

# On the Connectivity and Multihop Delay of Ad Hoc Cognitive Radio Networks

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**Abstract**—We analyze the multihop delay of ad hoc cognitive radio networks, where the transmission delay of each hop consists of the propagation delay and the waiting time for the availability of the communication channel (*i.e.*, the occurrence of a spectrum opportunity at this hop). Using theories and techniques from continuum percolation and ergodicity, we establish the scaling law of the minimum multihop delay with respect to the source-destination distance in cognitive radio networks. We show the starkly different scaling behavior of the multihop delay in *connected* networks as compared to networks that are only *intermittently connected* due to scarcity of spectrum opportunities.

**Index Terms**—Cognitive radio network, multihop delay, connectivity, intermittent connectivity, continuum percolation.

## I. INTRODUCTION

The basic idea of opportunistic spectrum access is to achieve spectrum efficiency and interoperability through a hierarchical access structure with primary and secondary users [1]. Secondary users, equipped with cognitive radios [2] capable of sensing and learning the communication environment, identify and exploit instantaneous and local spectrum opportunities without causing unacceptable interference to primary users [1].

Using theories and techniques from continuum percolation and ergodicity, we analytically characterize the connectivity and multihop delay of the secondary network. Specifically, we consider a Poisson distributed secondary network<sup>1</sup> overlaid with a Poisson distributed primary network in an infinite two-dimensional Euclidean space<sup>2</sup>. Due to the hierarchical structure of spectrum sharing, the transmission delay of each hop in the secondary network consists of two components: the propagation delay and the waiting time for the availability of the communication channel (*i.e.*, the occurrence of a spectrum opportunity at this hop).

### A. Main Results

The contribution of this paper is twofold. First, we analytically characterize the connectivity of the secondary network which depends on not only the topology of the secondary

network but also the transmitting and receiving activities of the privileged primary network. The connectivity of the secondary network is thus determined by two critical parameters: the density  $\lambda_S$  of the secondary users and the density  $\lambda_{PT}$  of the primary transmitters representing the traffic load of the primary network. As illustrated in Fig. 1, we show that according to the connectivity of the secondary network, the  $(\lambda_S, \lambda_{PT})$  plane can be partitioned into three regions: disconnected, connected, and intermittently connected, which are all interpreted in the percolation sense. The secondary network is disconnected if there does not exist almost surely (a.s.) an infinite connected component formed by topological links connecting two secondary users within each other's transmission range, and the secondary network is connected if there exists a.s. an infinite connected component formed by communication links, where communication links are those topological links experiencing the spectrum opportunities. Since the set of communication links is a subset of topological links, we define the intermittent connectivity for a secondary network that is not connected as the a.s. existence of an infinite connected component consisting of topological links. The above three concepts are detailed in Sec. III.

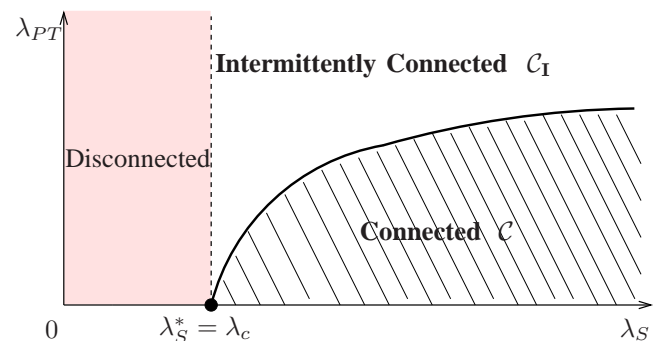


Fig. 1. The shaded region is the set of  $(\lambda_S, \lambda_{PT})$  under which the secondary network is connected (*i.e.*, the connectivity region  $\mathcal{C}$ ), the white region is the set of  $(\lambda_S, \lambda_{PT})$  under which the secondary network is intermittently connected (*i.e.*, the intermittent connectivity region  $\mathcal{C}_I$ ), and the colored region is the set of  $(\lambda_S, \lambda_{PT})$  under which the secondary network is disconnected. The critical density  $\lambda_S^*$  of the secondary users is equal to the critical density  $\lambda_c$  of a homogeneous network.

Second, we establish the scaling law of the minimum multihop delay in the secondary network with respect to the source-destination distance. When the secondary network is disconnected, there exist only finite topologically connected components. If we randomly choose two secondary users,

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<sup>1</sup>The notions of cognitive radio networks and secondary networks are used interchangeably in this paper.

<sup>2</sup>This infinite network model is equivalent in distribution to the limit of a sequence of finite networks with a fixed density as the area of the network increases to infinity, *i.e.*, the so-called *extended network* [3].

then they belong to two different topologically connected component a.s., which implies that they are not reachable from each other. We thus focus on the multihop delay between two secondary users in the infinite topologically connected component when the secondary network is either connected or intermittently connected.

To highlight the impact of the waiting time for spectrum opportunities on multihop delay, we first study the scaling law of the minimum multihop delay assuming that the propagation delay is negligible. We show that the scaling law of the minimum multihop delay with respect to the source-destination distance has two distinct regimes, corresponding to whether the secondary network is connected or intermittently connected. Specifically, let  $\mu$  be the source,  $\nu$  the destination,  $t(\mu, \nu)$  the minimum multihop delay from  $\mu$  to  $\nu$ , and  $d(\mu, \nu)$  the distance between  $\mu$  and  $\nu$ , then we show that, a.s.

$$\lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{d(\mu, \nu)} \begin{cases} = 0, & \text{if connected;} \\ > 0, & \text{if intermittently connected.} \end{cases}$$

When the secondary network is connected, a much stronger statement is actually shown, that is,

$$\lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{g(d(\mu, \nu))} = 0 \text{ a.s.,}$$

where  $g(d(\mu, \nu))$  is any monotonically increasing function of  $d(\mu, \nu)$  satisfying  $\lim_{d(\mu, \nu) \rightarrow \infty} g(d(\mu, \nu)) = \infty$ . It implies that the minimum multihop delay  $t(\mu, \nu)$  is asymptotically independent of the distance  $d(\mu, \nu)$  as  $d(\mu, \nu) \rightarrow \infty$ . Thus when the propagation delay is negligible, a connected cognitive radio (CR) network behaves almost the same as a homogeneous ad hoc network, in the sense that the waiting time for the spectrum opportunities does not affect the scaling law of the multihop delay with respect to the source-destination distance.

The above scaling law of the multihop delay may be illustrated with an analogy of traveling from a place  $\mu$  to another place  $\nu$ , where the waiting time for the spectrum opportunities is likened to the waiting time for traffic lights. Suppose that we can move fast enough such that (s.t.) the driving time on the road is negligible. When the secondary network is connected, there exists an infinite connected component consisting of communication links a.s. which can be regarded as a highway without traffic lights between  $\mu$  and  $\nu$ . Given that both  $\mu$  and  $\nu$  are within a finite distance to the highway (independent of the distance between  $\mu$  and  $\nu$ ), the traveling time from  $\mu$  to  $\nu$ , which is exactly the waiting time for traffic lights before entering the highway and after leaving the highway, is independent of the distance between  $\mu$  and  $\nu$ . When the secondary network is intermittently connected, there does not exist an infinite connected component formed by communication links a.s., *i.e.*, such a highway between  $\mu$  and  $\nu$  can not be found. Then we have to use local paths and wait for traffic lights from time to time, leading to the linear scaling of the traveling time with respect to the distance between  $\mu$  and  $\nu$ .

We also study the impact of the propagation delay on multihop delay. When the propagation delay  $\tau$  is nonnegligible,

we show that the minimum multihop delay scales linearly with the source-destination distance in both connected and intermittently connected regimes, but with different rates for the linear scaling. In particular, the limiting behavior of the rate as  $\tau \rightarrow 0$  is distinct in the two regimes, *i.e.*, a.s.

$$\lim_{\tau \rightarrow 0} \lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{d(\mu, \nu)} \begin{cases} = 0, & \text{if connected;} \\ > 0, & \text{if intermittently connected.} \end{cases}$$

It indicates that when the propagation delay is sufficiently small, the scaling rate of the multihop delay for a connected network is much smaller than the one for an intermittently connected network.

## B. Related Work

As a fundamental issue for the feasibility and efficiency of large-scale wireless networks, the scaling law has raised increasing interest in the research community since the seminal work of P. Gupta and P. R. Kumar [4]. The capacity scaling law of CR networks has been analyzed in [5–7]. To our best knowledge, the scaling law of the multihop delay with respect to the source-destination distance in a CR network has not been characterized analytically or experimentally in the literature.

The scaling law of the multihop delay in homogeneous ad hoc networks has been well studied in [8–16]. As the number of users in the network increases to infinity, the multihop delay for a specific routing algorithm is analyzed in [8–10], and the capacity-delay tradeoff is revealed under a given network and mobility model in [11–13]. Based on continuum percolation theory, the scaling law of the multihop delay with respect to the source-destination distance is established in [14–16].

## II. NETWORK MODEL

We consider a Poisson distributed secondary network overlaid with a Poisson distributed primary network in an infinite two dimensional Euclidean space. The primary network adopts a synchronized slotted structure with a slot length  $T_S$ . The realizations of active primary transmitters vary from slot to slot and are assumed to be i.i.d. across slots<sup>3</sup>. Thus  $T_S$  can be considered as the time constant of the spectrum opportunities which are determined by the transmitting and receiving activities of the primary users. Without loss of generality, we assume that  $T_S = 1$ .

At the beginning of each slot, the primary transmitters are distributed according to a two-dimensional Poisson point process  $X_{PT}$  with density  $\lambda_{PT}$ . To each primary transmitter, its receiver is uniformly distributed within its transmission range  $R_p$ . Here we have assumed that all the primary transmitters use the same transmission power and the transmitted signals undergo an isotropic path loss. Based on the displacement theorem [17, Chapter 5], it is easy to see that the primary receivers form another two-dimensional Poisson point process

<sup>3</sup>The different realizations of active primary transmitters in different slots can be caused by the mobility of these users or changes in the traffic pattern or both.

$X_{PR}$  with density  $\lambda_{PT}$ . Note that the two Poisson processes  $X_{PT}$  and  $X_{PR}$  are correlated.

The secondary users are distributed according to a two-dimensional Poisson point process  $X_S$  with density  $\lambda_S$ , independent of  $X_{PT}$  and  $X_{PR}$ . The locations of the secondary users are static over time. Based on the scaling argument [18, Chapter 2], we can set the transmission range  $r_p$  of the secondary users to 1 without loss of generality.

### III. CONNECTIVITY VS. INTERMITTENT CONNECTIVITY

A secondary network is disconnected if there does not exist an infinite connected component formed by topological links, where a topological link exists between two secondary users if they are within the transmission range of each other. Notice that this condition for the existence of a topological link is equivalent to the one for the existence of a communication link in homogeneous ad hoc networks. As discussed in [18, chapter 3], the connectivity of homogeneous networks, which is defined as the a.s. existence of an infinite connected component, is uniquely determined by its density. Thus, the secondary network with density  $\lambda_S$  is disconnected if and only if  $\lambda_S \leq \lambda_c$ , where  $\lambda_c$  is the critical density of homogeneous networks.

Due to the hierarchical structure of spectrum sharing, besides the density  $\lambda_S$  of the secondary users, the density  $\lambda_{PT}$  of the primary transmitters affects the connectivity of the secondary network. Specifically, in contrast to the case in homogeneous ad hoc networks, the existence of a communication link between two secondary users depends on not only the distance between them (at most  $r_p$ ) but also the availability of the communication channel (*i.e.*, the presence of a spectrum opportunity). The latter is determined by the transmitting and receiving activities of the primary network as described below.

#### A. Spectrum Opportunity

As illustrated in Fig. 2, where we consider the disk signal propagation and interference model, there exists an opportunity from  $\mu$ , the secondary transmitter, to  $\nu$ , the secondary receiver, if the transmission from  $\mu$  does not interfere with nearby *primary receivers* in the solid circle, and the reception at  $\nu$  is not affected by nearby *primary transmitters* in the dashed circle [19]. Referred to as the interference range of the secondary users, the radius  $r_I$  of the solid circle at  $\mu$  depends on the transmission power of  $\mu$  and the interference tolerance of the primary receivers, whereas the radius  $R_I$  of the dashed circle (the interference range of the primary users) depends on the transmission power of the primary users and the interference tolerance of  $\nu$ .

It is clear from the above discussion that spectrum opportunities are *asymmetric*. Specifically, a channel that is an opportunity when  $\mu$  is the transmitter and  $\nu$  the receiver may not be an opportunity when  $\nu$  is the transmitter and  $\mu$  the receiver. We consider applications with guaranteed delivery where acknowledgements are required to complete communications. Hence, bidirectional spectrum opportunities between  $\mu$  and  $\nu$  are needed. As a result, the single-hop transmission

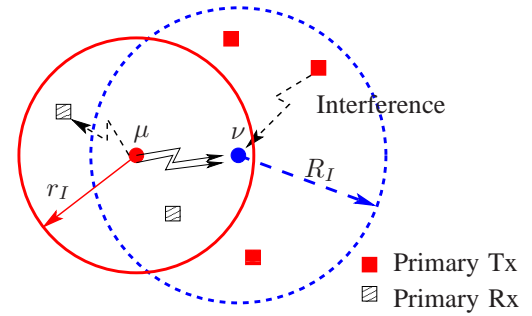


Fig. 2. Definition of spectrum opportunity.

delay from  $\mu$  to  $\nu$  is the waiting time for the presence of the first bidirectional opportunity plus the propagation delay  $\tau$ .

#### B. Connectivity and Connectivity Region

In a primary slot  $t$ , based on the conditions for the existence of a communication link, we can obtain an undirected random graph  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$  which represents the connectivity of the secondary network in this slot. As illustrated in Fig. 3, this graph  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$  is determined by the three Poisson point processes in slot  $t$ :  $X_S$ ,  $X_{PT}$ , and  $X_{PR}$ , where  $X_{PT}$  and  $X_{PR}$  are correlated.

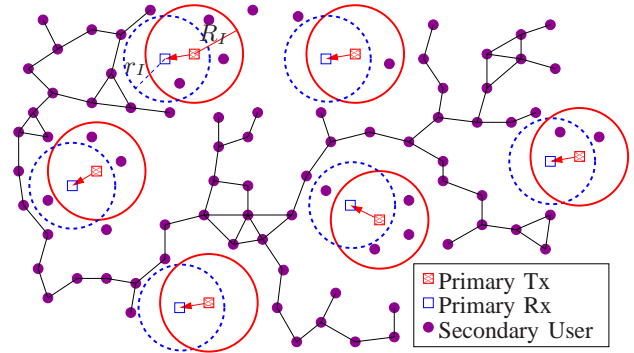


Fig. 3. A realization of the random graph  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$  which consists of all the secondary users and all the communication links in the primary slot  $t$  (denoted by solid lines). The solid circles denote the interference regions of the primary transmitters within which secondary users can not successfully receive, and the dashed circles denote the required protection regions for the primary receivers within which secondary users should refrain from transmitting.

We define the connectivity of the secondary network as the a.s. existence of an infinite connected component in  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$  for all  $t$ . Given the transmission power and the interference tolerance of both the primary and the secondary users (*i.e.*,  $R_p$ ,  $R_I$ ,  $r_p$ , and  $r_I$  are fixed), the connectivity region  $\mathcal{C}$  is defined as

$$\mathcal{C} \triangleq \{(\lambda_S, \lambda_{PT}) : \text{the secondary network is connected}\}.$$

A detailed analytical characterization of  $\mathcal{C}$  is given in [20, 21].

Referred to as the critical density of the secondary users,  $\lambda_S^*$  is the infimum density of the secondary users to ensure connectivity under a positive density of active primary transmitters:

$$\lambda_S^* \triangleq \inf\{\lambda_S : \exists \lambda_{PT} > 0 \text{ s.t. secondary network is connected}\}.$$

It is shown in [20, 21] that  $\lambda_S^*$  equals the critical density  $\lambda_c$  of a *homogeneous* ad hoc network.

### C. Intermittent Connectivity

By connecting two secondary users which are within the transmission range of each other via a topological link, we derive an undirected random graph  $\mathcal{G}_S(\lambda_S)$  which depends only on the Poisson point process  $X_S$  of the secondary network. Similarly, we define the connectivity of  $\mathcal{G}_S(\lambda_S)$  as the a.s. existence of an infinite connected component in it. It follows from the classic result on homogeneous networks [18, Chapter 3] that  $\mathcal{G}_S(\lambda_S)$  is connected if and only if  $\lambda_S > \lambda_c$ , where  $\lambda_c$  is the critical density of homogeneous networks.

$\mathcal{G}_S(\lambda_S)$  can also be obtained by adding topological links that do not see the opportunities in slot  $t$  to the random graph  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ . Thus, even if the secondary network is not connected, it is still possible that  $\mathcal{G}_S(\lambda_S)$  is connected. On the other hand, given a connected  $\mathcal{G}_S(\lambda_S)$ , there may not be enough topological links which experience the opportunities *simultaneously* to form an infinite connected component in  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$ , but for any two secondary users  $\mu$  and  $\nu$  in the infinite connected component of  $\mathcal{G}_S(\lambda_S)$ , packets from  $\mu$  can reach  $\nu$  along a path in  $\mathcal{G}_S(\lambda_S)$  with a finite multihop delay. In this case, although the transmission of the packets may not be completed in one primary slot, the packets can stay whenever the absence of the spectrum opportunities blocks their transmission and wait for some finite time to get through. The finiteness of the waiting time is guaranteed by the following two facts: (i) for each topological link in the secondary network, the probability of the spectrum opportunity is strictly positive no matter how large the density of the primary transmitters is (see [22, Appendix A]); (ii) the spectrum opportunities are time-varying due to the i.i.d. distribution of the primary network across slots. We thus define the intermittent connectivity for a secondary network that is not connected as the connectivity of  $\mathcal{G}_S(\lambda_S)$ . We also define the intermittent connectivity region  $\mathcal{C}_I$  as

$$\mathcal{C}_I \triangleq \{(\lambda_S, \lambda_{PT}) \notin \mathcal{C} : \lambda_S > \lambda_c\}.$$

### IV. MULTIHOP DELAY

In this section, we analytically characterize the asymptotic behavior of the minimum multihop delay as the source-destination distance tends to infinity. Let  $C(\mathcal{G}_S(\lambda_S))$  be the infinite connected component in  $\mathcal{G}_S(\lambda_S)$  when  $\lambda_S > \lambda_c$ , *i.e.*, the secondary network is either connected or intermittently connected. The question we aim to answer here is the scaling law of the minimum multihop delay between two arbitrary users in  $C(\mathcal{G}_S(\lambda_S))$  with respect to the distance between them. As shown in the following two theorems which consider the two cases when the propagation delay  $\tau = 0$  and  $\tau > 0$ , the connectivity of the secondary network determines the scaling law of the minimum multihop delay, where the highway provided by the infinite connected component consisting of communication links plays an indispensable role.

*Theorem 1:* Assume that  $\tau = 0$ . For any two secondary users  $\mu, \nu \in C(\mathcal{G}_S(\lambda_S))$ , where  $C(\mathcal{G}_S(\lambda_S))$  is the infinite

connected component of  $\mathcal{G}_S(\lambda_S)$ , let  $t(\mu, \nu)$  denote the minimum multihop delay from  $\mu$  to  $\nu$  and  $d(\mu, \nu)$  the distance between  $\mu$  and  $\nu$ , then

T1.1 if  $(\lambda_S, \lambda_{PT}) \in \mathcal{C}$ ,

$$\lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{g(d(\mu, \nu))} = 0 \text{ a.s.},$$

where  $g(d)$  is any monotonically increasing function of  $d$  with  $\lim_{d \rightarrow \infty} g(d) = \infty$ ;

T1.2 if  $(\lambda_S, \lambda_{PT}) \in \mathcal{C}_I$ ,  $\exists 0 < \beta < \infty$  s.t.

$$\lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{d(\mu, \nu)} = \beta \text{ a.s.}, \quad (1)$$

where the value of  $\beta$  depends on  $(\lambda_S, \lambda_{PT})$ .

*Proof Sketch:* To show T1.1, we use the infinite connected component<sup>4</sup> in  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t_0)$  during some primary slot  $t_0$  to construct a path from  $\mu$  to  $\nu$  such that the multihop delay along this path is independent of the distance  $d(\mu, \nu)$  (see Fig. 4 for an illustration). Let  $t_0$  be the first primary slot such that  $\mu$  belongs to the infinite connected component  $C(t_0)$  of  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t_0)$ , and  $w_\nu$  be the user in  $C(t_0)$  which is closest to  $\nu$ . Since the propagation delay  $\tau = 0$ , the multihop delay from  $w_\mu$  to  $w_\nu$  is zero. It follows that

$$t(\mu, \nu) = t_0 + t(w_\nu, \nu).$$

Then it suffices to show that  $t_0$  and  $t(w_\nu, \nu)$  are independent of  $d(\mu, \nu)$ , which can be done by using continuum percolation theory and ergodic theory.

To show T1.2, we first prove the existence of  $\lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{d(\mu, \nu)}$  based on the Subadditive Ergodic Theorem [23] and then derive a lower bound on  $\frac{t(\mu, \nu)}{d(\mu, \nu)}$  by considering the fact that the message from  $\mu$  can traverse only a finite distance towards  $\nu$  during each primary slot. Due to the page limit, we omit the details, which can be found in [22]. ■

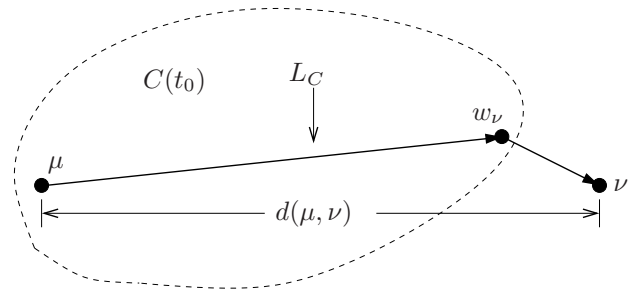


Fig. 4. An illustration of the constructed path  $L_C$  from  $\mu$  to  $\nu$  when  $(\lambda_S, \lambda_{PT}) \in \mathcal{C}$ .  $C(t_0)$  is the infinite connected component of  $\mathcal{G}(\lambda_S, \lambda_{PT}, t_0)$  which first contains  $\mu$ , and  $w_\nu$  is the user in  $C(t_0)$  which is closest to  $\nu$ .

*Theorem 2:* Assume that  $\tau > 0$ . For any two secondary users  $\mu, \nu \in C(\mathcal{G}_S(\lambda_S))$ , where  $C(\mathcal{G}_S(\lambda_S))$  is the infinite connected component of  $\mathcal{G}_S(\lambda_S)$ , let  $t^\tau(\mu, \nu)$  denote the

<sup>4</sup>It is shown in [20, 21] that there exists either zero or one infinite connected component in  $\mathcal{G}_H(\lambda_S, \lambda_{PT}, t)$  a.s. for any given  $t$ .

minimum multihop delay from  $\mu$  to  $\nu$  and  $d(\mu, \nu)$  the distance between  $\mu$  and  $\nu$ , then  $\exists \gamma = \gamma(\tau) > 0$  s.t.

$$\lim_{d(\mu, \nu) \rightarrow \infty} \frac{t^\tau(\mu, \nu)}{d(\mu, \nu)} = \gamma \geq \tau \text{ a.s.} \quad (2)$$

Furthermore, if  $(\lambda_S, \lambda_{PT}) \in \mathcal{C}$ ,

$$\lim_{\tau \rightarrow 0} \lim_{d(\mu, \nu) \rightarrow \infty} \frac{t^\tau(\mu, \nu)}{d(\mu, \nu)} = 0 \text{ a.s.}; \quad (3)$$

if  $(\lambda_S, \lambda_{PT}) \in \mathcal{C}_I$ ,

$$\lim_{\tau \rightarrow 0} \lim_{d(\mu, \nu) \rightarrow \infty} \frac{t^\tau(\mu, \nu)}{d(\mu, \nu)} \geq \beta > 0 \text{ a.s.}, \quad (4)$$

where  $\beta = \lim_{d(\mu, \nu) \rightarrow \infty} \frac{t(\mu, \nu)}{d(\mu, \nu)}$  is defined in (1).

*Proof Sketch:* The equality in (2) is based on the Subadditive Ergodic Theorem [23], while the inequality in (2) is established via a simple lower bound on  $t^\tau(\mu, \nu)$ . The basic idea behind establishing (3) is to consider the multihop delay along the path constructed in Fig. 4. Eqn. (4) follows immediately from the fact that  $t^\tau(\mu, \nu) \geq t(\mu, \nu)$ , where  $t(\mu, \nu)$  is the minimum multihop delay when  $\tau = 0$ . The details are omitted due to the page limit, and are given in [22]. ■

## V. CONCLUSION

We have studied the connectivity and multihop delay of ad hoc cognitive radio networks. The criterion for connectivity is the occurrence of percolation, *i.e.*, the almost sure existence of an infinite connected component. The impact of connectivity on the multihop delay has been examined by establishing the asymptotic behavior of the minimum multihop delay as the source-destination distance tends to infinity. Specifically, depending on whether the cognitive radio network is connected or intermittently connected, the scaling of the minimum multihop delay behaves distinctly, in terms of either the scaling law when the propagation delay is negligible or the scaling rate when the propagation delay is nonnegligible. This result on scaling is independent of the random positions of the source and the destination, and it only depends on the network parameters (e.g., the density of the secondary users and the traffic load of the primary network). In establishing these results, we have used theories and techniques from continuum percolation and ergodicity including the concept of critical density and the Subadditive Ergodic Theorem.

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