Connectivity of Cognitive Radio Networks: Proximity vs. Opportunity

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ABSTRACT

We address the connectivity of large-scale ad hoc cognitive radio networks, where secondary users exploit channels temporarily and locally unused by primary users and the existence of a communication link between two secondary users depends not only on the distance between them but also on the transmitting and receiving activities of nearby primary users. We introduce the concept of *connectivity region* defined as the set of density pairs — the density of the secondary users and the density of the primary transmitters — under which the secondary network is connected. Using theories and techniques from continuum percolation, we analytically characterize the connectivity region of the secondary network by showing its three basic properties and analyzing its two critical parameters. Furthermore, we reveal the tradeoff between proximity (the number of neighbors) and the occurrence of spectrum opportunities by studying the impact of the secondary users' transmission power on the connectivity region of the secondary network, and design the transmission power of the secondary users to maximize their tolerance to the primary traffic load.

Categories and Subject Descriptors

H.1.0 [**Information Systems**]: Models and Principles—*General*; F.0 [**Theory of Computation**]: General

General Terms

Algorithms, Design, Theory

Keywords

Cognitive radio, connectivity, continuum percolation

1. INTRODUCTION

The basic idea of opportunistic spectrum access (OSA) is to adopt a dynamic and hierarchical structure for spectrum sharing and interference management. Specifically, a secondary network is overlaid with a primary network, where secondary users identify and exploit temporarily and locally unused channels without causing unacceptable interference to primary users [18].

1.1 Connectivity and Connectivity Region

While the connectivity of homogeneous ad hoc networks consisting of equal-priority users has been well studied (see, for example, [1–3, 6, 12, 13]), little is known about the connectivity of large heterogeneous networks with interdependent, interactive, and hierarchical network components with different priorities such as cognitive radio (CR) networks. The problem is fundamentally different from its counterpart in homogeneous networks. In particular, the connectivity of the low priority network component depends on the characteristics (traffic pattern/load, topology, interference tolerance, etc.) of the high priority component, thus creating a much more diverse and complex design space.

Using theories and techniques from continuum percolation, we analytically characterize the connectivity of large-scale ad hoc CR networks[1]. Specifically, we consider a Poisson distributed secondary network overlaid with a Poisson distributed primary network in an infinite two dimensional Euclidean space[2]. We define *network connectivity* as the existence of an infinite connected component almost surely (a.s.), *i.e.,* the occurrence of percolation. We say that the secondary network is strongly connected when it contains a *unique* infinite connected component a.s.

Due to the hierarchical structure of spectrum sharing, a communication link exists between two secondary users if the following two conditions hold: (C1) they are within each other's transmission range; (C2) they see a spectrum opportunity determined by the transmitting and receiving activities of nearby primary users (see Sec. 2.2.1). Thus, given the transmission power and the interference tolerance[3] of both the primary and the secondary users, the connectivity of the secondary network depends on the density of the secondary users (due to (C1)) and the traffic load of the primary

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[1]The notions of cognitive radio networks and secondary networks are used interchangeably in this paper.

[2]This infinite network model is equivalent in distribution to the limit of a sequence of finite networks with a fixed density as the area of the network increases to infinity, *i.e.,* the so-called *extended network* [10]. It follows from the arguments similar to the ones used in [4, Chapter 3] for homogeneous ad hoc networks that this infinite ad hoc CR network model represents the limiting behavior of large-scale ad hoc CR networks.

[3]The interference tolerance of users is defined as the maximum allowable interference power received by a user such that the user can successfully decode the message transmitted by another user at the farthest distance (*i.e.,* the transmission range) to the receiver.

users (due to (C2)).

We thus introduce the concept of *connectivity region* $\mathcal{C}$, defined as the set of density pairs $(\lambda_S, \lambda_{PT})$ under which the secondary network is connected, where λ_S denotes the density of the secondary users and λ_{PT} the density of primary transmitters (representing the traffic load of the primary users). As illustrated in Fig. 1, a secondary network with a density pair $(\lambda_S, \lambda_{PT})$ inside this region is connected: the network has a "giant" connected component which includes infinite secondary users. The existence of the "giant" connected component enables bidirectional communications between distant secondary users via multihop relaying. On the other hand, a secondary network with a density pair $(\lambda_S, \lambda_{PT})$ outside this region is not connected: the network is separated into an infinite number of *finite* connected components. Consequently, any secondary user can only communicate with users within a limited range.

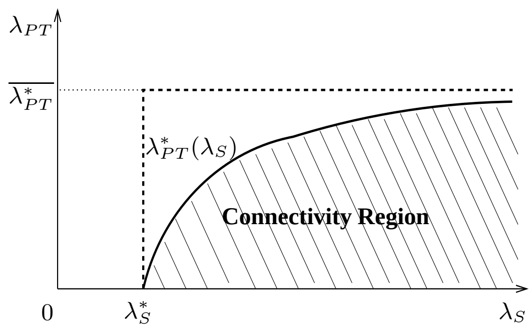

Figure 1: The connectivity region $\mathcal{C}$ (the upper boundary $\lambda^*_{PT}(\lambda_S)$ is defined as the supremum density of the primary transmitters to ensure connectivity with a *fixed* density of the secondary users; the critical density λ^*_S of the secondary users is defined as the infimum density of the secondary users to ensure connectivity under a *positive* density of the primary transmitters; the critical density $\overline{\lambda^*_{PT}}$ of the primary transmitters the supremum density of the primary transmitters to ensure connectivity with a *finite* density of the secondary users).

We first establish three basic properties of the connectivity region: contiguity, monotonicity of the boundary, and uniqueness of the infinite connected component. Specifically, based on a coupling argument, we show that the connectivity region is a contiguous area bounded below by the λ_S-axis and bounded above by a monotonically increasing function $\lambda^*_{PT}(\lambda_S)$ (see Fig. 1), where the upper boundary $\lambda^*_{PT}(\lambda_S)$ is defined as

$$\lambda^*_{PT}(\lambda_S) \triangleq \sup\{\lambda_{PT} : \ \mathcal{G}(\lambda_S, \lambda_{PT}) \text{ is connected.}\},$$

with $\mathcal{G}(\lambda_S, \lambda_{PT})$ denoting the secondary network of density λ_S overlaid with a primary network specified by the density λ_{PT} of the primary transmitters. The uniqueness of the infinite connected component is established based on the ergodic theory and certain combinatorial results. It shows that once the secondary network is connected, it is strongly connected.

Second, we define and analyze two critical parameters of the connectivity region: λ^*_S and $\overline{\lambda^*_{PT}}$. They jointly specify the profile as well as an outer bound on the connectivity region. Referred to as the critical density of the secondary users, λ^*_S is the infimum density of the secondary users to ensure connectivity under a positive density of the primary transmitters:

$$\lambda^*_S \triangleq \inf\{\lambda_S : \exists \lambda_{PT} > 0 \text{ s.t. } \mathcal{G}(\lambda_S, \lambda_{PT}) \text{ is connected}\}.$$

We show that λ^*_S equals the critical density λ_c of a *homogeneous* ad hoc network (*i.e.*, in the absence of primary users), which has been well studied [11]. This result shows that the "takeoff" point in the connectivity region is completely determined by the effect of proximity—the number of neighbors (nodes within the transmission range of a secondary user).

Referred to as the critical density of the primary transmitters, $\overline{\lambda^*_{PT}}$ is the supremum density of the primary transmitters to ensure the connectivity of the secondary network with a finite density of the secondary users:

$$\overline{\lambda^*_{PT}} \triangleq \sup\{\lambda_{PT} : \exists \lambda_S < \infty \text{ s.t. } \mathcal{G}(\lambda_S, \lambda_{PT}) \text{ is connected}\}.$$

We obtain an upper bound on $\overline{\lambda^*_{PT}}$ which is shown to be achievable in simulations. More importantly, this result shows that when the density of the primary transmitters is higher than the (finite) value given by this upper bound, the secondary network cannot be connected no matter how dense it is. This parameter $\overline{\lambda^*_{PT}}$ thus characterizes the impact of opportunity occurrence on the connectivity of the secondary network: when the density of the primary transmitters is beyond a certain level, there are simply not enough spectrum opportunities for any secondary network to be connected.

1.2 Impact of Transmission Power: Proximity vs. Opportunity

Following the analytical characterizations of the connectivity region, we study the impact of system design parameters, in particular, the transmission power p_{tx} of the secondary users on the connectivity region. We reveal an interesting tradeoff between proximity and opportunity in the design of CR networks. As illustrated in Fig. 2, we show that increasing p_{tx} enlarges the connectivity region $\mathcal{C}$ in the λ_S-axis (*i.e.*, better proximity leads to a smaller "takeoff" point), but at the price of reducing $\mathcal{C}$ in the λ_{PT}-axis. Specifically, with a large p_{tx}, few secondary users experience spectrum opportunities due to their large interference range with respect to the primary users. This leads to a poor tolerance to the primary traffic load parameterized by λ_{PT}.

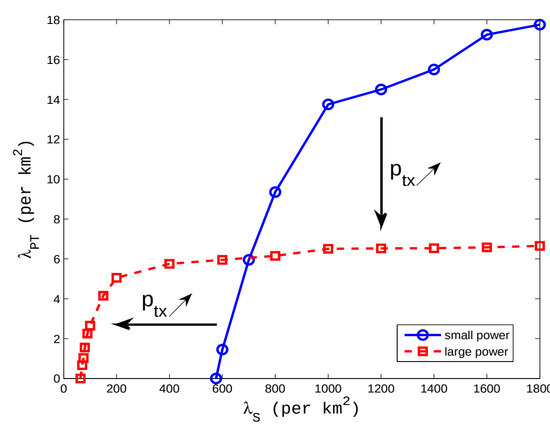

Figure 2: Simulated connectivity regions for two different transmission powers (p_{tx} denotes the transmission power of secondary users, and the large p_{tx} is 3^α times the small p_{tx}, where α is the path-loss exponent).

The transmission power p_{tx} of the secondary users should thus be chosen according to the operating point of the CR network given by the density of the secondary users and the traffic load of the co-existing primary users. Using the tolerance to the primary traffic load as the performance measure, we show that the interference range r_I of the secondary users should be equal to the interference range R_I of the primary users in order to maximize the upper bound on the critical density $\overline{\lambda^*_{PT}}$ of the primary transmitters. Given the interference tolerance of both the primary and the secondary users, we can then design the optimal transmission power p_{tx} of the secondary users based on that of the primary users.

1.3 Related Work

To our best knowledge, the connectivity of large-scale ad hoc CR networks has not been characterized analytically or experimentally in the literature. There are a number of classic results on the connectivity of homogeneous ad hoc networks. For example, it has been shown that to ensure either 1-connectivity (there exists a path between any pair of nodes) [6, 7, 13] or k-connectivity (there exist at least k node-disjoint paths between any pair of nodes) [1], the average number of neighbors of each node must increase with the network size. On the other hand, to maintain a weaker connectivity – p-connectivity (*i.e.*, the probability that any pair of nodes is connected is at least p), the average number of neighbors is only required to be above a certain 'magic number' which does not depend on the network size [12].

The theory of continuum percolation has been used by Dousse *et al.* in analyzing the connectivity of a homogeneous ad hoc network under the worst case mutual interference [2, 3]. In [9], the connectivity and the transmission delay in a homogeneous ad hoc network with statically or dynamically on-off links are investigated from a percolation-based perspective.

The optimal power control in CR networks has been studied in [16], which focuses on a single pair of secondary users in a Poisson network of primary users. The impacts of secondary users' transmission power on the occurrence of spectrum opportunities and the reliability of opportunity detection are analytically characterized.

2. NETWORK MODEL

We consider a Poisson distributed secondary network overlaid with a Poisson distributed primary network in an infinite two dimensional Euclidean space. The models of the primary and secondary networks are specified in the following two subsections.

2.1 The Primary Network

The primary transmitters are distributed according to a two dimensional Poisson point process with density λ_{PT}. To each primary transmitter, its receiver is uniformly distributed within its transmission range R_p. Here we have assumed that every primary transmitter uses the same transmission power and the transmitted signal undergoes an isotropic path loss. Based on the displacement theorem [8, Chapter 5], it is easy to see that the primary receivers also form a two dimensional Poisson point process with density λ_{PT}. Note that the two Poisson processes formed by the primary transmitters and receivers are correlated.

2.2 The Secondary Network

The secondary users are distributed according to a two dimensional Poisson point process with density λ_S, independent of the two Poisson processes of the primary transmitters and receivers. The transmission range of the secondary users is denoted by r_p.

2.2.1 Communication Links

In contrast to a homogeneous ad hoc network, the existence of a communication link between two secondary users depends not only on the distance between them but also on the availability of the communication channel (*i.e.*, the presence of a spectrum opportunity). The latter is determined by the transmitting and receiving activities in the primary network as described below.

As illustrated in Fig. 3, where we consider the disk signal propagation and interference model, there exists an opportunity from A, the secondary transmitter, and B, the secondary receiver, if the transmission from A does not interfere with nearby *primary receivers* in the solid circle, and the reception at B is not affected by nearby *primary transmitters* in the dashed circle [17]. Referred to as the interference range of the secondary users[4], the radius r_I of the solid circle at A depends on the transmission power of A and the interference tolerance of the primary receivers, whereas the radius R_I of the dashed circle (the interference range of the primary users[5]) depends on the transmission power of the primary users and the interference tolerance of B.

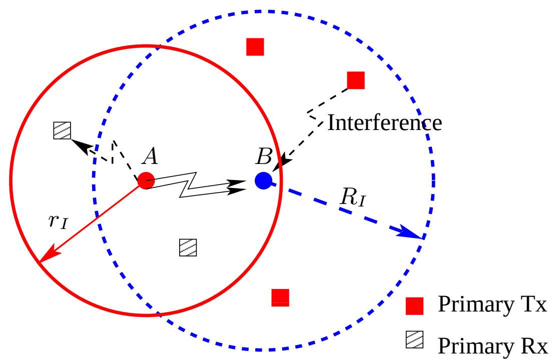

Figure 3: Definition of spectrum opportunity.

It is clear from the above discussion that spectrum opportunities depend on both transmitting and receiving activities of the primary users. Furthermore, spectrum opportunities are *asymmetric*. Specifically, a channel that is an opportunity when A is the transmitter and B the receiver may not be an opportunity when B is the transmitter and A the receiver. In other words, there exist unidirectional communication links in the secondary network. Since unidirectional links are difficult to utilize in wireless networks [14], we only consider bidirectional links in the secondary network when we define connectivity. As a consequence, when we determine whether there exists a communication link between two secondary users, we need to check the existence of spectrum opportunities in both directions.

To summarize, under the disk signal propagation and interference model, there is a (bidirectional) link between A and B if and only if (i) the distance between A and B is at most r_p; (ii) there exists a bidirectional spectrum opportunity between A and B, $i.e.$, there are no primary transmitters within distance R_I of either A or B and no primary receivers within distance r_I of either A or B.

[4]The interference range of the secondary users is defined as the maximum distance from a secondary transmitter to a primary user such that the interference of the secondary transmitter to the primary user is above the interference tolerance of the primary users. By considering the definition of the interference tolerance, we have that as long as the primary receiver is not within the interference range of the secondary transmitter and it is within the transmission range of the primary transmitter, the primary receiver can successfully decode the message from the primary transmitter no matter the distance between the primary transmitter and the primary receiver.

[5]The interference range of the primary users is defined similarly to the interference range of the secondary users.

2.2.2 Connectivity

We interpret the connectivity of the secondary network in the percolation sense: the secondary network is connected if there exists an infinite connected component a.s.

Based on the above conditions (i, ii) for the existence of a communication link, we can obtain an undirected random graph $\mathcal{G}(\lambda_S, \lambda_{PT})$ corresponding to the secondary network, which is determined by three Poisson point processes: the secondary users with density λ_S, the primary transmitters with density λ_{PT}, and the primary receivers with density λ_{PT} (correlated to the process of the primary transmitters)[6]. See Fig. 4 for an illustration of $\mathcal{G}(\lambda_S, \lambda_{PT})$.

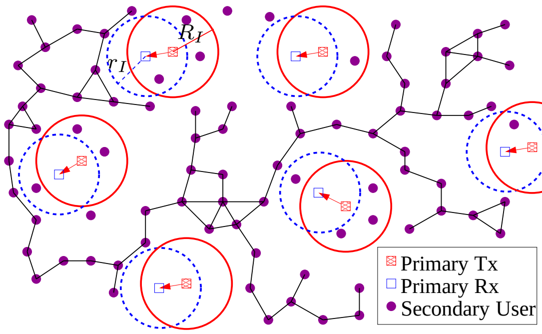

Figure 4: A realization of the CR network. The random graph $\mathcal{G}(\lambda_S, \lambda_{PT})$ consists of all the secondary nodes and all the bidirectional links denoted by solid lines. The solid circles denote the interference regions of the primary transmitters within which secondary users can not successfully receive, and the dashed circles denote the required protection regions for the primary receivers within which secondary users should refrain from transmitting.

The question we aim to answer in this paper is the connectivity of the secondary network, *i.e.*, the percolation in $\mathcal{G}(\lambda_S, \lambda_{PT})$.

3. ANALYTICAL CHARACTERIZATIONS OF THE CONNECTIVITY REGION

Given the transmission power and the interference tolerance of both the primary and the secondary users ($i.e.$, R_p, R_I, r_p, and r_I are fixed), the connectivity of the secondary network depends on the density λ_S of the secondary users and the density λ_{PT} of the primary transmitters. We thus introduce the concept of connectivity region $\mathcal{C}$ of a CR network, which is defined as the set of density pairs $(\lambda_S,\ \lambda_{PT})$ under which the secondary network $\mathcal{G}(\lambda_S, \lambda_{PT})$ is connected.

$$\mathcal{C} \triangleq \{(\lambda_S,\ \lambda_{PT}) :\ \mathcal{G}(\lambda_S, \lambda_{PT}) \text{ is connected.}\}.$$

3.1 Basic Properties of the Connectivity Region

We establish in Theorem 1 below three basic properties of the connectivity region.

[6]The two Poisson point processes of the primary transmitters and receivers are essentially a snap shot of the realizations of the primary transmitters and receivers. In different slots, different sets of primary users become active transmitters/receivers. Thus, even if a secondary user is isolated at one time due to the absence of spectrum opportunities, it may experience an opportunity at a different time and be connected to other secondary users.

THEOREM 1. *Basic Properties of Connectivity Region*

T1.1 *The connectivity region $\mathcal{C}$ is contiguous, that is, for any two points $(\lambda_{S1}, \lambda_{PT1}), (\lambda_{S2}, \lambda_{PT2}) \in \mathcal{C}$, there exists a continuous path in $\mathcal{C}$ connecting the two points.*

T1.2 *The lower boundary of the connectivity region $\mathcal{C}$ is the λ_S-axis. Let $\lambda^*_{PT}(\lambda_S)$ denote the upper boundary of the connectivity region $\mathcal{C}$, i.e.,*

$$\lambda^*_{PT}(\lambda_S) \triangleq \sup\{\lambda_{PT} :\ \mathcal{G}(\lambda_S, \lambda_{PT}) \text{ is connected.}\},$$

then we have that $\lambda^*_{PT}(\lambda_S)$ is monotonically increasing with λ_S.

T1.3 *There exists either zero or one infinite connected component in $\mathcal{G}(\lambda_S, \lambda_{PT})$ a.s.*

PROOF SKETCH. The proofs for T1.1 and T1.2 are based on the coupling argument which is a technique frequently used in continuum percolation [11, Section 2.2]. To prove T1.3, we first show the ergodicity[7] of the random model driven by the three Poisson point processes of the primary transmitters and the primary receivers and the secondary users. Let K denote the (random) number of infinite connected components in $\mathcal{G}(\lambda_S, \lambda_{PT})$, then it is obvious that the event $\{K = k\}$ is invariant under the group of shift transformations, for all $k \geq 0$. It follows from the ergodicity of the random model that the event occurs with probability 0 or 1. Consequently, we have that K is an constant a.s. Then it suffices to exclude the possibility of $K \geq 2$ and $K = \infty$. The details of all the proofs can be found in [15]. □

T1.1 and T1.2 specify the basic structure of the connectivity region, as illustrated in Fig. 1. T1.3 implies the occurrence of a phase transition phenomenon, that is, there exists either a unique infinite connected component a.s. or no infinite connected component a.s. This uniqueness of the infinite connected component also establishes the strong connectivity of the secondary network: once it is connected, it is strongly connected.

3.2 Critical Densities

In this subsection, we study the critical densities of the secondary users and the primary transmitters.

THEOREM 2. *Critical Densities*
Given R_p, R_I, r_p, and r_I, we have

T2.1 $\lambda^*_S = \lambda_c(r_p)$, *where $\lambda_c(r_p)$ is the critical density for a conventional homogeneous ad hoc network with transmission range r_p (i.e., in the absence of the primary network).*

T2.2 $\overline{\lambda^*_{PT}} \leq \frac{\lambda_c(1)}{4\max\{R_I^2, r_I^2\} - r_p^2}$, *where the constant $\lambda_c(1)$ is the critical density for a conventional homogeneous ad hoc network with a unit transmission range.*

PROOF SKETCH. The basic idea of the proof for T2.1 is to approximate the secondary network $\mathcal{G}(\lambda_S, \lambda_{PT})$ by a discrete edge-percolation model on the grid, and then apply a 'Peierls argument' [5, Chapter 1] to the discrete edge-percolation model. This discretization technique is often used to convert a continuum percolation

[7]A model is said to be ergodic if the group of shift transformations $\{S_x :\ x \in \mathbb{R}^d$ or $\mathbb{Z}^d\}$ acts ergodically on the probability space $(\Omega, \mathcal{F}, \mu)$ of the model, where the shift transformation S_x is to shift the realization $\omega \in \Omega$ by x. A group of transformations $\{S_x :\ x \in \mathbb{R}^d$ or $\mathbb{Z}^d\}$ is said to be act ergodically if the σ-algebra of events invariant under the whole group is trivial, *i.e.*, any invariant event has measure either zero or one.

model to a discrete site/edge percolation model (see, e.g., [11, Chapter 3], [3]).

The proof for T2.2 is based on the argument that if there is an infinite connected component in the secondary network, then an infinite vacant component must exist in the two Poisson Boolean models driven by primary transmitters and primary receivers, respectively. The key point is to carefully choose the radii of the two Poisson Boolean models. The details of the two proofs can be found in [15]. □

Fig. 5 shows a simulation example of the connectivity region, where this upper bound appears to be achievable.

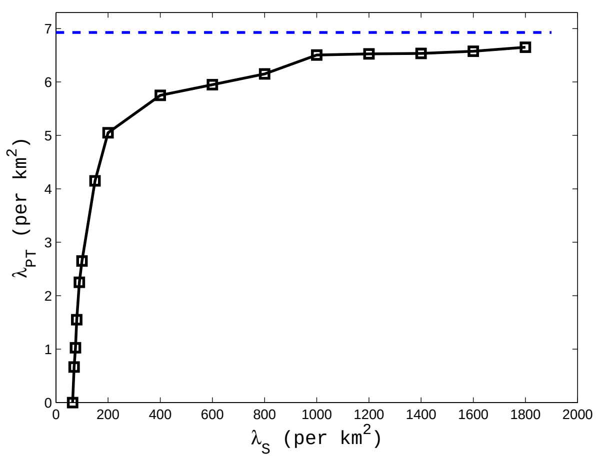

Figure 5: Simulated connectivity regions when $r_p = 150$m, $r_I = 240$m, $R_p = 100$m, and $R_I = 120$m. The blue dashed line is the upper bound on the critical density of primary transmitters.

4. IMPACT OF TRANSMISSION POWER: PROXIMITY VS. OPPORTUNITY

In this section, we study the impact of the secondary users' transmission power on the connectivity region. As has been illustrated in Fig. 2, there exists a tradeoff between proximity and opportunity. Specifically, increasing the transmission power p_{tx} of the secondary users leads to a smaller critical density λ_S^* of the secondary users, but at the same time, a smaller critical density $\overline{\lambda_{PT}^*}$ of the primary transmitters.

From the scaling relation of the critical density [11, Proposition 2.11], we know that in a homogeneous two dimensional network,

$$\lambda_c(r_p) = \lambda_c(1)\,(r_p)^{-2} \propto (p_{tx})^{-\frac{2}{\alpha}}\,,$$

where α is the path-loss exponent, and $\lambda_c(r_p)$ is the critical density for a homogeneous ad hoc network with transmission range r_p. Thus, if each secondary user adopts a high transmission power, then $\lambda_c(r_p)$ reduces. It follows from T2.1 that the critical density λ_S^* of the secondary users for connectivity reduces due to enhanced proximity (increased number of direct neighbors).

Using the tolerance to the primary traffic load as the performance measure, we address in the following theorem the problem of how to choose the transmission power of secondary users based on that of primary users in order to maximize the upper bound on the critical density $\overline{\lambda_{PT}^*}$ of primary transmitters.

THEOREM 3. *Let r_I and R_I denote the interference range of the secondary and the primary users, respectively. For a fixed R_I, the upper bound on $\overline{\lambda_{PT}^*}$ given in T2.2 is maximized when $r_I = R_I$.*

PROOF SKETCH. Since under the disk signal propagation and interference model[8], $r_p = \beta r_I$ for some $\beta \in (0, 1)$, this theorem can be proven by considering two cases: $r_I \le R_I$ and $r_I > R_I$. Details can be found in [15]. □

This theorem shows that in order to achieve the best tolerance to the primary traffic in terms of connectivity, the secondary network should choose its transmission power such that its interference range r_I is equal to the interference range R_I of the primary network. An example of the upper bound $\overline{\lambda_{PT}^*}$ is plotted as a function of r_I in Fig. 6.

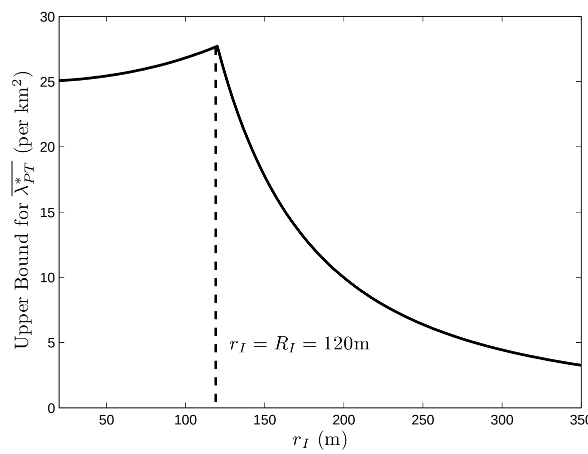

Figure 6: An example of the upper bound for $\overline{\lambda_{PT}^*}$ as a function of r_I (Parameters are given by $R_I = 120$m, $r_p = 0.625 r_I$).

5. CONCLUSION

We have studied the connectivity of a large-scale cognitive radio network in terms of the occurrence of the percolation phenomenon. We have introduced the concept of connectivity region to specify the dependency of connectivity on the density of the secondary users and the traffic load of the primary users. By using the coupling argument, the ergodic theory, and certain combinatorial results, we have shown three basic properties of the connectivity region: the contiguity, the monotonicity of the boundary, and the uniqueness of the infinite connected component. Furthermore, we have analytically characterized the critical density of the secondary users and the critical density of the primary transmitters; they jointly specify the profile as well as an outer bound on the connectivity region. By examining the impact of the secondary users' transmission power on the connectivity region, we have demonstrated the tradeoff between proximity and spectrum opportunity in the design of the optimal transmission power in cognitive radio networks.

[8]Since the minimum transmission power for successful reception is, in general, higher than the maximum allowable interference power, it follows that the transmission range r_p of the secondary users is smaller than the interference range r_I of the secondary users. Furthermore, under the disk signal propagation and interference model, we have $r_p = \beta r_I$ $(0 < \beta < 1)$.

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