JOINT PHY-MAC DESIGN FOR OPPORTUNISTIC SPECTRUM ACCESS WITH MULTI-CHANNEL SENSING

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ABSTRACT

This paper addresses the design of opportunistic spectrum access (OSA) where secondary users are allowed to sense and access multiple channels in the spectrum without causing unacceptable interference to primary users. Integrated in OSA design are a spectrum sensor at the physical (PHY) layer and a sensing and an access strategy at the MAC layer. Within the framework of partially observable Markov decision process, we develop a separation principle for the joint OSA design, leading to an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy when spectrum sensor and access strategy are designed independently across channels. We also propose two heuristic approaches that exploit the correlation among channel occupancies, one at the PHY layer and the other at the MAC layer. Simulation results indicate that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer, and that the detection capability of the spectrum sensor can be improved by exploiting MAC layer information.

1. INTRODUCTION

Opportunistic spectrum access (OSA) exploits temporal and spatial spectrum opportunities resulting from the bursty traffic of primary users and guard bands in space. It has captured increasing attention recently due to its potential in improving spectrum efficiency [1]. As shown in [2, 3], basic design components of OSA include (i) a spectrum sensor at the physical (PHY) layer, which identifies spectrum opportunities; (ii) a sensing strategy at the MAC layer, which specifies which channels in the spectrum to sense; and (iii) an access strategy, also at the MAC layer, which determines whether to access based on potentially erroneous sensing outcomes. The design objective is to maximize the throughput of secondary users under the constraint that the probability of colliding with primary users is capped below a certain threshold.

Most existing work on OSA focuses on the cognitive MAC design under the ideal assumption that the spectrum sensor is perfect [4, 5]. Recently, within the framework of partially observable Markov decision process (POMDP), the design of the PHY layer spectrum sensor has been integrated into the MAC design for optimal OSA in the presence of sensing errors [2,3]. In [2], a separation principle is established for OSA with single-channel sensing, which leads to an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy.

This paper extends the results in [2] to the scenario where secondary users can sense and access multiple channels simultaneously. We show that when the spectrum sensor and the access strategy are designed independently across channels, the separation principle developed in [2] still holds for OSA with multi-channel sensing. We, however, note that such independent design is suboptimal since it ignores the correlation among channel occupancies. We thus propose two heuristic approaches to exploit channel correlation, one at the PHY layer and the other at the MAC layer. Simulation results indicate that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer. We also find that the performance of the PHY layer spectrum sensor improves over time by incorporating the MAC layer sensing and access decisions. These observations illustrate the two-way interaction between the PHY and the MAC layers: the necessity of incorporating the sensor operating characteristics into the MAC design and the benefit of exploiting the MAC layer information in the PHY layer design.

2. PROBLEM STATEMENT AND FORMULATION

2.1. Network Model

Consider a spectrum of N orthogonal channels licensed to a slotted primary network. Let $S_n(t) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$ denote the occupancy of channel n in slot t. We assume that the spectrum occupancy $\mathbf{S}(t) \triangleq [S_1(t), \ldots, S_N(t)]$ follows a discrete-time homogeneous Markov process with finite state space $\mathbb{S} \triangleq \{0, 1\}^N$. The transition probabilities are denoted as $\{P_{\mathbf{s},\mathbf{s}'}\}_{\mathbf{s},\mathbf{s}' \in \mathbb{S}}$, where $P_{\mathbf{s},\mathbf{s}'} \triangleq \Pr\{\mathbf{S}(t+1) = \mathbf{s}' \,|\, \mathbf{S}(t) = \mathbf{s}\}$ is the probability that the spectrum occupancy state transits from $\mathbf{s} \in \mathbb{S}$ to $\mathbf{s}' \in \mathbb{S}$. We assume that the transition probabilities are known and remain unchanged in T slots[1].

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[1]The robustness of the optimal OSA design to inaccurate transition probabilities has been demonstrated in [2]. When the transition probabilities are unknown, formulations and algorithms for POMDP with an unknown model exist in the literature [6] and can be applied to this problem.

We consider a secondary ad hoc network whose users independently search for and exploit instantaneous spectrum opportunities in these N channels[2]. Specifically, at the beginning of slot t, a secondary user with data to transmit chooses a set $\mathcal{A}(t) \subset \{1, \ldots, N\}$ of channels to sense, where $|\mathcal{A}(t)| = L$ and $1 \leq L \leq N$. Based on the imperfect sensing outcomes $\mathbf{\Theta}_{\mathcal{A}}(t) \stackrel{\Delta}{=} \{\Theta_n(t)\}_{n \in \mathcal{A}(t)} \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}^L$, the secondary user decides whether to access each sensed channel: $\mathbf{\Phi}_{\mathcal{A}}(t) \stackrel{\Delta}{=} \{\Phi_n(t)\}_{n \in \mathcal{A}(t)} \in \{0 \text{ (no access)}, 1 \text{ (access)}\}^L$. A collision with primary users happens when the secondary user accesses a busy channel. Collisions among secondary users are resolved via carrier sensing, *i.e.,* the secondary user with the smallest backoff time transmits. At the end of this slot, the receiver acknowledges every successful transmission. We let $\mathbf{K}_{\mathcal{A}}(t) \stackrel{\Delta}{=} \{K_n(t)\}_{n \in \mathcal{A}(t)} \in \{0 \text{ (no ACK)}, 1 \text{ (ACK)}\}^L$. We assume that the acknowledgement is error-free. That is, acknowledgement $K_n(t) = 1$ is received if and only if the secondary user accesses an idle channel. Our goal is to design an OSA strategy for the secondary user, which sequentially specifies which channels in the spectrum to sense, which spectrum sensor to use, and which sensed channels to access.

2.2. Basic Components of OSA

Integrated in the design of OSA are three basic components: a spectrum sensor, a sensing strategy, and an access strategy.
Spectrum Sensor Suppose that a set $\mathcal{A}(t)$ of channels are chosen at the beginning of slot t, where $|\mathcal{A}(t)| = L \geq 1$. The spectrum sensor detects the occupancies of chosen channels by performing a 2^L-ary hypothesis test:

$$\mathcal{H}_0 : \mathbf{S}_{\mathcal{A}}(t) = [1, \ldots, 1], \ \ldots\ , \mathcal{H}_{2^L-1} : \ \mathbf{S}_{\mathcal{A}}(t) = [0, \ldots, 0],$$

where $\mathbf{S}_{\mathcal{A}}(t) \stackrel{\Delta}{=} \{S_n(t)\}_{n \in \mathcal{A}(t)} \in \{0, 1\}^L$ denotes the occupancies of the chosen channels $\mathcal{A}(t)$ in the current slot. The probabilities of these hypotheses are learned from the entire decision and observation history of the secondary user.

Sensing errors occur if the spectrum sensor mistakes one hypothesis for another. Since there are a total of 2^L hypotheses, the performance of the spectrum sensor can be specified by a set of $2^L \times (2^L - 1)$ error probabilities. In the presence of noise and fading, perfect sensing cannot be attained. Hence, the optimal design of the spectrum sensor should achieve a tradeoff among these $2^L \times (2^L - 1)$ error probabilities.
Sensing and Access Strategies A sensing strategy specifies, in each slot t, a set $\mathcal{A}(t)$ of channels to be sensed, where $\mathcal{A}(t) \subset \{1, \ldots, N\}$ and $|\mathcal{A}(t)| = L$. An access strategy decides a set of transmission probabilities $\{f_n(\boldsymbol{\theta}_{\mathcal{A}}, t)\}$, where

$$f_n(\boldsymbol{\theta}_{\mathcal{A}}, t) \stackrel{\Delta}{=} \Pr\{\Phi_n(t) = 1 | \mathbf{\Theta}_{\mathcal{A}}(t) = \boldsymbol{\theta}_{\mathcal{A}}\}$$

is the probability of accessing sensed channel $n \in \mathcal{A}(t)$ when the sensing outcomes are given by $\boldsymbol{\theta}_{\mathcal{A}}(t) = \{0, 1\}^L$ in slot t.

[2]The design of cooperative OSA strategies for secondary users can be formulated by a decentralized POMDP problem [7]. We point out that while cooperation among secondary users might improve the overall network throughput, such cooperative design is much more complex.

2.3. POMDP Formulation

The joint design of OSA with multi-channel sensing can be formulated as a POMDP problem.
Reward Assume that the expected number of bits that can be delivered by the secondary user in a slot is proportional to the channel bandwidth. Given sensing action $\mathcal{A}(t)$, we define the immediate reward $R^{(\mathcal{A}(t))}_{\mathbf{K}_{\mathcal{A}}(t)}$ as

$$R^{(\mathcal{A}(t))}_{\mathbf{K}_{\mathcal{A}}(t)} = \sum_{n \in \mathcal{A}(t)} K_n(t) B_n. \tag{1}$$

Hence, the total expected reward represents overall throughput, the total expected number of bits that can be delivered by the secondary user in T slots.
Observation As detailed in [9], the secondary user and its desired receiver must have the same history of observations so that they make the same channel selection decisions without exchanging extra control message. Since sensing errors may cause different sensing outcomes at the transmitter and the receiver, the acknowledgement $\mathbf{K}_{\mathcal{A}}(t)$ should be used as the common observation in each slot.
Belief Vector Within the framework of POMDP, the secondary user's knowledge of the spectrum occupancy based on its decision and observation history can be encoded in a belief vector $\mathbf{\Lambda}(t) \stackrel{\Delta}{=} \{\lambda_{\mathbf{s}}(t)\}_{\mathbf{s} \in \mathbb{S}}$ [8], where $\lambda_{\mathbf{s}}(t)$ is the conditional probability (given the entire observation history) that the spectrum occupancy is in state $\mathbf{s} \in \mathbb{S}$ at the beginning of slot t prior to the state transition. We point out here that based on the belief vector $\mathbf{\Lambda}(t)$ at the beginning of slot t, the secondary user can calculate the distribution of the current spectrum occupancy $\mathbf{S}(t)$ as

$$\Pr\{\mathbf{S}(t) = \mathbf{s}\} = \sum_{\mathbf{s}' \in \mathbb{S}} \lambda_{\mathbf{s}'}(t) P_{\mathbf{s}', \mathbf{s}}, \qquad \forall \mathbf{s} \in \mathbb{S}. \tag{2}$$

The probabilities of the hypothesis used in the design of the spectrum sensor can thus be determined from (2).
Objective The design objective is to maximize the total expected reward in T slots under the constraint that the probability $P_n(t)$ of collision perceived by the primary network in any channel n and any slot t is capped below a threshold ζ:

$$\begin{aligned} &\max \mathbb{E}[\sum_{t=1}^{T} R^{(\mathcal{A}(t))}_{\mathbf{K}_{\mathcal{A}}(t)} | \mathbf{\Lambda}(1)] \\ &\text{s.t. } P_n(t) \stackrel{\Delta}{=} \Pr\{\Phi_n(t) = 1 \,|\, S_n(t) = 0\} \leq \zeta, \quad \forall t, n, \end{aligned} \tag{3}$$

where $\mathbf{\Lambda}(1)$ is the initial belief vector, which represents the information on the initial spectrum occupancy. Note that when $\Pr\{S_n(t) = 0\} = 0$, no collision will occur and the optimal access decision is straightforward: $\Phi_n(t) = 1$.

3. SEPARATION PRINCIPLE

The constrained POMDP given in (3) appears to be intractable due to the high-dimensional and uncountable action space. In this section, we show that under certain conditions, there exists a separation principle for the optimal joint design of OSA, which enables us to obtain an explicit optimal design of the spectrum sensor and a closed-form optimal access strategy.

Theorem 1: When the spectrum sensor and the access strategy are designed independently across channels, the optimal joint design of OSA with multi-channel sensing can be carried out in two steps:

1. *Choose the spectrum sensor and the access strategy to maximize the expected immediate reward subject to the collision constraint.*
2. *Choose the sensing strategy to maximize the expected total reward.*

In this case, the optimal spectrum sensor is given by the optimal Neyman-Pearson (NP) detector with probability of miss detection equal to ζ, which detects the channel occupancy by using only the measurements from this channel, and the optimal access decision is to trust the sensing outcome from this channel. The optimal sensing strategy can be obtained by solving an unconstrained POMDP.

Proof: See [9]. □□□

We emphasize that the extension of the separation principle developed in [2] to the multi-channel sensing scenarios is based on the assumption that the spectrum sensor and the access strategy are designed independently across channels. Specifically, we assume that the occupancy of a channel is detected without taking into account the measurements of other channels and the access decision on a channel is made independently of the sensing outcomes from other channels. Intuitively, under the above assumption, the design of the spectrum sensor and the access strategy for the multi-channel sensing case can be treated as L independent design problems, one for each chosen channel. Hence, the optimal design for the single-channel $L = 1$ sensing case can be extended to $L > 1$.

Theorem 1 provides sufficient conditions under which the design given by the separation principle (referred to as the SP approach for simplicity) is optimal. In Proposition 1, we show that the SP approach is locally optimal (*i.e.,* maximizes the expected immediate reward) under certain relaxed conditions.

Proposition 1: Suppose that the spectrum sensor is designed independently across channels while the access strategy jointly exploits the sensing outcomes from all channels. The SP approach is locally optimal when channel occupancies are independent.

Proof: See [9]. □□□

4. HEURISTIC APPROACHES

While simplifying the joint design of OSA with multi-channel sensing, the condition that the spectrum sensor and the access strategy are designed independently across channels result in throughput degradation since the correlation among channel occupancies is ignored. We propose two heuristic approaches to exploit the channel correlation: the PHY layer and the MAC layer approaches.

4.1. The PHY Layer Approach

When the spectrum occupancy states are correlated across channels, we have correlated channel measurements at the PHY layer. Hence, the channel correlation can be exploited by using the measurements of all chosen channels in occupancy detection. With this in mind, we propose a heuristic design of the spectrum sensor: it adopts the optimal NP detector with probability of miss detection equal to ζ, which is designed to uses all channel measurements. We note that the structure of the optimal NP detector adopted by this sensor relies on the joint distribution of the spectrum occupancy states, which is given by the belief vector (see Section 5 for an example). That is, this heuristic sensor design is affected by the observation and decision history of the secondary user and thus improves over time due to accumulated observations (see Figure 1).

Based on the sensing outcomes given by the above spectrum sensor, the secondary user can adopt the access strategy of the SP approach: to access if and only if the channel is sensed as idle. We refer this approach as the PHY layer approach. Proposition 2 provides a sufficient condition under which this PHY layer approach is locally optimal.

Proposition 2: Suppose that the access strategy is designed independently across channels while the spectrum sensor jointly exploits the measurements taken from all chosen channels. The PHY layer approach is locally optimal. When channel occupancies are independent, the PHY layer approach reduces to the SP approach.

Proof: See [9]. □□□

4.2. The MAC Layer Approach

When channel occupancies are correlated, so are the sensing outcomes given by the spectrum sensor. Hence, the channel correlation can also be exploited at the MAC layer by making access decisions jointly across channels. A heuristic MAC layer approach is to adopt the SP sensor, which detects the channel occupancy by using only the measurements of this channel, and then choose the access decisions that exploit sensing outcomes from all chosen channels to maximize the expected immediate reward. Specifically, for any chosen channels $\mathcal{A}(t)$ and any belief vector $\mathbf{\Lambda}(t)$ in slot t, we choose the set of transmission probabilities as follows

$$
\begin{aligned}
\{\hat{f}_n(\boldsymbol{\theta}_\mathcal{A}, t)\} &= \arg \max_{f_n(\boldsymbol{\theta}_\mathcal{A}) \in [0,1]} \mathbb{E}[R^{(\mathcal{A}(t))}_{\mathbf{K}_\mathcal{A}(t)} | \mathbf{\Lambda}(t)] \\
&= \arg \max_{f_n(\boldsymbol{\theta}_\mathcal{A}) \in [0,1]} \sum_{n \in \mathcal{A}(t)} B_n \Pr\{S_n(t) = 1\} \\
&\quad \times \sum_{\boldsymbol{\theta}_\mathcal{A}, \mathbf{s}_\mathcal{A}} h_{\mathbf{S}_\mathcal{A}|S_n}(\mathbf{s}_\mathcal{A} \,|\, 1)\, l_{\mathbf{\Theta}_\mathcal{A}|\mathbf{S}_\mathcal{A}}(\boldsymbol{\theta}_\mathcal{A} \,|\, \mathbf{s}_\mathcal{A})\, f_n(\boldsymbol{\theta}_\mathcal{A}), \\
\text{s.t. } P_n(t) &= \sum_{\boldsymbol{\theta}_\mathcal{A}, \mathbf{s}_\mathcal{A}} h_{\mathbf{S}_\mathcal{A}|S_n}(\mathbf{s}_\mathcal{A} \,|\, 0) \\
&\quad \times l_{\mathbf{\Theta}_\mathcal{A}|\mathbf{S}_\mathcal{A}}(\boldsymbol{\theta}_\mathcal{A} \,|\, \mathbf{s}_\mathcal{A})\, f_n(\boldsymbol{\theta}_\mathcal{A}) \leq \zeta, \ \ \forall n \in \mathcal{A}(t),
\end{aligned}
$$

where $h_{\mathbf{S}_\mathcal{A}|S_n}(\mathbf{s}_\mathcal{A} \,|\, i) \stackrel{\Delta}{=} \Pr\{\mathbf{S}_\mathcal{A}(t) = \mathbf{s}_\mathcal{A} \,|\, S_n(t) = i\}$, $i = 0, 1$, is the conditional distribution of the channel occupancy states $\mathbf{S}_\mathcal{A}(t)$, which can be obtained from the belief vector via (2), and $l_{\mathbf{\Theta}_\mathcal{A}|\mathbf{S}_\mathcal{A}}(\boldsymbol{\theta}_\mathcal{A} \,|\, \mathbf{s}_\mathcal{A}) \stackrel{\Delta}{=} \Pr\{\mathbf{\Theta}_\mathcal{A}(t) = \boldsymbol{\theta}_\mathcal{A} \,|\, \mathbf{S}_\mathcal{A}(t) = \mathbf{s}_\mathcal{A}\}$

is the sensing error probability determined by the operating characteristics of the SP sensor.

We can obtain the above access strategy via linear programming. Proposition 3 shows that this MAC layer approach is equivalent to the SP approach when channel occupancies are independent. This agrees with our intuition that when channels are independent, so are the sensing outcomes from the chosen channels. Hence, independent access decision-making performs as well as the joint one in terms of immediate reward.

Proposition 3: Suppose that the spectrum sensor is designed independently across channels while the access strategy jointly exploits the sensing outcomes from all chosen channels. When channel occupancies are independent, the MAC layer approach reduces to the SP approach and hence is locally optimal.

Proof: See [9]. □□□

5. SIMULATION EXAMPLES

In this section, we compare the performance of the SP, the PHY layer, and the MAC layer approaches. Since these three approaches differ in the spectrum sensor and the access strategy, we can employ any sensing strategy to compare their performance. For simplicity, we adopt a myopic sensing strategy that chooses, in each slot, the set $\mathcal{A}$ of channels that maximizes the expected immediate reward under perfect sensing,

$$\mathcal{A} = \arg\max_{\mathcal{A}\in\mathbb{A}_s} \sum_{n\in\mathcal{A}} B_n \Pr\{S_n(t)=1\}. \tag{4}$$

We model both the background noise and the primary signal as white Gaussian processes. Let σ_0^2 and σ_1^2 denote the noise power and the primary signal power, respectively. At the beginning of each slot, the spectrum sensor takes M independent measurements $\mathbf{Y}_n \stackrel{\Delta}{=} [Y_{n,1},\ldots,Y_{n,M}]$ from each chosen channel $n\in\mathcal{A}$.

For each chosen channel $n\in\mathcal{A}$, the SP sensor uses the corresponding measurements $\mathbf{Y}_n$ and performs the following hypothesis test:

$$\begin{aligned}\mathcal{H}_0(S_n=1): &\qquad \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, \sigma_0^2\mathbf{I}_M),\\ \mathcal{H}_1(S_n=0): &\qquad \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, (\sigma_1^2+\sigma_0^2)\mathbf{I}_M),\end{aligned} \tag{5}$$

where $\mathcal{N}(\mathbf{0}_M,\sigma^2\mathbf{I}_M)$ denotes an M-dimensional Gaussian distribution with identical mean 0 and variance σ^2 in each dimension. It can be readily shown that the optimal NP detector is given by an energy detector [10, Sec. 2.6.2]:

$$||\mathbf{Y}_n||_2 = \sum_{i=1}^{M} Y_{n,i}^2 \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \eta_n. \tag{6}$$

The probabilities of false alarm ϵ_n and miss detection δ_n of the energy detector can be calculated by [10, Sec. 2.6.2]:

$$\delta_n = \gamma\left(\frac{M}{2}, \frac{\eta_n}{2(\sigma_0^2+\sigma_1^2)}\right),\ \ \epsilon_n = 1-\gamma\left(\frac{M}{2}, \frac{\eta_n}{2\sigma_0^2}\right),$$

where $\gamma(m,a) = \frac{1}{\Gamma(m)}\int_0^a t^{m-1}e^{-t}\,dt$ is the incomplete gamma function. The optimal decision threshold η_n^* of the energy detector is chosen so that the probability of miss detection is fixed at $\delta_n=\zeta$.

On the other hand, the sensor of the PHY layer approach uses all channel measurements $\{\mathbf{Y}_n\}_{n\in\mathcal{A}}$ and performs a composite hypothesis test for each chosen channel $n\in\mathcal{A}$:

$$\begin{aligned}&\mathcal{H}_0(S_n(t)=1): \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, \sigma_0^2\mathbf{I}_M),\\ &\qquad \mathbf{Y}_m \sim \mathcal{N}(\mathbf{0}_M, (\sigma_0^2+1_{[S_m=0]}\sigma_1^2)\mathbf{I}_M),\ \forall m\in\mathcal{A}\backslash\{n\}\\ &\mathcal{H}_1(S_n(t)=0): \mathbf{Y}_n \sim \mathcal{N}(\mathbf{0}_M, (\sigma_0^2+\sigma_1^2)\mathbf{I}_M),\\ &\qquad \mathbf{Y}_m \sim \mathcal{N}(\mathbf{0}_M, (\sigma_0^2+1_{[S_m=0]}\sigma_1^2)\mathbf{I}_M),\ \forall m\in\mathcal{A}\backslash\{n\},\end{aligned} \tag{7}$$

The distribution of the channel occupancy states $\mathbf{S}_{\mathcal{A}}(t)$ under each hypothesis is given by $h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}\,|\,i)$. The optimal NP detector for (7) is a likelihood ratio test [10, Sec. 2.5]:

$$\frac{\sum_{\mathbf{s}_{\mathcal{A}}} h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}\,|\,0)\prod_{m\in\mathcal{A}} p_m(\mathbf{Y}_m|s_m)}{\sum_{\mathbf{s}_{\mathcal{A}}} h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}\,|\,1)\prod_{m\in\mathcal{A}} p_m(\mathbf{Y}_m|s_m)} \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \tau_n, \tag{8}$$

where $p_n(\mathbf{Y}_n|s_n)$ is the probability density function of independent Gaussian channel measurements $\mathbf{Y}_n$:

$$p_n(\mathbf{Y}_n|s_n) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi(\sigma_0^2+1_{[s_n=0]}\sigma_1^2)}} e^{-\frac{Y_{n,i}^2}{2(\sigma_0^2+1_{[s_n=0]}\sigma_1^2)}}.$$

Note that when channel occupancies are independent, the sensor employed by the PHY layer approach is equivalent to the SP sensor since $h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}}\,|\,0) = h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}'_{\mathcal{A}}\,|\,1)$ for any $\mathbf{s}_{\mathcal{A}}$ and $\mathbf{s}'_{\mathcal{A}}$ such that $s_m = s'_m$ when $m\neq n$. The error probabilities of this sensor can be evaluated via simulation. In each slot, the optimal detection threshold τ_n^* is chosen according to the belief vector so that the resulting probability of miss detection is fixed at ζ.

As proven in Propositions 2 and 3, the PHY layer and the MAC layer approaches are equivalent to the SP approach when channel occupancies are independent. We thus compare below the performance of these three approaches in correlated channels. Specifically, we consider $N=4$ correlated channels, each with bandwidth $B_n=1$. The transition probabilities of the spectrum occupancy are given by

$$P_{[0000],[0111]} = 0.6, \quad P_{[0000],[0000]} = 0.4,$$
$$P_{[0111],[0000]} = P_{[1011],[0000]} = P_{[1101],[0000]} = P_{[1110],[0000]} = 0.2,$$
$$P_{[0111],[1011]} = P_{[1011],[1101]} = P_{[1101],[1110]} = P_{[1110],[0111]} = 0.8.$$

For simplicity, we assume that there is only one secondary user seeking instantaneous spectrum availability. The initial belief vector is set to the stationary distribution of the underlying Markov process. The maximum allowable probability of collision is $\zeta=0.05$. In each slot, $L=3$ channels are chosen. The spectrum sensor takes $M=1$ measurement of each chosen channel and the noise and the primary signal powers are given by $\sigma_0^2 = 0$ dB and $\sigma_1^2 = 10$ dB.

5.1. Comparison of Sensor Performance

In Figure 1, we plot the receiver operating characteristic (ROC) curves (probability of false alarm vs. probability of detection) of the SP sensor and the sensor employed by the PHY layer approach. Note that the sensor of the MAC layer approach is the same as the SP sensor. We see that the sensor of the PHY layer approach outperforms the SP sensor. Specifically, for a fixed probability of miss detection, the probability of false alarm of the sensor of the PHY layer approach is much smaller than that of the SP sensor. This is because

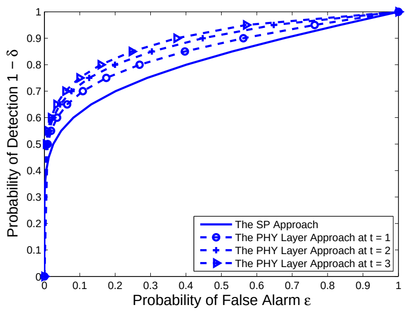

Fig. 1. Comparison of ROC curves.

the PHY layer approach exploits the correlation among channel measurements in occupancy detection while the SP sensor does not. We also observe that the ROC curve of the sensor employed by the PHY layer approach improves in each slot while that of the SP sensor remains the same. This observation can be explained by comparing the optimal detectors (6) and (8). Clearly, the energy detector (6) used by the SP approach is static and so is its performance. However, as seen from (8), the decision variable of the sensor employed by the PHY layer approach depends on the conditional distribution $h_{\mathbf{S}_{\mathcal{A}}|S_n}(\mathbf{s}_{\mathcal{A}} \,|\, i)$ of the channel occupancies. Hence, its performance varies over time according to the belief vector. As time t increases, the belief vector and hence the sensor performance improves due to the accumulated observations. Figure 1 demonstrates that the performance of the PHY layer sensor can be improved by incorporating the MAC layer sensing and access decisions encoded in the belief vector.

5.2. Comparison of Throughput Performance

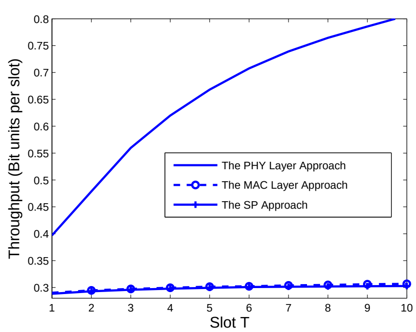

Fig. 2. Comparison of normalized throughput.

In Figure 2, we compare the throughput, measured by the total expected reward per slot, of these three approaches. As expected, the SP approach, which ignores the channel correlation, performs the worst. By jointly exploiting the sensing outcomes in access decision-making, the MAC layer approach can improve throughput performance. A much larger performance gain is achieved by the PHY layer approach which jointly exploits the channel measurements in occupancy detection. We can thus see that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer. In other words, independent channel occupancy detection at the PHY layer hurts throughput more than independent access decision-making at MAC layer. This agrees with our intuition because the spectrum sensor of the MAC layer approach makes a hard-decision on whether the channel is idle or not. The resulting sensing outcomes are thus less informative than the original channel measurements, leading to throughput degradation.

6. CONCLUSION

In this paper, we provided sufficient conditions for the existence of separation principle for optimal design of OSA with multi-channel sensing. We also proposed two heuristic approaches that exploit the correlation among channel occupancies. Simulation examples demonstrated the performance improvement of the PHY layer spectrum sensor over time resulting from the incorporation of the MAC layer sensing and access decisions. We also found that the exploitation of channel correlation at the PHY layer is more effective than that at the MAC layer in improving throughput.

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