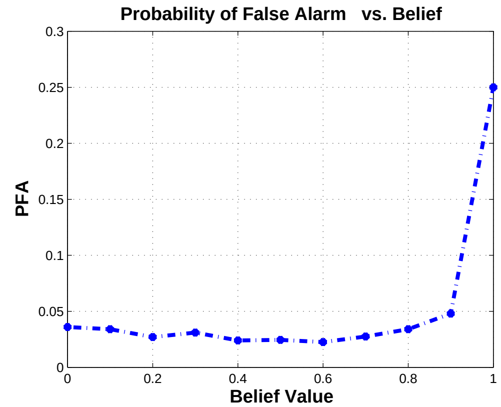

Fig. 2. Probability of False Alarm ϵ vs. belief ω ($L = 1$, $L_s = 0.05$, $\sigma_0^2 = 1$, $\sigma_1^2 = 10$, $\zeta = 0.4$, $P_{11} = 0.8$, $P_{01} = 0.2$)

Fig. 2 shows *PFA* as function of belief value from 0 to 1. In here we consider a simple spectrum sensing scenario where the background noise and the primary signal are modeled as white Gaussian processes. At the beginning of each slot, the spectrum sensor takes two samples from the chosen channel and performs the following binary hypothesis test:

$$\begin{aligned}\mathcal{H}_0(O_n(k) = 1) &: \mathbf{Y} \sim \mathcal{N}(\mathbf{0}_2, \sigma_0^2 \mathbf{I}_2) \\ \mathcal{H}_1(O_n(k) = 0) &: \mathbf{Y} \sim \mathcal{N}(\mathbf{0}_2, (\sigma_0^2 + \sigma_1^2)\mathbf{I}_2). \quad (18)\end{aligned}$$

for an energy detector under the *NP* test.

Note that in here we only want to consider the variation of *PFA* as a function of belief and that is why we have relaxed the collision constraint $\zeta = 0.4$ in order to have a better and faster simulation results for our two samples.

As We observe, for a wide range of belief values from 0 to about 0.9, *PFA* changes very slowly as a function of belief meaning that our conditions on the variation (first and the second derivative) of *PFA* are hopefully always satisfied, but as the belief value increases from 0.9 to 1 it varies much faster than before, so conditions may not be satisfied anymore, and in order to make sure that separation principle holds, we should use suboptimal sensor operating point policies that do not vary very fast for high belief values.

So for a wide range of belief values conditions are hopefully satisfied.

VII. Conclusion

In this paper, we have developed a POMDP formulation for the joint design of OSA in unslotted primary network. We have shown that the separation principle developed in [3] for OSA in slotted primary systems can be extended to unslotted primary systems under certain conditions. This result leads to simple closed-form solutions to the joint design of OSA in unslotted primary systems.

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