On the Lifetime of Wireless Sensor Networks

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***Abstract*— We derive a general formula for the lifetime of wireless sensor networks which holds independently of the underlying network model including network architecture and protocol, data collection initiation, lifetime definition, channel fading characteristics, and energy consumption model. This formula identifies two key parameters at the physical layer that affect the network lifetime: the channel state and the residual energy of sensors. As a result, it provides not only a gauge for performance evaluation of sensor networks but also a guideline for the design of network protocols. Based on this formula, we propose a medium access control protocol that exploits both the channel state information and the residual energy information of individual sensors. Referred to as the max-min approach, this protocol maximizes the minimum residual energy across the network in each data collection.**

***Index Terms*— Network lifetime, medium access control, wireless sensor network.**

I. INTRODUCTION

A WIRELESS sensor network (WSN) consists of low-cost, low-power, and energy-constrained sensors responsible for monitoring a physical phenomenon and reporting to access points (APs) where the end-user can access the data. In many applications, it is undesirable or infeasible to replace or recharge sensors. Hence, the network lifetime becomes a critical concern in the design of WSNs. While various energy-efficient protocols have been proposed to prolong network lifetime, lifetime analysis is notoriously difficult since the network lifetime depends on many factors including network architecture and protocols, data collection initiation, lifetime definition, channel characteristics, and energy consumption model. Upper bounds on lifetime are thus derived for various WSNs (see [1]–[4] and references therein). To our best knowledge, accurate analysis of network lifetime is not available in the literature.

In this letter, we derive a general formula for network lifetime which holds independently of the underlying network model. This formula reveals that two physical layer parameters are crucial to network lifetime: the channel state and the residual energy of sensors. It indicates that lifetime-maximizing protocols should exploit both the channel state information (CSI) and the residual energy information (REI) of individual sensors. Based on this formulation, we propose a greedy approach to medium access for lifetime maximization. Using both CSI and REI, the proposed protocol maximizes the minimum residual energy across the network in each data collection.

II. A GENERAL MODEL OF WSNS

We list below important network characteristics that affect the network lifetime.

Network Architecture. Network architecture specifies how sensors should report their data to the APs. Three types of network architecture have been considered in the literature: the flat ad hoc, the hierarchical ad hoc, and the SEnsor Network with Mobile Access (SENMA). Under the flat ad hoc architecture, sensors relay each other's data in multiple hops to the APs. In hierarchical WSNs, sensors form clusters and report their data to the cluster heads who are responsible for sending the aggregated data to the APs. In SENMA, sensors communicate directly with mobile APs moving around the sensor field.

Data Collection Initiation. According to the applications, data collections in a WSN can be initiated by the internal clock of sensors, the event of interest, or the demand of the end-user. In clock-driven WSNs, sensors collect and transmit data at pre-determined time intervals. In event-driven or demand-driven WSNs, data collections are triggered by an event of interest or a request from the APs.

Channel and Energy Consumption Model. The energy consumption model characterizes the sources of energy consumption in the network. According to the rate of energy expenditure, we classify energy consumption into two general categories: the continuous energy consumption and the reporting energy consumption. The continuous energy consumption is the minimum energy needed to sustain the network during its lifetime without data collection. It includes, for example, battery leakage and sensor sleeping energy. The reporting energy consumption is the additional energy consumed in data collections. It depends on the rate of data collection as well as the channel model and the network architecture and protocols. It includes the energy consumed in transmission, reception, and possibly channel acquisition. We point out that energy consumption may come from other sources such as network maintenance whose energy expenditure rate is neither continuous nor related to data collections. As shown in Section III, we can easily accommodate these energy consumption sources in the derived lifetime formula.

Lifetime Definition. Network lifetime is the time span from the deployment to the instant when the network is considered nonfunctional. When a network should be considered nonfunctional is, however, application-specific. It can be, for example, the instant when the first sensor dies, a percentage of sensors die, the network partitions, or the loss of coverage occurs.

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Our goal here is to derive a general formula for network lifetime which holds independently of the underlying network model. It should allow us to identify key parameters that affect network lifetime without worrying about specific network settings. As a result, it can provide design guidelines applicable to various types of sensor networks.

III. A GENERAL FORMULA FOR NETWORK LIFETIME

In this section, we study the average lifetime of WSNs in a general setting. We do not specify the network architecture, the data collection initiation, or the channel and the energy consumption model. Moreover, the obtained formula applies to any definition of the network lifetime.

Theorem 1 *For a WSN with total non-rechargeable initial energy* $\mathcal{E}_0$*, the average network lifetime* $\mathbb{E}[\mathcal{L}]$*, measured as the average amount of time until the network dies, is given by*

$$\mathbb{E}[\mathcal{L}] = \frac{\mathcal{E}_0 - \mathbb{E}[E_w]}{P_c + \lambda\,\mathbb{E}[E_r]}, \tag{1}$$

where P_c *is the constant continuous power consumption over the whole network,* $\mathbb{E}[E_w]$ *is the expected wasted energy (i.e., the total unused energy in the network when it dies),* λ *is the average sensor reporting rate defined as the number of data collections per unit time, and* $\mathbb{E}[E_r]$ *is the expected reporting energy consumed by all sensors in a randomly chosen data collection.*

Proof: The derivation of (1) is based on the strong law of large numbers (SLLN). Suppose we perform M independently and identically distributed (i.i.d.) trials on the same WSN to record the network lifetime $\mathcal{L}$, the wasted energy E_w, and the energy consumption in each data collection E_i. For the m-th trial ($1 \leq m \leq M$), we can write the total energy consumed during the whole lifetime as

$$\mathcal{E}_0 - E_w^{(m)} = P_c \mathcal{L}^{(m)} + \sum_{i=1}^{N^{(m)}} E_i^{(m)}, \tag{2}$$

where $N^{(m)}$ is the number of data collections during the network lifetime of the m-th trial. Summing (2) up over the M trials and dividing both sides by M, we obtain

$$\mathcal{E}_0 - \frac{1}{M}\sum_{m=1}^{M} E_w^{(m)} = \frac{1}{M}\sum_{m=1}^{M}\mathcal{L}^{(m)}\left[P_c + \left(\frac{\sum_{m=1}^{M} N^{(m)}}{\sum_{m=1}^{M}\mathcal{L}^{(m)}}\right) \times \left(\frac{\sum_{m=1}^{M}\sum_{i=1}^{N^{(m)}} E_i^{(m)}}{\sum_{m=1}^{M} N^{(m)}}\right)\right]. \tag{3}$$

Note that $\lim_{M\to\infty} \frac{\sum_{m=1}^{M} N^{(m)}}{\sum_{m=1}^{M}\mathcal{L}^{(m)}} = \lambda$ is the average sensor reporting rate. Next, we will show that

$$\mathbb{E}[E_r] \overset{\Delta}{=} \mathbb{E}_i\{\mathbb{E}[E_i]\} = \lim_{M\to\infty} \frac{\sum_{m=1}^{M}\sum_{i=1}^{N^{(m)}} E_i^{(m)}}{\sum_{m=1}^{M} N^{(m)}}, \tag{4}$$

where $\mathbb{E}[E_i]$ is the average reporting energy consumed in the i-th data collection, $\mathbb{E}_i\{\cdot\}$ denotes the expectation over the randomly chosen data collection index i.

The average reporting energy consumed in the i-th data collection can be written as

$$\mathbb{E}[E_i] = \lim_{M\to\infty} \frac{\sum_{m=1}^{M} E_i^{(m)}\chi_m(i)}{D_i}, \quad 1 \leq i \leq T, \tag{5}$$

where $\chi_m(i) = 1$ for $1 \leq i \leq N^{(m)}$ and 0 otherwise, $D_i = \sum_{m=1}^{M}\chi_m(i)$ is the total number of the occurrence of the i-th data collection among the M trials, and $T = \max_m\{N^{(m)}\}$ is the maximum number of data collections during the network lifetime[1]. The probability that the randomly chosen data collection happens to be the i-th data collection is given by

$$p_i = \lim_{M\to\infty} \frac{D_i}{\sum_{m=1}^{M} N^{(m)}} \quad 1 \leq i \leq T. \tag{6}$$

Averaging (5) over the randomly chosen data collection index i, we obtain the expected reporting energy consumed in a randomly chosen data collection as given in (4). Let M go to infinity in (3), we obtain the average network lifetime given in (1) based on SLLN. □□□

The lifetime formula given in (1) provides a quantitative characterization of key components that affect network lifetime under a general network setting. Specifically, a lifetime-maximizing protocol should aim at reducing the average wasted energy $\mathbb{E}[E_w]$ and the average reporting energy $\mathbb{E}[E_r]$. To reduce $\mathbb{E}[E_w]$, the protocol should exploit the REI of individual sensors to achieve balanced energy consumption across the network. To reduce $\mathbb{E}[E_r]$, the protocol should exploit the CSI to prioritize sensors with better channels for transmission thus reduce the energy consumed in transmission. In Section IV, we propose, based on the design guideline provided by (1), a medium access control (MAC) protocol that uses both CSI and REI for lifetime maximization.

We point out that (1) can be easily extended to include other energy consumption sources. For example, to include the energy consumed in network maintenance, we obtain the following formula via a derivation similar to that given above.

$$\mathbb{E}[\mathcal{L}] = \frac{\mathcal{E}_0 - \mathbb{E}[E_w]}{P_c + \lambda\,\mathbb{E}[E_r] + \eta\,\mathbb{E}[E_m]}, \tag{7}$$

where η is the maintenance rate of the network which shows how often the maintenance is performed, and $\mathbb{E}[E_m]$ is the expected energy consumed in a randomly chosen network maintenance.

IV. A GREEDY APPROACH TO LIFETIME-MAXIMIZING MEDIUM ACCESS

In this section, we apply (1) to the MAC design in a specific WSN. We demonstrate that exploiting the two physical layer parameters (channel state and residual energy) identified by (1) in the design of MAC protocols leads to improved network performance.

Consider a clock-driven WSN with S homogeneous sensors, each powered by a non-rechargeable battery with E_0 initial energy. In each data collection, one out of S sensors is selected to transmit its measurement to a mobile AP through a wireless fading channel. We seek the answer to the following question:

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[1] Since the initial energy $\mathcal{E}_0$ is finite and the reporting energy is lower-bounded by the energy consumed in the transceiver circuitry of reporting sensors, the maximum number T of data collections during the network lifetime is finite.

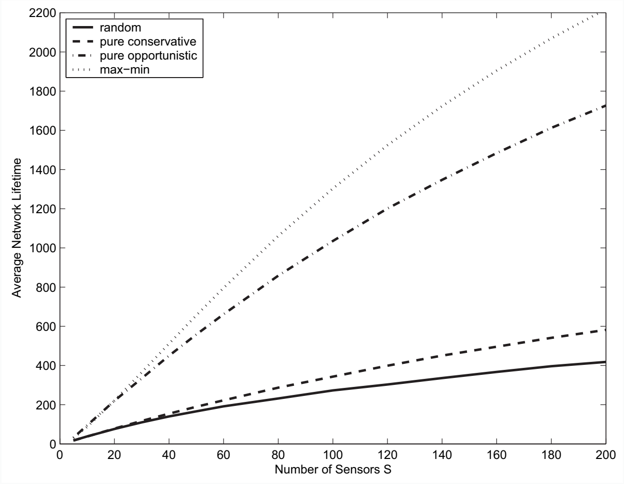

Fig. 1. Comparison of the network lifetime. $E_0 = 5$, $E_c = 0.01$, $E_{es} = 0.001$.

which sensor should be enabled in each data collection in order to maximize the network lifetime.

We assume that sensor measurements are in the form of equal-sized packets. The channel between the mobile AP and a sensor follows a block fading model with the block length equal to the transmission time of one packet. The required reporting energy $E_r(c_i)$ of sensor i as a function of its fading gain c_i can be modelled as

$$E_r(c_i) = E_{tc} + \frac{\overline{E}}{c_i} \tag{8}$$

where E_{tc} is the energy consumed in the transmitter circuitry and $\overline{E}$ is the required transmission energy to achieve an acceptable received SNR at the AP in the absence of channel fading. Clearly, the better the channel gain c_i, the smaller the required transmission energy $E_r(c_i)$. A sensor is considered dead if its residual energy drops below E_{tc}, *i.e.,* it does not have enough energy for transmission under any channel condition. We ignore the continuous energy consumption in the network and define the network lifetime as the time span until any sensor in the network dies (the first death) or no sensor has enough energy for transmission during a data collection (the first failure in data collection), whichever occurs first[2].

Applying (1) to the current network setting, we have

$$\mathbb{E}[\mathcal{L}] = \frac{SE_0 - \mathbb{E}[E_w]}{\mathbb{E}[E_r]}, \tag{9}$$

where we have assumed, without loss of generality, that $\lambda = 1$. Equation (9) shows that the network lifetime $\mathbb{E}[\mathcal{L}]$ increases as $\mathbb{E}[E_r]$ or $\mathbb{E}[E_w]$ decreases. To prolong the network lifetime, the MAC protocol should strike a balance between $\mathbb{E}[E_r]$ and $\mathbb{E}[E_w]$. With this goal in mind, we propose a MAC protocol which selects the sensor with the maximum energy-efficiency index γ_i defined as

$$\gamma_i = e_i - E_r(c_i), \tag{10}$$

where e_i is the residual energy of sensor i at the beginning of a data collection. It is clear from (10) that the proposed protocol maximizes the minimum residual energy across the network in each data collection. We can see that this protocol, referred to as the max-min protocol, presents a greedy approach to lifetime maximization by exploiting both CSI and REI of individual sensors. A distributed implementation of the max-min protocol, which allows each sensor to determine whether to transmit based on its own channel state and residual energy, can be found in [5].

Fig. 1 provides simulation result on the lifetime comparison of several MAC protocols in i.i.d. Rayleigh fading channel. All the energy quantities are normalized by the required transmission energy $\overline{E}$ in the absence of channel fading. The "random" protocol which utilizes neither CSI nor REI randomly chooses a sensor for transmission. The pure conservative protocol which selects the sensor with the most residual energy $\max_i\{e_i\}$ aims to reduce $\mathbb{E}[E_w]$ by exploiting REI. On the other hand, the pure opportunistic protocol which selects the sensor with the best channel $\max_i\{c_i\}$ focuses solely on minimizing the reporting energy $\mathbb{E}[E_r]$ by utilizing CSI. To compare the lifetime performance on a fair basis, we consider the energy E_{es} required for channel acquisition in the pure opportunistic and the max-min protocols. Fig. 1 shows that by exploiting both CSI and REI, the max-min protocol improves the network lifetime performance, and the gain in lifetime increases with the size S of the network.

V. CONCLUSION

In this letter, we derive a general expression for the lifetime of WSNs which holds regardless of the underlying network model. This formula provides insights on lifetime-maximizing protocol design. It reveals that a lifetime-maximizing protocol should exploit both CSI and REI of individual sensors. Based on this formula, we propose a greedy approach to lifetime maximization which achieves considerable improvement in lifetime performance.

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[2] We realize that this lifetime definition may not apply to many WSN applications. It, however, provides insights on protocol design and makes analysis tractable.