Connectivity of Cognitive Radio Networks:

Proximity vs. Opportunity

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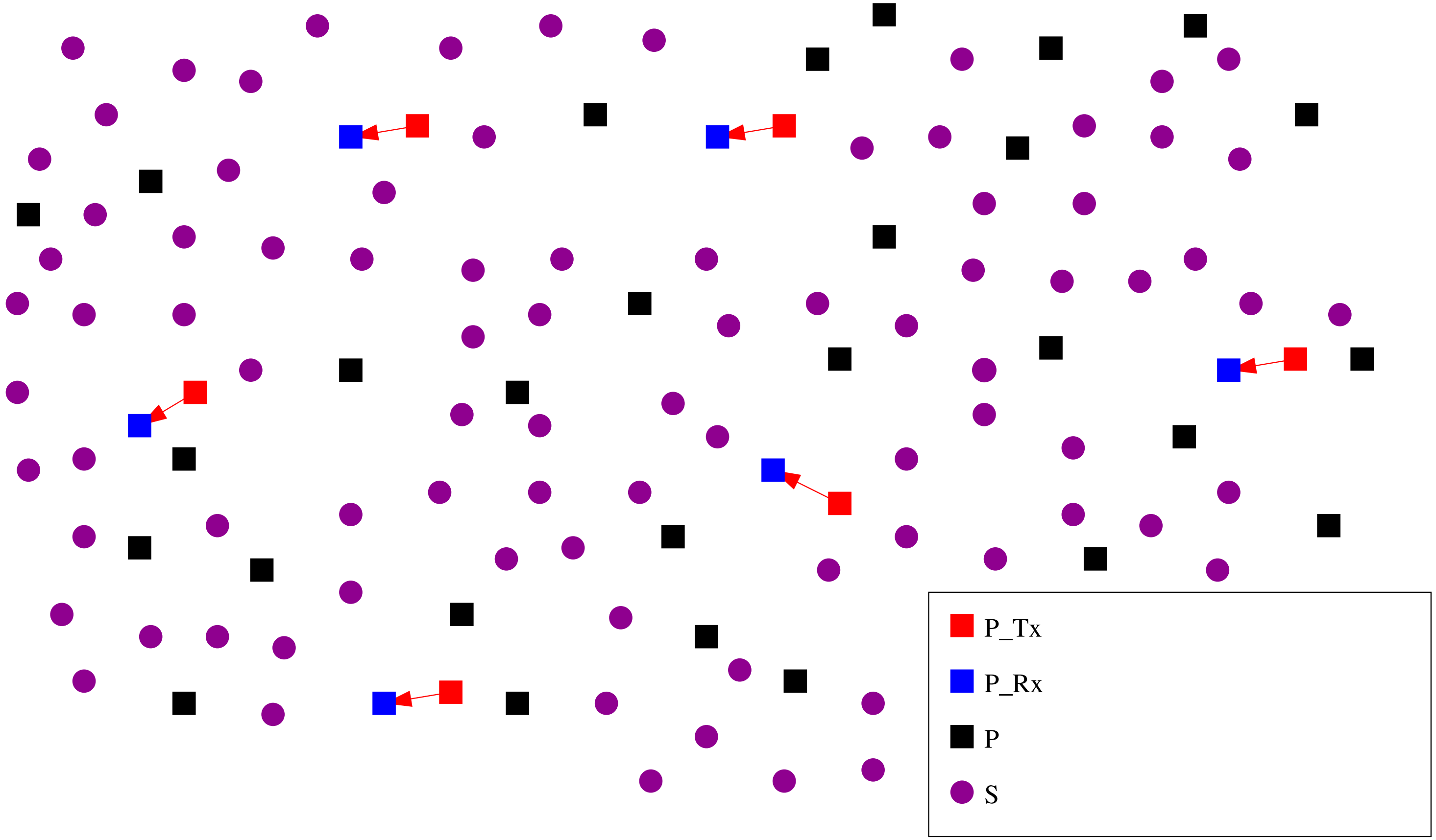
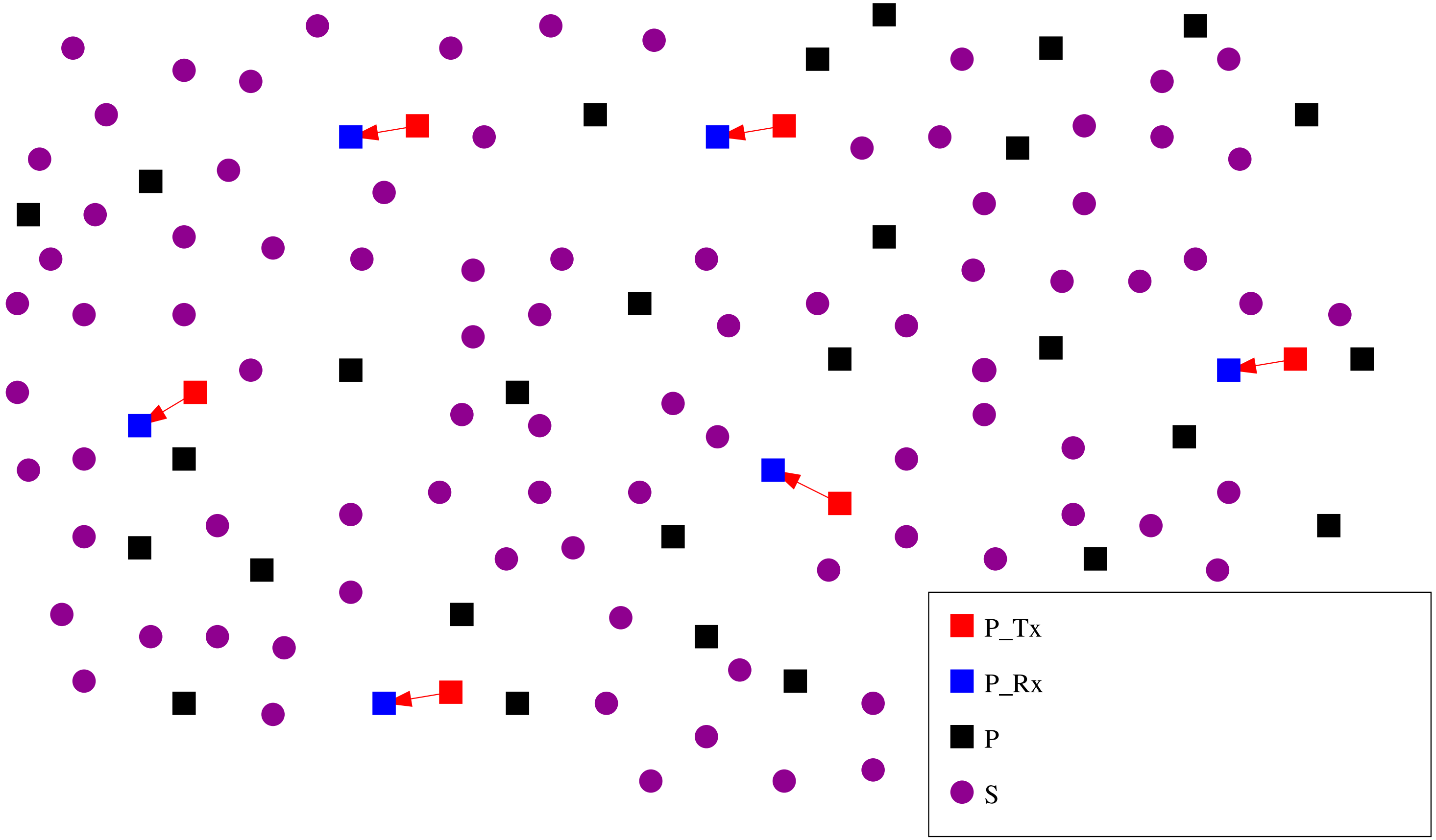
Supported by ARL, ARO, and NSF.

Connectivity: Poisson Primary + Poisson Secondary



Connectivity: the existence of an infinite connected component almost surely.

Connectivity

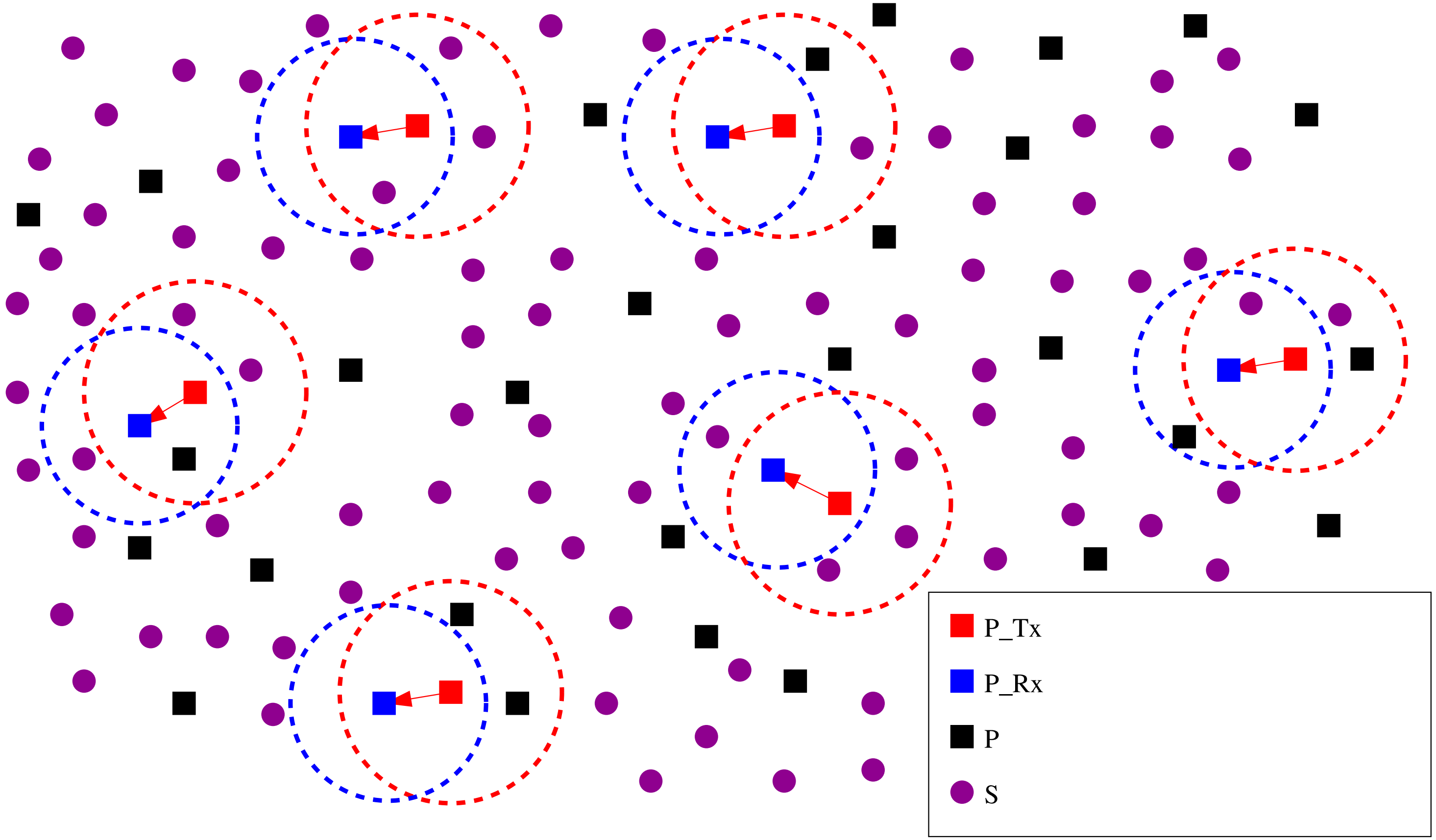


Connectivity: the existence of an infinite connected component almost surely.

Existence of A Link between Two Secondary Users:

- they are within Tx range;
- they see a bidirectional opportunity.

Who Sees An Opportunity Who Doesn't?



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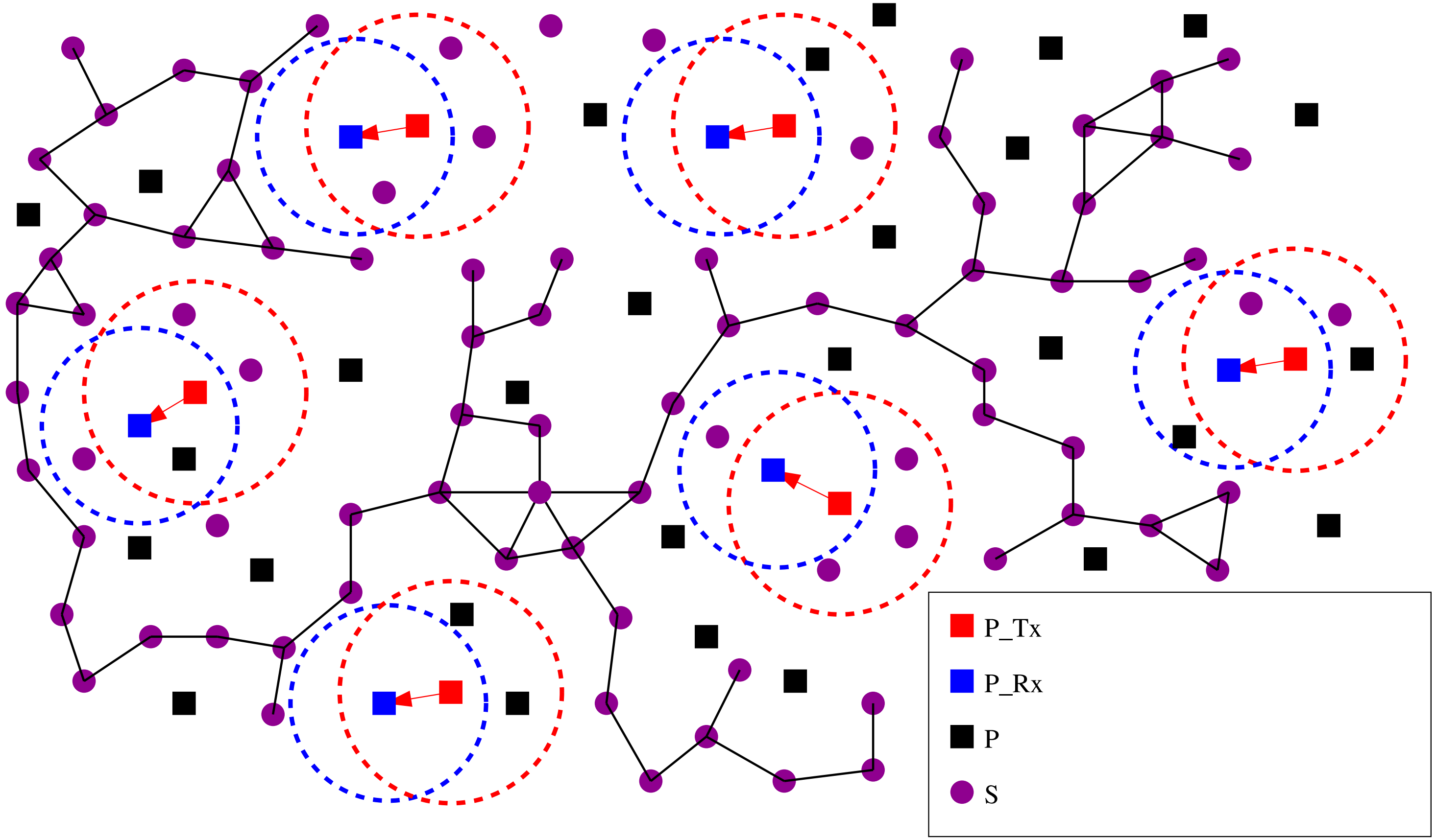

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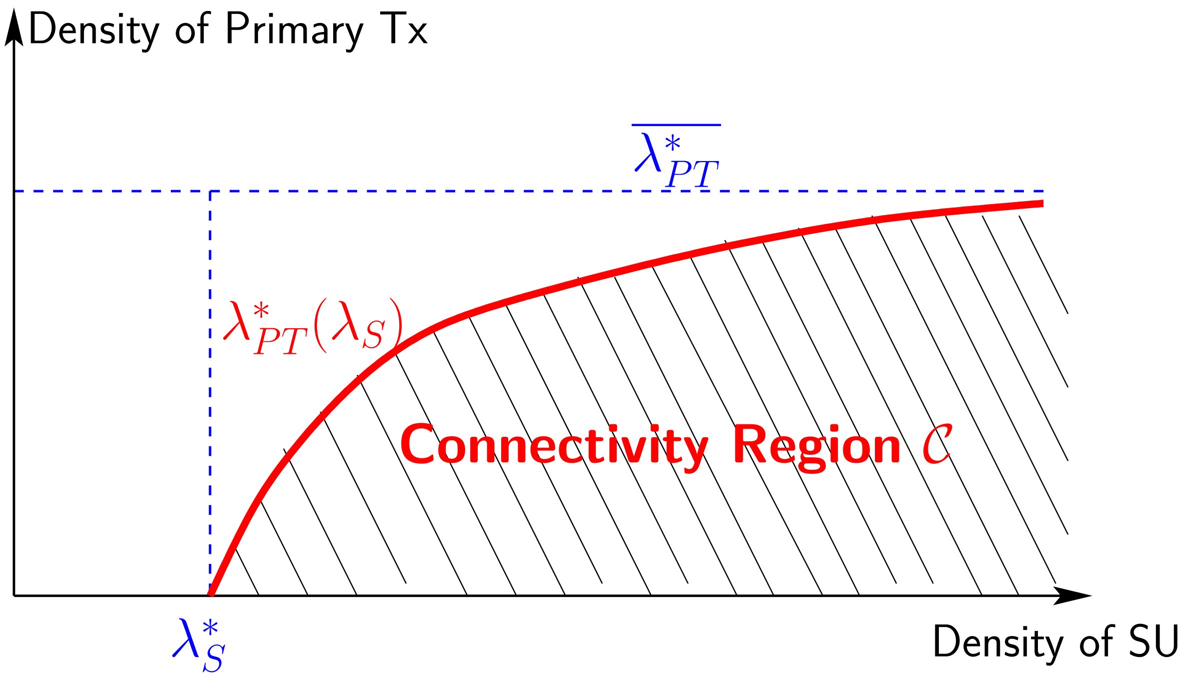
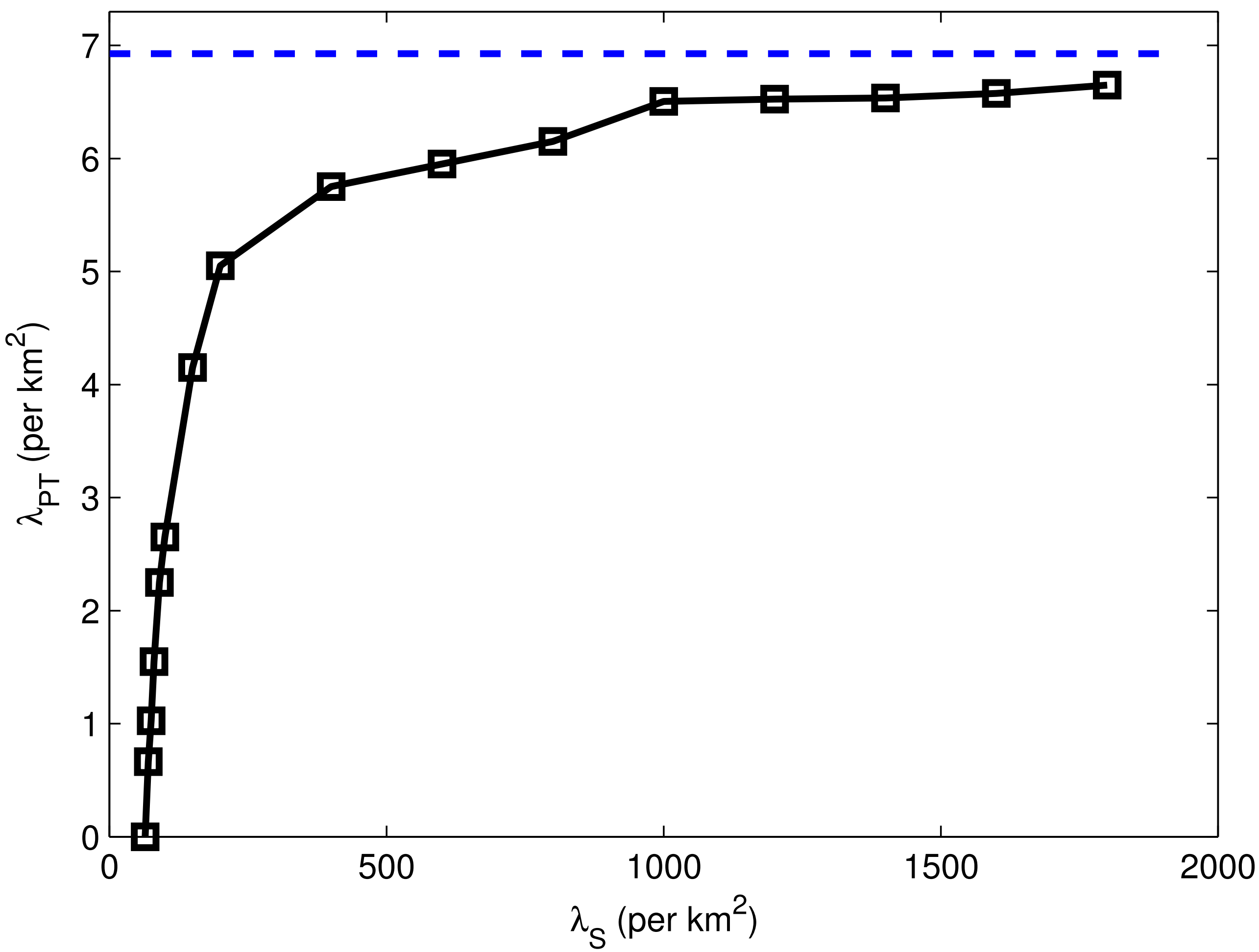
Connectivity Region



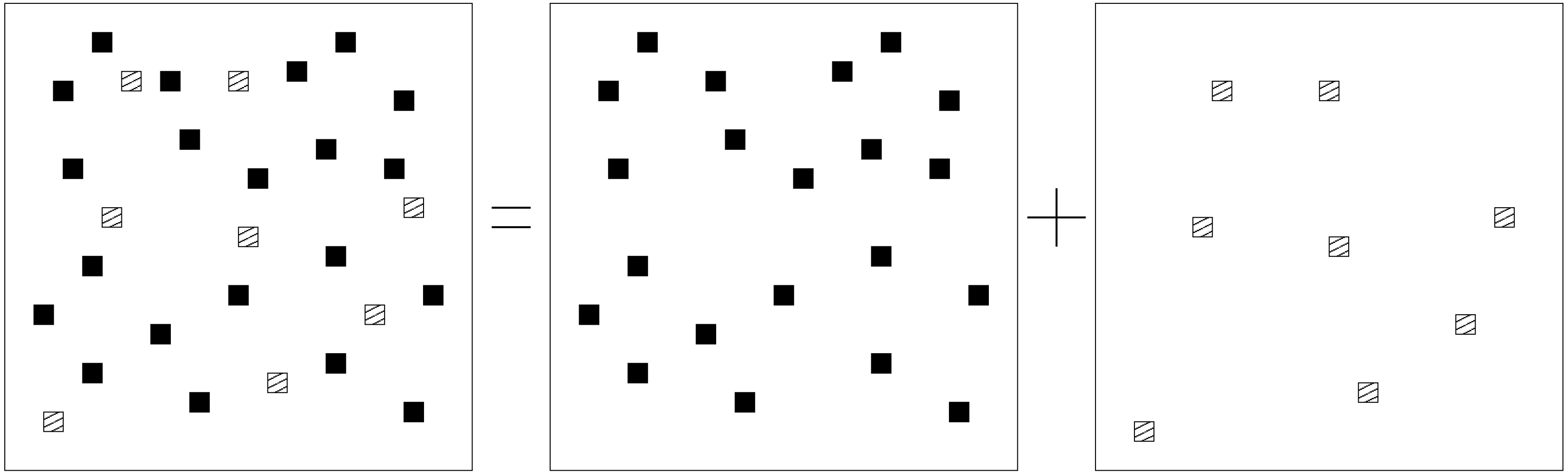
- The connectivity region $\mathcal{C}$ is contiguous.
- $\lambda^*_{PT}(\lambda_S)$ monotonically increases with λ_S.
- $\forall(\lambda_S, \lambda_{PT}) \in \mathcal{C}$, there exists a *unique* infinite connected component.
- The critical density of secondary users: $\lambda^*_S = \lambda_c(r_p)$ (*CD of homogenous networks*).
- The critical density of primary transmitters: $\overline{\lambda^*_{PT}} \leq \frac{\lambda_c(1)}{4\max\{R_I^2, r_I^2\} - r_p^2}$.

Critical Density of Primary Transmitters



Proof Techniques

- Contiguity and monotonicity: coupling argument



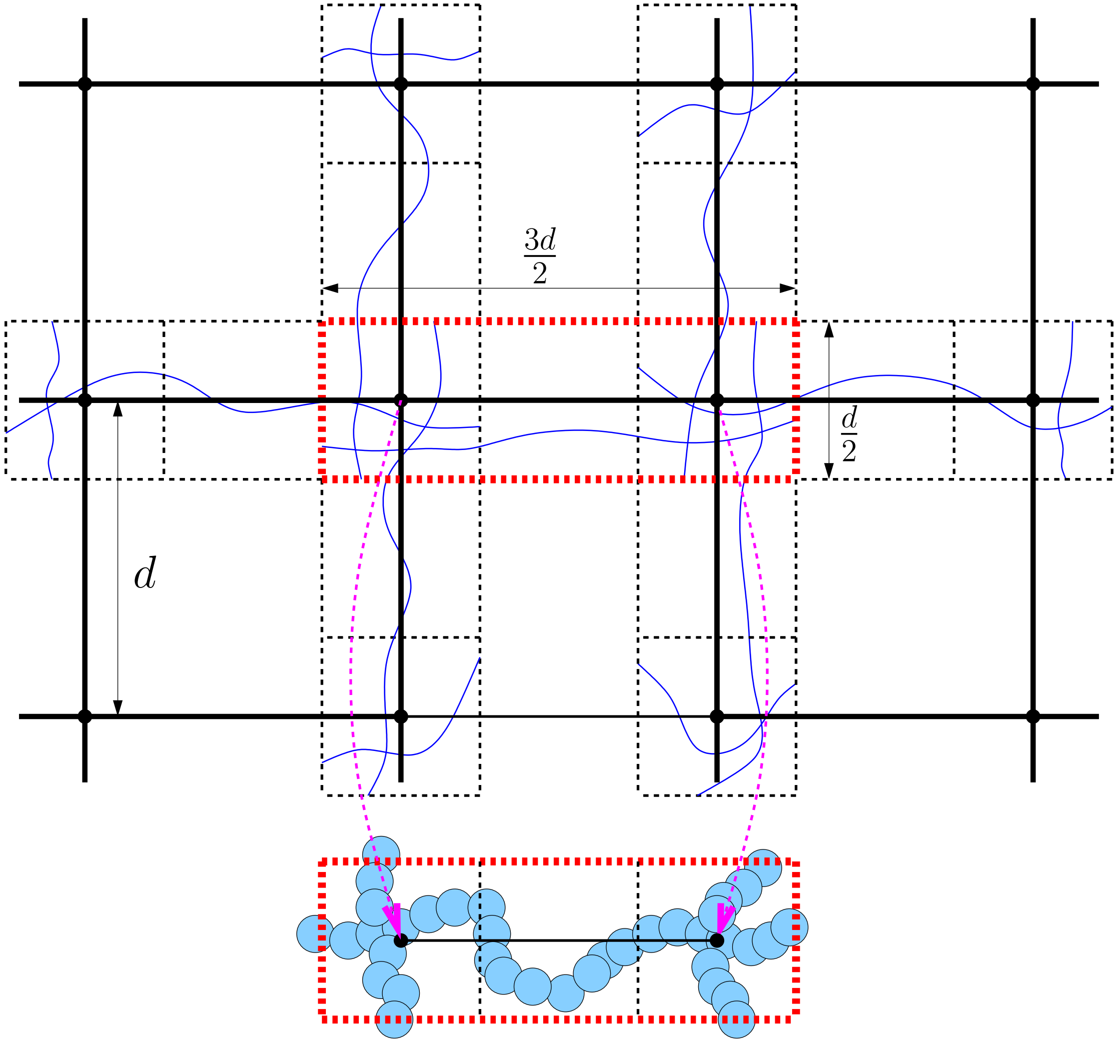
Poisson(λ_2) = Poisson(λ_1) + Poisson($\lambda_2 - \lambda_1$)

- Uniqueness of the infinite connected component
  - Ergodic theory $\Rightarrow$ # of infinite connected components $=$ constant a.s.
  - Contradiction $\Rightarrow$ # of infinite connected components $\in \{0, 1, \infty\}$ a.s.
  - Combinatorics $\Rightarrow$ # of infinite connected components $\neq \infty$ a.s.

Proof Techniques

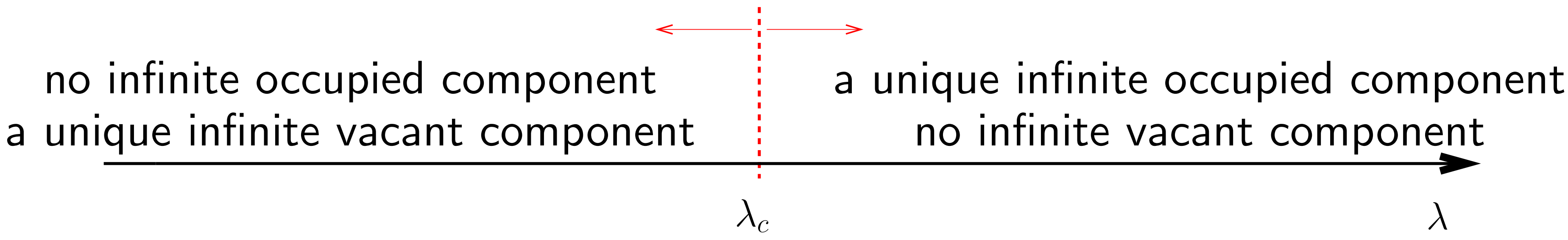
- Critical density of secondary users

Discretize the continuum model into a dependent edge-percolation model.



Proof Techniques

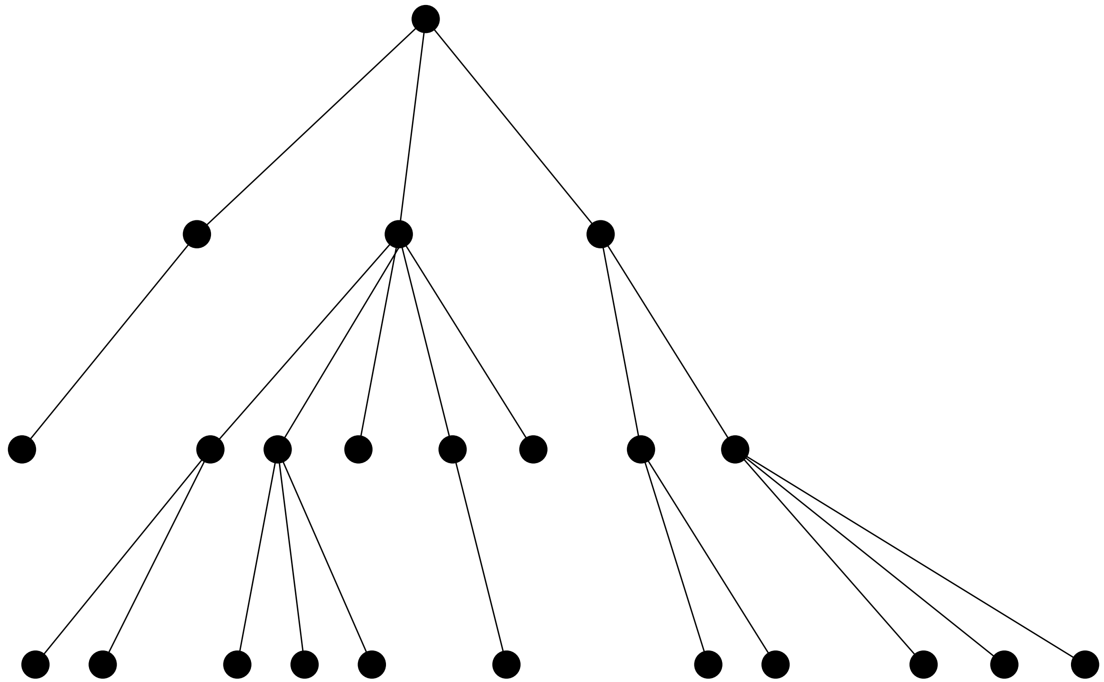
- Critical density of primary transmitters
  - An infinite connected component in the secondary network ⇒ An infinite vacant component in the two Poisson Boolean models driven by the primary transmitters and receivers.
  - Sharp transitions for two-dimensional Poisson Boolean models



Outer Bound on Connectivity Region

- Necessary condition for connectivity: conditional average degree $\mu > 1$.

Proof: construct a branching process, where $\mu = \mathbb{E}[\#\text{offspring}]$.



- Conditional average degree μ

$$\begin{aligned} \mu &= \mathbb{E}[\text{degree} | \text{ the secondary user sees the opportunity}] \\ &= (\lambda_S \pi r_p^2) \cdot \Pr\{\text{opportunity} | \text{ one secondary user sees the opportunity}\}. \end{aligned}$$

Inner Bound on Connectivity Region

- Sufficient condition for connectivity:

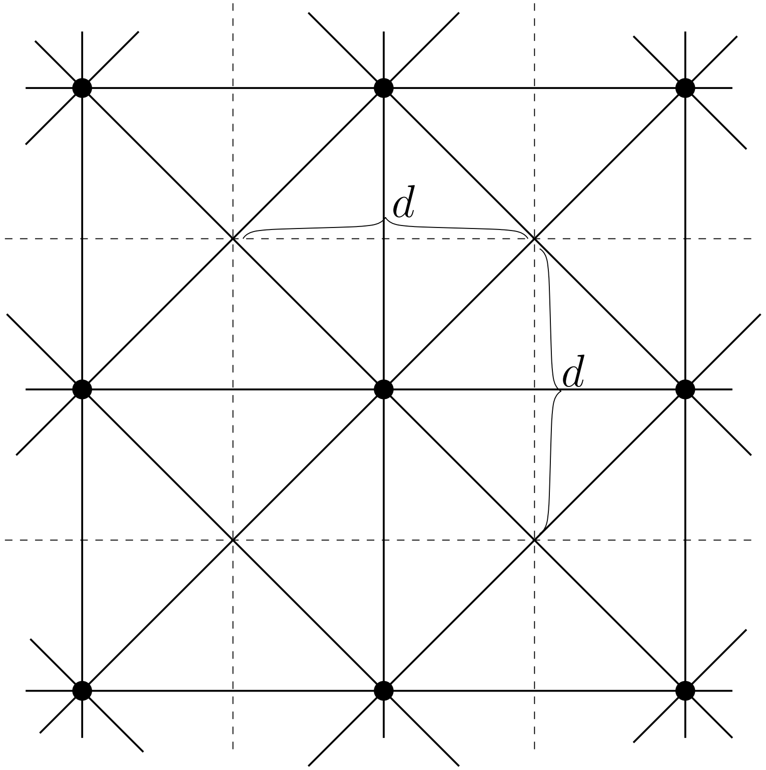
$$\left[1 - \exp\left(-\frac{\lambda_S r_p^2}{8}\right)\right] \exp\left\{-\lambda_{PT}\pi\left[R_I^2 + r_I^2 - I\left(R_I, R_p, r_I\right)\right]\right\} > p_c,$$

where

$$I(r, R_p, r_I) = 2\int_0^r t\frac{S_I(t, R_p, r_I)}{\pi R_p^2}\mathrm{d}t,$$

$S_I(t, R_p, r_I)$ is the common area of two circles with radii R_p and r_I and centered t apart, and p_c is the upper critical probability of a constructed dependent site-percolation model $\mathcal{L}$.

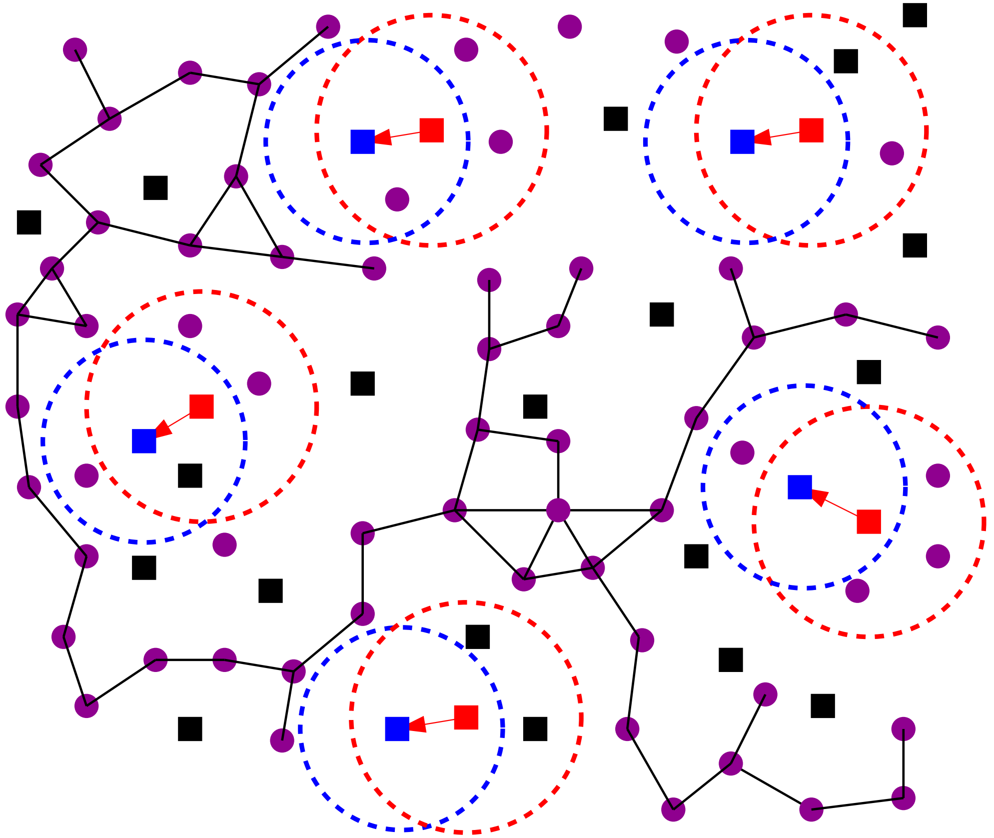
Proof: the ergodicity of the network model and its relation with $\mathcal{L}$.



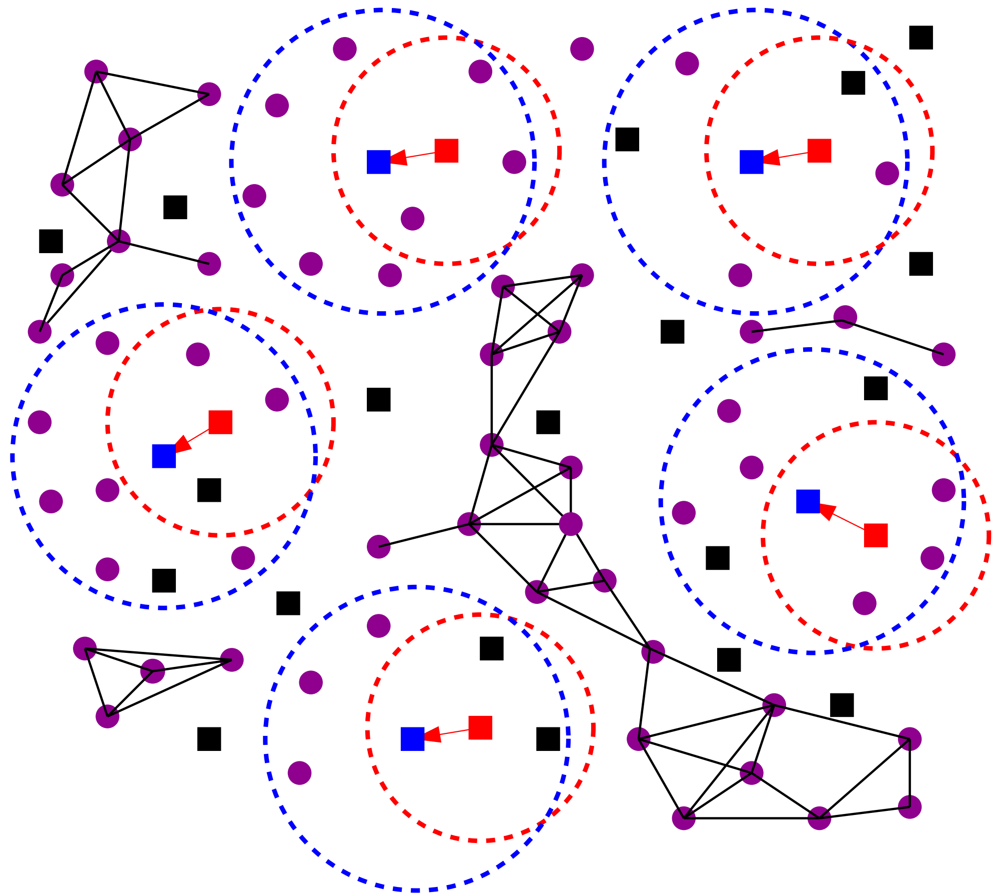
Proximity vs. Opportunity

Increasing p_{tx} leads to more neighbors but fewer opportunities.

Small p_{tx}



Large p_{tx}



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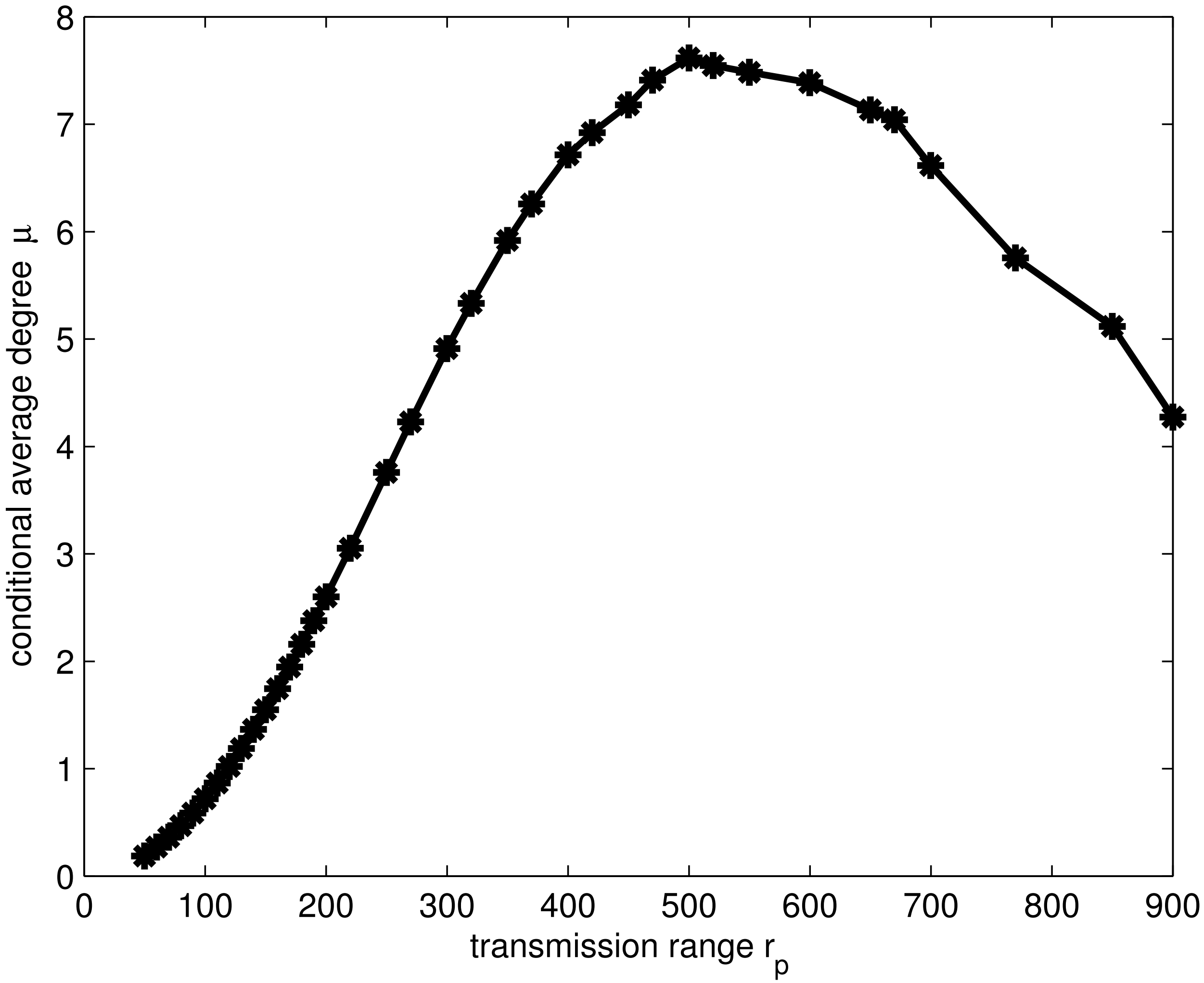

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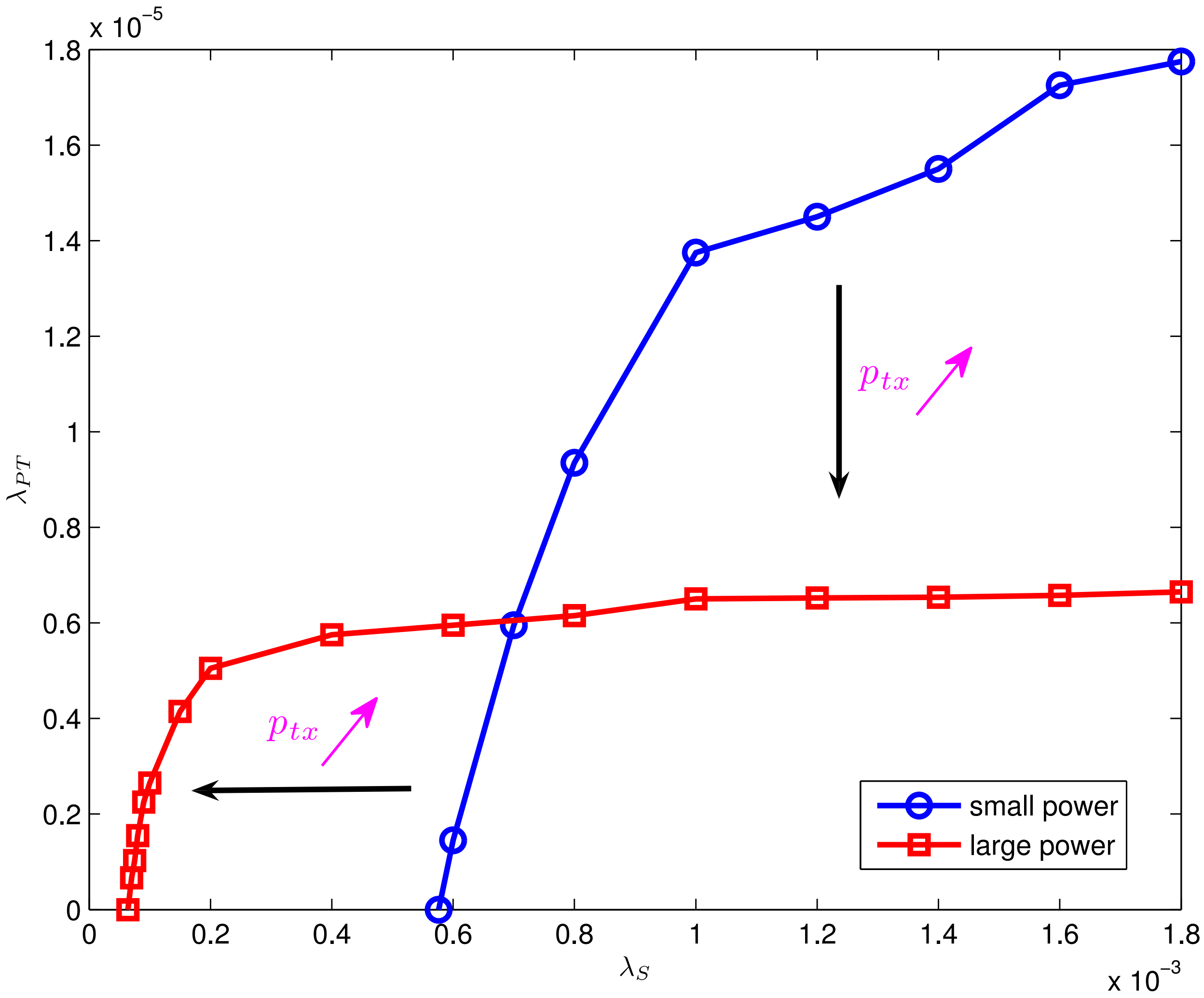
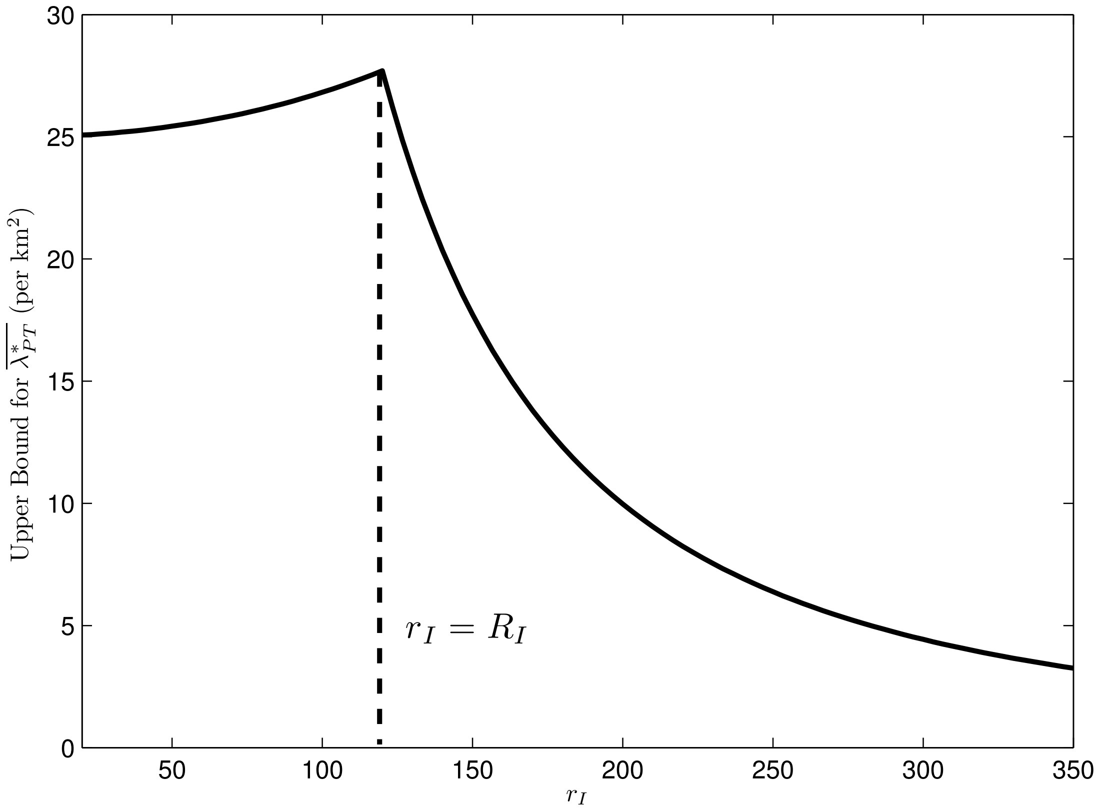
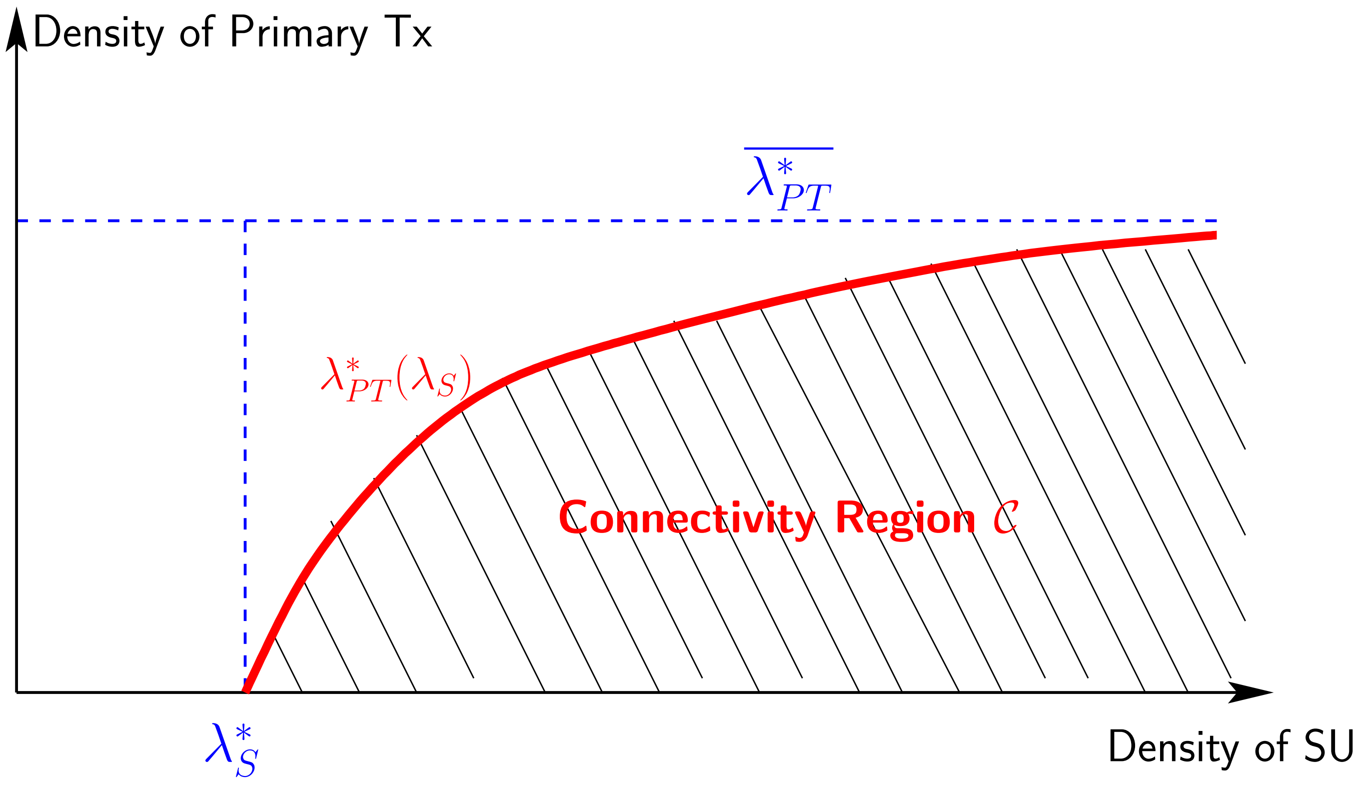


Optimal Transmission Power



To coexist with heavy traffic load: Tx power matching between primary and secondary.

Conclusion


Density of Primary Tx
$\overline{\lambda_{PT}^*}$
$\lambda_{PT}^*(\lambda_S)$
Connectivity Region $\mathcal{C}$
λ_S^*
Density of SU