

Power Control in Spectrum Overlay Networks: How to Cross a Multi-Lane Highway

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Abstract—We consider power control in spectrum overlay networks (also referred to as opportunistic spectrum access) where secondary users identify and exploit instantaneous and local spectrum opportunities without causing unacceptable interference to primary users. We quantify the impact of the transmission power of secondary users on the occurrence of spectrum opportunities and the reliability of opportunity detection. We demonstrate that the probability of spectrum opportunity decreases exponentially with the transmission power and reliable opportunity detection is achieved in the two extreme regimes of the ratio between the transmission power of secondary users and that of primary users. Such analytical characterizations allow us to study power control for optimal transport throughput under constraints on the interference to primary users.

Index Terms—Cognitive radio, opportunistic spectrum access, spectrum overlay, power control, spectrum opportunity detection.

I. INTRODUCTION

The tradeoff between long-distance direct transmissions and multi-hop relaying, both in terms of energy efficiency and network capacity, is now well understood in conventional wireless networks [1], [2].

This tradeoff in spectrum overlay networks is, however, much more complex. In spectrum overlay networks (also referred to as opportunistic spectrum access), secondary users identify and exploit instantaneous and local spectrum opportunities under constraints on interference to primary users [3]. The transmission power of secondary users not only determines the communication range but also affects the availability of spectrum opportunities. If a secondary user is to use a high transmission power to reach its intended receiver directly, it must wait for the opportunity that no primary receiver is within its relatively large interference region, which occurs less often. If, on the other hand, it uses low transmission power, it sees more transmission opportunities, but must rely on multi-hop relaying, and each hop must wait for its own opportunity to emerge.

Perhaps one may liken delivering a packet from source to destination in spectrum overlay networks as crossing a multi-lane highway, each lane having a different traffic load. Should we wait until all lanes are clear and dash through or cross one lane at a time whenever an opportunity arises? Clearly, the answer depends on the traffic load.

The problem becomes more intriguing when our ability to detect traffic in multiple lanes varies with the number of lanes in question. As shown in this paper, this is indeed the case in spectrum overlay networks: the transmission power of a secondary user affects its opportunity detection performance. Optimal power control for spectrum overlay thus requires a careful analysis of the impact of the transmission power p_{tx} of secondary users on the occurrence of opportunities and the reliability of opportunity detection.

In this paper, to quantify the impact of p_{tx} on the occurrence of spectrum opportunities, we derive a closed-form expression for the

probability of opportunity and demonstrate that it decreases exponentially with p_{tx} . The performance of spectrum opportunity detection, represented by the Receiver Operating Characteristics (ROC), is then analyzed as a function of p_{tx} to demonstrate its impact on the reliability of opportunity detection.

An interesting finding is that reliable spectrum opportunity detection is achieved in the two extreme regimes of the ratio between the transmission power p_{tx} of secondary users and the transmission power P_{tx} of primary users: $\frac{p_{tx}}{P_{tx}} \rightarrow 0$ and $\frac{p_{tx}}{P_{tx}} \rightarrow \infty$. Even though these two extreme regimes are ideal for opportunity detection, they may not result in an efficient communication system. Specifically, for $\frac{p_{tx}}{P_{tx}} \rightarrow 0$, the number of hops required to reach the destination may become unacceptable, while for $\frac{p_{tx}}{P_{tx}} \rightarrow \infty$, the probability of opportunity approaches 0. Optimal power control in spectrum overlay networks thus requires a carefully chosen performance measure. In this paper, we adopt the metric of *transport throughput* that takes into account both the distance covered by a transmission and the rate of successful transmission. The latter is determined by the probability of opportunity, the reliability of opportunity detection, as well as the constraint on the interference to primary users.

II. SPECTRUM OPPORTUNITY: DEFINITION AND DETECTION

A. Spectrum Opportunity and Its Implications

The concept of spectrum opportunity is more involved than it at first may appear [4]. As illustrated in Fig. 1, a channel is an opportunity to a pair of secondary users A and B if they can communicate successfully while limiting the interference to primary receivers. More specifically, B can not be affected by primary transmitters, and A can not interfere with any primary receivers. In other words, there is no primary receiver located within distance r_I to A , and no primary transmitter within distance R_I to B , where r_I is the interference range of secondary users and is monotonically increasing with the transmission power p_{tx} , R_I is the interference range of primary users. Here we have assumed a deterministic and homogeneous signal propagation model. The use of circle to illustrate interference region is, however, immaterial to the definition of spectrum opportunity. This definition applies to a general signal propagation and interference model by replacing the solid and dashed circles with, respectively, the subset of primary receivers who are potential victims of A 's transmission and the subset of primary transmitters who can interfere with the reception at B [5].

It is clear from Fig. 1 that a higher transmission power (larger r_I) of secondary users requires the absence of primary receivers over a larger area, which occurs less often.

B. Spectrum Opportunity Detection

The first step in spectrum opportunity detection is for the secondary transmitter A to detect the presence of nearby primary receivers. Without assuming cooperation from primary users, however, primary receivers are much more difficult to detect than primary transmitters.

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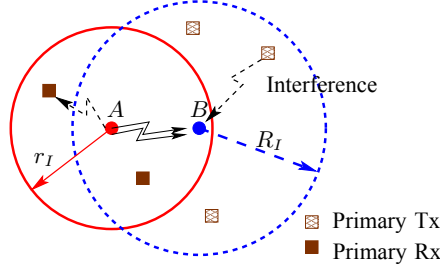


Fig. 1. Definition of spectrum opportunity.

A common approach is to transform the problem of detecting primary receivers to detecting primary transmitters, the so-called "listen-before-talk" (LBT). As shown in Fig. 2, A infers the presence of primary receivers within distance r_I from the presence of primary transmitters within distance r_D , where r_D denotes the detection range and can be adjusted by changing, for example, the threshold of an energy detector.

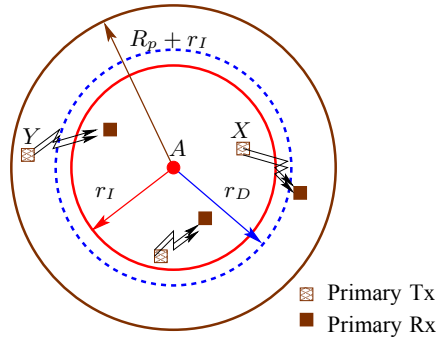


Fig. 2. Spectrum opportunity detection at the secondary tx

Unfortunately, detection errors occur even if A listens with perfect ears, *i.e.*, it can detect the presence of primary transmitters within r_D perfectly. As shown in Fig. 2, the transmission from X may prevent A from accessing an opportunity. On the other hand, A may cause interference to the receiver of Y . By adjusting the detection range r_D , secondary users achieve different tradeoffs between overlooked opportunities and collisions with primary users. The largest detection range we need to consider is $R_p + r_I$, where R_p is the transmission range of primary users, *i.e.*, all primary receivers are within distance R_p to their transmitters.

The fundamental deficiency of LBT resembles the hidden and exposed terminal problem in the conventional ad hoc networks of peer users. It is thus natural to consider the use of RTS/CTS handshaking signaling to enhance the detection performance of LBT. As we will see in Section III and V, RTS/CTS-enhanced LBT offers different performance from LBT.

For best-effort delivery applications such as media streaming and network gaming [6], acknowledgements are not required to confirm the completion of data transmissions. In this case, the probability of successful transmission of LBT can be significantly improved. It, however, does not affect the performance of RTS/CTS-enhanced LBT due to the reception of CTS signals.

III. PERFORMANCE OF SPECTRUM OPPORTUNITY DETECTION

Optimal power control in spectrum overlay networks requires a careful analysis of the impact of transmission power on the

performance of spectrum opportunity detection. In this section, we introduce the figures of merit for assessing the performance of LBT and RTS/CTS-enhanced LBT and derive general expressions for them.

A. Figures of Merit

Spectrum opportunity detection can be formulated as a binary hypothesis testing problem. Let $\mathbb{I}(A, d, \text{rx})$ denote the presence of primary receivers within distance d to the secondary transmitter A , and $\mathbb{I}(B, d, \text{tx})$ the presence of primary transmitters within distance d to the secondary receiver B . Let $\overline{\mathbb{I}(\cdot, \cdot, \cdot)}$ denote the complement of $\mathbb{I}(\cdot, \cdot, \cdot)$. The two hypotheses are given by

$$\begin{aligned} \mathcal{H}_0 &: \text{opportunity exists, } i.e., \overline{\mathbb{I}(A, r_I, \text{rx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})}. \\ \mathcal{H}_1 &: \text{no opportunity, } i.e., \mathbb{I}(A, r_I, \text{rx}) \cup \mathbb{I}(B, R_I, \text{tx}). \end{aligned}$$

The performance of a binary detector is characterized by the false alarm probability P_F and the miss detection probability P_{MD} :

$$P_F = \Pr\{\text{decides } \mathcal{H}_1 \mid \mathcal{H}_0\}, \quad P_{MD} = \Pr\{\text{decides } \mathcal{H}_0 \mid \mathcal{H}_1\}.$$

Since the goal of our design is to maximize the throughput of secondary users while limiting the interference to the primary users, two more metrics are necessary to characterize the detection performance: the probability P_S of successful data transmission, and the probability P_C of colliding with primary users.

$$P_S = \Pr\{\text{successful data transmission}\}, \quad (1)$$

$$P_C = \Pr\{A \text{ transmits data} \mid \mathbb{I}(A, r_I, \text{rx})\}. \quad (2)$$

Note that P_C is conditioned on $\mathbb{I}(A, r_I, \text{rx})$ instead of \mathcal{H}_1 . We have assumed that the interference caused by the RTS, CTS, and ACK signals are negligible due to their short durations.

B. Performance of LBT

For LBT, the observation space consists of the detection outcome of primary transmitters within distance r_D of A . It thus follows that

$$P_F = \Pr\{\mathbb{I}(A, r_D, \text{tx}) \mid \mathcal{H}_0\}, \quad (3)$$

$$P_{MD} = \Pr\{\overline{\mathbb{I}(A, r_D, \text{tx})} \mid \mathcal{H}_1\}. \quad (4)$$

Similarly, we have

$$P_C = \Pr\{\overline{\mathbb{I}(A, r_D, \text{tx})} \mid \mathbb{I}(A, r_I, \text{rx})\}, \quad (5)$$

$$P_S = \Pr\{S = 1 \mid \mathcal{H}_0\} \cdot \Pr\{\mathcal{H}_0\} + \Pr\{S = 1 \mid \mathcal{H}_1\} \cdot \Pr\{\mathcal{H}_1\}, \quad (6)$$

where $S = 1(0)$ denotes the random event of a successful (failed) data transmission.

When the immediate acknowledgement (ACK) is required, we have

$$\Pr\{S = 1 \mid \mathcal{H}_0\} = \Pr\{\overline{\mathbb{I}(A, r_E, \text{tx})} \mid \mathcal{H}_0\}, \quad (7)$$

$$\Pr\{S = 1 \mid \mathcal{H}_1\} = \Pr\{\overline{\mathbb{I}(A, r_E, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})} \mid \mathcal{H}_1\}. \quad (8)$$

For best-effort delivery, we have

$$\Pr\{S = 1 \mid \mathcal{H}_0\} = 1 - P_F, \quad (9)$$

$$\Pr\{S = 1 \mid \mathcal{H}_1\} = \Pr\{\overline{\mathbb{I}(A, r_D, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})} \mid \mathcal{H}_1\}. \quad (10)$$

If we substitute (7), (8) or (9), (10) into (6), we can obtain a much simpler expression for P_S :

$$P_S = \begin{cases} \Pr\{\overline{\mathbb{I}(A, r_E, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})}\} & \text{with ACK} \\ \Pr\{\overline{\mathbb{I}(A, r_D, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})}\} & \text{without ACK} \end{cases} \quad (11)$$

It may appear that the success probability P_S does not depend on r_I . However, since the collision probability P_C depends on r_I , and there is a constraint on P_C ($P_C \leq \zeta$), it follows that for different r_I , we need to choose different r_D to satisfy the constraint on P_C , resulting in different P_S .

C. Performance of RTS/CTS-enhanced LBT

Define the effective range $r_E = \max\{r_D, R_I\}$. Since for RTS/CTS-enhanced LBT, the observation space comprises the RTS and CTS signals, we have the following.

$$\begin{aligned} P_F &= \Pr\{\text{failed RTS-CTS exchange} \mid \mathcal{H}_0\} \\ &= \Pr\{\mathbb{I}(A, r_D, \text{tx}) \cup \mathbb{I}(B, R_I, \text{tx}) \cup \mathbb{I}(A, R_I, \text{tx}) \mid \mathcal{H}_0\} \\ &= \Pr\{\mathbb{I}(A, r_E, \text{tx}) \mid \mathcal{H}_0\}, \end{aligned} \quad (12)$$

where the last step follows from $\Pr\{\mathbb{I}(B, R_I, \text{tx}) \cap \mathcal{H}_0\} = 0$.

$$\begin{aligned} P_{MD} &= \Pr\{\text{successful RTS-CTS exchange} \mid \mathcal{H}_1\} \\ &= \Pr\{\overline{\mathbb{I}(A, r_D, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})} \cap \overline{\mathbb{I}(A, R_I, \text{tx})} \mid \mathcal{H}_1\} \\ &= \Pr\{\overline{\mathbb{I}(A, r_E, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})} \mid \mathcal{H}_1\} \end{aligned} \quad (13)$$

$$P_C = \Pr\{\overline{\mathbb{I}(A, r_E, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})} \mid \mathbb{I}(A, r_I, \text{rx})\} \quad (14)$$

$$\begin{aligned} P_S &= \Pr\{S = 1 \mid \mathcal{H}_0\} \cdot \Pr[\mathcal{H}_0] + \Pr\{S = 1 \mid \mathcal{H}_1\} \cdot \Pr[\mathcal{H}_1] \\ &= (1 - P_F) \cdot \Pr[\mathcal{H}_0] + P_{MD} \cdot \Pr[\mathcal{H}_1] \end{aligned} \quad (15)$$

Note that (15) follows from the fact that miss detections always lead to successful data transmission for RTS/CTS-enhanced LBT. The reason behind this is that miss detections can only occur after a successful RTS-CTS exchange. Then B can receive data successfully as it can receive RTS.

Similarly to (11), P_S can be simplified as follows:

$$P_S = \Pr\{\overline{\mathbb{I}(A, r_E, \text{tx})} \cap \overline{\mathbb{I}(B, R_I, \text{tx})}\}. \quad (16)$$

Notice that P_S of RTS/CTS-enhanced LBT is identical to that of LBT for guaranteed delivery in (11) (see [8]). Due to the requirement on the successful reception of CTS in opportunity detection, the success probability P_S of RTS/CTS-enhanced LBT is independent of the application, *i.e.*, whether acknowledgement is required.

IV. IMPACT OF TRANSMISSION POWER

In this section, we quantitatively characterize the impact of the transmission power p_{tx} of secondary users on the occurrence of opportunities and the reliability of opportunity detection. We consider a Poisson primary network. The basic ideas presented here, however, can be generalized.

Consider a decentralized primary network with slotted transmission structure. Assume that users are distributed according to a two-dimensional homogeneous Poisson process with density λ . At the beginning of each slot, each primary user has a probability p to transmit data to a receiver that is uniformly distributed within its transmission range R_p . Based on the Thinning Theorem and the Displacement Theorem for marked Poisson processes [7], both primary transmitters and receivers form a two-dimensional homogeneous Poisson process with density $p\lambda$. Note that these two Poisson processes are not independent.

A. Impact of Transmission Power on the Probability of Opportunity

Let $S_I(d, r_1, r_2)$ denote the intersecting area of two circles with radius r_1 and r_2 and centered d apart. Let $S_c(d, r_1, r_2)$ denote the complement of $S_I(d, r_1, r_2)$ within the circle of radius r_1 , *i.e.*, the area of $S_c(d, r_1, r_2)$ is given by the difference between πr_1^2 and the area of $S_I(d, r_1, r_2)$. Here d is the distance between A and B . If we pick A as the origin of the polar coordinate system, then the probability of opportunity can be shown to be

$$\Pr[\mathcal{H}_0] = \exp\left[-p\lambda \left(\iint_{S_c(d, r_I + R_p, R_I)} \frac{S_I(r, R_p, r_I)}{\pi R_p^2} r dr d\theta + \pi R_I^2 \right)\right]$$

The detailed derivation of $\Pr[\mathcal{H}_0]$ and the specific expressions for $S_I(d, r_1, r_2)$ and $S_c(d, r_1 + R_p, R_I)$ can be found in [8]. We point out that the double integral can be reduced to a single integral, which significantly simplifies the computation.

The following properties characterize the impact of transmission power (represented by r_I) on the probability of opportunity.

Property 1: Monotonicity of $\Pr[\mathcal{H}_0]$ with respect to r_I : $\Pr[\mathcal{H}_0]$ is a monotonically decreasing function of r_I .

The following property characterizes the rate at which $\Pr[\mathcal{H}_0]$ decreases with r_I . A numerical example is given in Fig. 3.

Property 2: Asymptotically Achievable Lower and Upper Bounds:

$$\exp[-p\lambda\pi(r_I^2 + R_I^2)] < \Pr[\mathcal{H}_0] \leq \exp(-p\lambda\pi r_I^2) \text{ for all } r_I > 0,$$

and $\Pr[\mathcal{H}_0] = \exp[-p\lambda\pi r_I^2]$ when $r_I \geq d + R_I + R_p$.

Property 2 shows that $\Pr[\mathcal{H}_0]$ decreases exponentially with r_I . Due to space limit, the proof of Property 1 and 2 is omitted.

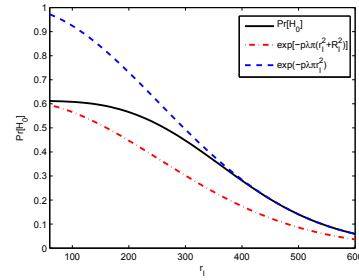


Fig. 3. $\Pr[\mathcal{H}_0]$ vs r_I ($p = 0.01$, $\lambda = 10/200^2$, $d = 50$, $R_p = 200$, $R_I = 250$)

B. Impact of Transmission Power on Detection Performance

By using techniques similar to those used in obtaining the expression for $\Pr[\mathcal{H}_0]$, we can obtain expressions for P_F and P_{MD} . The opportunity detection performance is given by the ROC, which gives the probability of detection $P_D = 1 - P_{MD}$ as a function of P_F . Each point on the ROC corresponds to a different detection range r_D . Let $P_F(r_D = R_I)$ and $P_D(r_D = R_I)$ denote, respectively, the false alarm probability and the detection probability achieved at $r_D = R_I$, we then have the following property whether the transmission range d of secondary users is proportional to the interference range r_I of secondary users or d is fixed.

Property 3: Asymptotic properties of ROC:

- P3.1) Point $(P_F(r_D = R_I), P_D(r_D = R_I)) \rightarrow (0, 1)$ when $\frac{r_I}{R_I} \rightarrow 0$.
- P3.2) Point $(P_F(r_D = r_I), P_D(r_D = r_I)) \rightarrow (0, 1)$ when $\frac{r_I}{R_I} \rightarrow \infty$.

The proof for Property 3 is omitted due to space limit. Property 3 shows us that at least one point of the ROC approaches $(0, 1)$ as $\frac{r_I}{R_I} \rightarrow 0$ or ∞ . Since the ROC is continuous, it follows that reliable opportunity detection is achieved in the two extreme regimes of the transmission power: $\frac{p_{tx}}{P_{tx}} \rightarrow 0$ and $\frac{p_{tx}}{P_{tx}} \rightarrow \infty$. Fig. 4 shows the ROC of LBT. We observe that the detection performance becomes nearly perfect as r_I goes to zero or infinity.

V. POWER CONTROL FOR OPTIMAL TRANSPORT THROUGHPUT

From Sections IV.A and IV.B, it seems that the transmission power p_{tx} should be chosen as small as possible to maximize the probability of opportunity and improve the detection quality. As discussed in Section I, by taking into account both the distance covered by a transmission and the rate of successful transmission, we should adopt the metric of transport throughput which is defined as

$$C_T(r_I, r_D) = d(r_I) \cdot P_S(r_D), \quad (17)$$

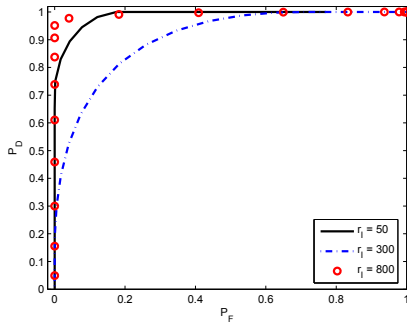


Fig. 4. ROC for LBT ($p = 0.01$, $\lambda = 10/200^2$, $R_p = 200$, $R_I = R_p/0.8 = 250$, $d = 0.9 \cdot r_I$)

where d is the transmission range of the secondary user which is proportional to r_I . Power control for optimal transport throughput can be formulated as a constrained optimization:

$$r_I^* = \arg \max_{r_I} \{C_T\} = \arg \max_{r_I} \{d(r_I) \cdot P_S(r_D)\} \text{ s.t. } P_C(r_I, r_D) \leq \zeta$$

where ζ denotes the constraint on the probability of collision with primary users. The above optimization can be solved numerically. Two numerical examples where we plot C_T as a function of r_I for different traffic loads of primary users are shown in Fig. 5 and 6.

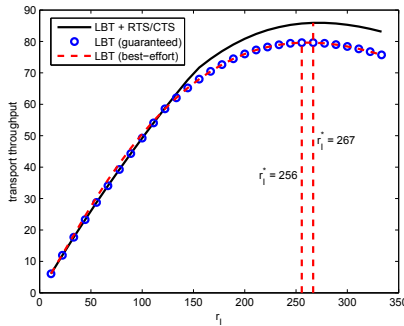


Fig. 5. Transport Throughput vs r_I ($p = 0.01$, $\lambda = 10/200^2$, $R_p = 200$, $R_I = R_p/0.8 = 250$, $d = 0.9 \cdot r_I$, $\zeta = 0.05$)

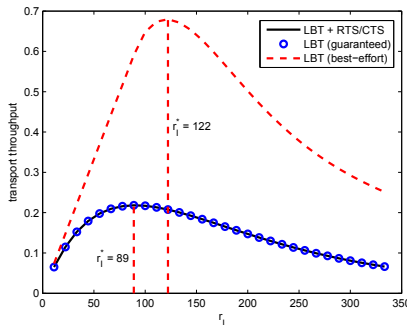


Fig. 6. Transport Throughput vs r_I ($p = 0.1$, $\lambda = 10/200^2$, $R_p = 200$, $R_I = R_p/0.8 = 250$, $d = 0.9 \cdot r_I$, $\zeta = 0.05$)

From Fig. 5 and Fig. 6, we observe the following: when the traffic load of the primary network is low, the optimum interference range

r_I^* of secondary users is close to the interference range R_I of primary transmitters; when the traffic load of the primary network is high, the optimum interference range r_I^* of secondary users is much smaller than the interference range R_I of primary transmitters.

We point out that when the traffic load of the primary network is relatively heavy, RTS/CTS signaling degrades the performance of LBT when acknowledgements are unnecessary (see Fig. 6). This rather surprising finding suggests that spectrum overlay in primary systems with relatively heavy traffic is more suitable for best-effort delivery applications.

This message is also conveyed by Fig. 7, where we plot the ratio of r_I^* to R_I as a function of $p\lambda$, where r_I^* is given by the optimal transmission power associated with the best detection scheme. We observe from Fig. 7 that the optimal transmission power of secondary users decreases monotonically with the traffic load of the primary network. The steep fall when we switch detectors is consistent with the previous figures.

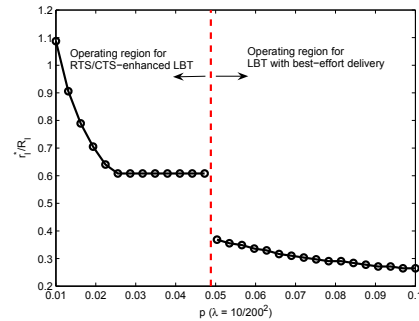


Fig. 7. r_I^* vs p ($\lambda = 10/200^2$)

VI. CONCLUSION

We have studied transmission power control in spectrum overlay systems for optimal transport throughput under constraints on interference to primary users. Findings in this paper include (i) the probability of spectrum opportunity decreases exponentially with the transmission power of secondary users; (ii) reliable opportunity detection is achieved in the two extreme regimes of the transmission power; (iii) the optimal transmission power of secondary users decreases monotonically with the traffic load of the primary network; (iv) spectrum overlay in primary systems with relatively heavy traffic is more suitable for best-effort delivery applications.

REFERENCES

- [1] M. Haenggi and D. Puccinelli, "Routing in Ad Hoc Networks: A Case for Long Hops," *IEEE Communications Magazine*, Oct. 2005.
- [2] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, March 2000.
- [3] Q. Zhao and B. M. Sadler, "A Survey of Dynamic Spectrum Access," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 79-89, May, 2007.
- [4] Q. Zhao, "Spectrum Opportunity and Interference Constraint in Opportunistic Spectrum Access," in *Proc. of ICASSP*, April, 2007.
- [5] Q. Zhao, W. Ren, A. Swami, "Spectrum Opportunity Detection: How Good Is Listen-Before-Talk?" in *Proc. of IEEE Asilomar Conference on Signals, Systems, and Computers*, November, 2007 (invited).
- [6] Tom Sheldon, "Encyclopedia of Networking & Telecommunications," Osborne/McGraw-Hill, 2001.
- [7] J.F.C. Kingman, "Poisson Processes," Clarendon Press, Oxford, 1993.
- [8] W. Ren, Q. Zhao, "Opportunity Detection at the Secondary Transmitter and Receiver," Technical Report (TR-07-05), Aug. 2007.