Joint Design and Separation Principle for Opportunistic Spectrum Access

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Abstract— **This paper develops optimal strategy for opportunistic spectrum access (OSA) by integrating the design of spectrum sensor at the physical layer with that of spectrum sensing and access policies at the medium access control (MAC) layer. The design objective is to maximize the throughput of secondary users while limiting their probability of colliding with primary users. By exploiting the rich structures of the problem, we establish a separation principle: the design of spectrum sensor and access policy can be decoupled from that of sensing policy without losing optimality. This separation principle enables us to obtain closed-form optimal sensor operating characteristic and access policy, leading to significant complexity reduction. It also allows us to study the inherent interaction between spectrum sensor and access policy and the tradeoff between false alarm and miss detection in opportunity identification.**

I. Introduction

The exponential growth in wireless services and the physical limit on usable radio frequencies have motivated the development of dynamic spectrum sharing and allocation technologies. Opportunistic spectrum access (OSA), first envisioned by Mitola [1] and then investigated by the DARPA XG program [2], has received great attention [3] due to its potential in improving spectrum efficiency. The idea of OSA is to allow secondary users to identify, search for, and use instantaneous spectrum opportunities in a manner that limits the level of interference perceived by primary users. Correspondingly, there are three basic components of OSA: 1) a spectrum sensor at the physical layer that identifies spectrum opportunities; 2) a sensing policy at the medium access control (MAC) layer that specifies which channels to sense; and 3) an access policy at the MAC layer that determines the subset of channels on which to transmit based on the sensing outcomes.

In this paper, we aim at optimal OSA by integrating the design of spectrum sensor at the physical layer with that of sensing and access policies at the MAC layer. The objective is to maximize the throughput of secondary users under the constraint that the probability of collision perceived by primary users is below a certain threshold. We formulate the joint OSA design as a constrained partially observable Markov decision process (POMDP), which often requires randomized policies to achieve optimality. By exploiting the underlying structure of the problem, we establish a separation principle: the design of spectrum sensor and access policy can be decoupled from that of sensing policy without losing optimality. This separation principle allows us to obtain closed-form optimal sensor operating point and access policy, and reduce a constrained POMDP to an unconstrained one, leading to deterministic optimal sensing policy. It also enables us to study the inherent interaction between spectrum sensor and access policy and obtain the best tradeoff between false alarm and miss detection in spectrum opportunity identification.

Related Work Differing from this paper that mainly addresses the exploitation of temporal spectrum opportunities resulting from the bursty traffic of primary users, a majority of existing work (see [4]–[6] and references therein) focus on geographic spectrum opportunities that are static or slowly varying in time. The application being considered is a network of geographically distributed secondary users, each affected by a different set of primary users whose spectrum access activities are considered static over a long period of time. The design objective is to allocate these spatially varying spectrum opportunities among secondary users so that the network-level spectrum efficiency is maximized subject to some regulatory constraint on interference to primary users. Due to the slow temporal variation of spectrum occupancy, opportunity identification is not as critical a component in this class of applications, and the existing work often assumes perfect knowledge of spectrum opportunities in the whole spectrum at any location.

Research efforts have also been made to exploit temporal spectrum opportunities [7]–[9] under perfect spectrum sensing. However, in the presence of noise and fading, sensing errors will occur. Hence, the design of sensing and access policies should take into account the operating characteristics of the spectrum sensor. A heuristic approach has been proposed in [9] for OSA in the presence of sensing errors. This paper develops a mathematical framework for the optimal joint design of spectrum sensor at the physical layer and sensing and access policies at the MAC layer.

II. Network Model

Consider a spectrum that consists of N channels, each with bandwidth B_n $(n = 1, \cdots, N)$. These N channels are licensed to a slotted primary network. We model the spectrum occupancy of primary users by a discrete-time Markov process

[0]This work was supported in part by the Army Research Laboratory CTA on Communication and Networks under Grant DAAD19-01-2-0011 and by the National Science Foundation under Grants CNS-0627090 and ECS-0622200.

1-4244-0785-0/06/$20.00

whose states are defined as $\mathbf{S}(t) \stackrel{\Delta}{=} [S_1(t), \ldots, S_N(t)]$, where $S_n(t) \in \{0 \text{ (busy)}, 1 \text{ (idle)}\}$ denotes the availability of channel n in slot t. The state space $\mathcal{S}$ of this Markov process is thus given by $\mathcal{S} = \{0,1\}^N$. The probability that the spectrum occupancy transits from state $\mathbf{s}$ to state $\mathbf{s}'$ is denoted by $P_{\mathbf{s},\mathbf{s}'}$.

We consider a group of secondary users seeking spectrum opportunities in these N channels. We focus on an ad hoc network where secondary users sense and access the spectrum independently. At the beginning of each slot, a secondary user with data to transmit chooses at most L channels to sense and then to decide whether to access these channels based on the sensing outcomes. Such spectrum sensing and access decisions are made to maximize its own throughput while limiting the interference to primary users by fully exploiting the sensing history and the spectrum occupancy dynamics. When the secondary user decides to transmit, it generates a random backoff time, and transmits when this timer expires and no other secondary user has already accessed the channel during the backoff time. At the end of the slot, the receiver acknowledges a successful data transmission. The basic slot structure is illustrated in Fig. 1. Details on implementation can be found in [9]. In this paper, we assume $L = 1$. Extension to $L > 1$ will be discussed in the upcoming journal paper.

Our goal is to jointly optimize the spectrum sensor and the sensing/access policies to maximize the throughput of the secondary user in T slots under the constraint that the probability $P_a(t)$ of collision perceived by the primary network in any channel and any slot is below a threshold ζ, *i.e.,* $P_a(t) \stackrel{\Delta}{=} \Pr\{\text{collision } | S_a(t) = 0\} \leq \zeta$ for any channel a and slot t. Note that the maximum allowed collision probability ζ is generally specified by the primary network.

III. Problem Formulation

In this section, we formulate the optimal joint OSA design as a constrained POMDP problem. Involved in the design are three basic components: a spectrum sensor, a sensing policy, and an access policy.

A. Spectrum Sensor

The spectrum sensor of a secondary user detects, at the beginning of each slot, the availability of the chosen channel. It can be considered as performing a binary hypotheses test: $\mathcal{H}_0$ (null hypothesis indicating that the sensed channel is idle) vs. $\mathcal{H}_1$ (alternative). Let Θ_a be the sensing outcome (the result of the hypotheses test): $\Theta_a = 1$ (idle) and $\Theta_a = 0$ (busy).

If the sensor mistakes $\mathcal{H}_0$ for $\mathcal{H}_1$, a false alarm occurs, and a spectrum opportunity is overlooked by the sensor. On the other hand, when the sensor mistakes $\mathcal{H}_1$ for $\mathcal{H}_0$, we have a miss detection. Let $\epsilon \stackrel{\Delta}{=} \Pr\{\Theta_a = 0 \,|\, S_a = 1\}$ and $\delta \stackrel{\Delta}{=} \Pr\{\Theta_a = 1 \,|\, S_a = 0\}$ denote, respectively, the probabilities of false alarm and miss detection. The performance of a sensor is specified by the receiver operating characteristics (ROC) curve which gives the probability of detection $1-\delta$ as a function of ϵ (an example is given in Fig. 2). We point out that analyzing the ROC curve of the spectrum sensor in a wireless network environment can be complex. We assume here that the ROC curve of the spectrum sensor has already been obtained, and we focus on the tradeoff between false alarm and miss detection. Specifically, we seek answer to the question which point δ on the given ROC curve the spectrum sensor should operate at.

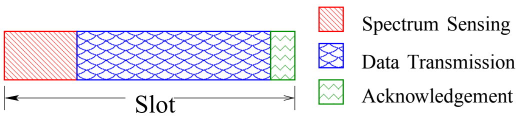

Fig. 1. The slot structure.

If the secondary user completely trusts the sensing outcome in decision-making, false alarms result in wasted spectrum opportunities while miss detections lead to collisions with primary users. To optimize the performance of the secondary user while limiting its interference to the primary network, we should carefully choose the sensor operating point. Meanwhile, the spectrum access decisions should be made by taking into account the sensor operating characteristics. A joint design of the spectrum sensor at the physical layer and the access policy at the MAC layer is thus necessary to achieve optimality.

B. Sensing and Access Policies

The sensing policy specifies, in each slot, which channel to sense, and the access policy determines whether to transmit based on the sensing outcome. At the beginning of a slot, a secondary user with data to transmit chooses a channel $a \in \{1, \ldots, N\}$ to sense. Based on the sensing outcome Θ_a, the secondary user decides whether to transmit over the sensed channel: $\Phi_a \in \{0 \text{ (no access)}, 1 \text{ (access)}\}$. At the end of the slot, the receiver acknowledges a successful data transmission: $K_a \in \{0 \text{ (unsuccessful)}, 1 \text{ (successful)}\}$. Note that an acknowledgement $K_a = 1$ is obtained if and only if the secondary user chooses to access $\Phi_a = 1$ and the channel is idle $S_a = 1$, *i.e.,* $K_a = 1_{[S_a=1,\Phi_a=1]}$. A reward $R_{K_a}^{(a,\Phi_a)}$ is accrued depending on K_a. Assuming that the number of information bits that can be transmitted is proportional to the channel bandwidth, we define the reward $R_{K_a}^{(a,\Phi_a)}$ obtained by choosing sensing and access action (a, Φ_a) as:

$$R_{K_a}^{(a,\Phi_a)} = K_a B_a. \tag{1}$$

Due to partial spectrum monitoring and sensing errors, the secondary user and the receiver cannot directly observe the current state of the spectrum occupancy. We thus have a POMDP. It has been shown in [11] that the knowledge of the current spectrum occupancy state based on all past decisions (*i.e.,* sensing and access actions) and observations (*i.e.,* acknowledgements) can be summarized by a belief state $\mathbf{\Lambda}(t) \stackrel{\Delta}{=} \{\lambda_{\mathbf{s}}(t)\}_{\mathbf{s}\in\mathcal{S}}$. Each element $\lambda_{\mathbf{s}}(t)$ of the belief state $\mathbf{\Lambda}(t)$ is the conditional probability (given the decision and observation history) that the current spectrum occupancy state is given by $\mathbf{S}(t) = \mathbf{s}$ prior to the state transition in slot t. Hence, a sensing policy π_s is given by a sequence of functions: $\pi_s = [\mu_1, \ldots, \mu_T]$ where $\mu_t : [0,1]^{|\mathcal{S}|} \rightarrow \{1, \ldots, N\}$ maps the belief state $\mathbf{\Lambda}(t) \in [0,1]^{|\mathcal{S}|}$ at the beginning of slot t to a channel $a \in \{1, \ldots, N\}$ to be sensed. An access policy π_c is given by a sequence of functions: $\pi_c = [\nu_1, \ldots, \nu_T]$ where $\nu_t : [0,1]^{|\mathcal{S}|} \times \{0,1\} \rightarrow \{0,1\}$ maps the belief state $\mathbf{\Lambda}(t) \in [0,1]^{|\mathcal{S}|}$ and the sensing outcome $\Theta_a \in \{0,1\}$ of the chosen channel a to an access action $\Phi_a \in \{0,1\}$.

In this paper, we assume that the state transition probabilities $\{P_{\mathbf{s},\mathbf{s}'}\}$ of the underlying Markov model are known. If the transition probabilities are unknown, formulations and algorithms for POMDP with an unknown model exist in the literature [12] and can be applied to our problem. In Section V, we study the impact of mismatched Markov model on the OSA performance.

C. Design Objective

We aim to determine the optimal sensor operating point δ and the optimal sensing and access policies $\{\pi_s, \pi_c\}$. The objective is to maximize the throughput of the secondary user (or equivalently the total expected reward) in T slots under the collision constraint:

$$\{\delta^*, \pi_s^*, \pi_c^*\} = \arg \max_{\delta, \pi_s, \pi_c} \mathbb{E}_{\{\delta, \pi_s, \pi_c\}} \left[\sum_{t=1}^{T} R_{K_a}^{(a,\Phi_a)}(t) \middle| \mathbf{\Lambda}(1) \right]$$
$$\text{s.t. } P_a(t) = \Pr\{\Phi_a(t) = 1 \,|\, S_a(t) = 0, \mathbf{\Lambda}(t)\} \leq \zeta \text{ holds}$$
$$\text{for any } a \text{ and } t \text{ such that } \Pr\{S_a(t) = 0 \,|\, \mathbf{\Lambda}(t)\} > 0, \quad (2)$$

where $\mathbb{E}_{\{\delta, \pi_s, \pi_c\}}$ is the expectation given that sensing and access policies $\{\pi_s, \pi_c\}$ are employed and sensor operates at point δ, $\mathbf{\Lambda}(1)$ is the initial belief state which is usually given by the stationary distribution of the spectrum occupancy states. Note that when $\Pr\{S_a(t) = 0 \,|\, \mathbf{\Lambda}(t)\} = 0$, *i.e.,* channel a is available with probability 1 in slot t, the constraint in (2) becomes irrelevant and the secondary user's access decision is simply $\Phi_a(t) = 1$. In the rest of this paper, we consider the non-trivial case where $\Pr\{S_a(t) = 0 \,|\, \mathbf{\Lambda}(t)\} > 0$ in any channel a and slot t.

IV. Separation Principle for Optimal Joint Design

The design objective given in (2) is a constrained POMDP, which usually requires randomized policies to achieve optimality. In this case, a sensing policy determines the mapping from the current belief state to the probability of choosing each channel and an access policy the mapping from the current belief state to the transmission probabilities under different sensing outcomes. Since there exist uncountably many probability distributions, randomized policies are computationally prohibitive. In this section, we establish a separation principle for the optimal joint design. This separation principle reveals the existence of deterministic optimal sensing and access policies, leading to significant complexity reduction. It also enables us to obtain closed-form optimal sensor operating point at which the best tradeoff between false alarms and miss detections is achieved.

A. The Impact of Sensor Operating Point on Access Policy

Let $f_a^\theta(\mathbf{\Lambda}(t), t)$ be the probability of transmitting over chosen channel a given sensing outcome $\Theta_a = \theta$ and belief state $\mathbf{\Lambda}(t)$ at the beginning of slot t. In Theorem 1, we derive closed-form optimal transmission probabilities $(f_a^1(\mathbf{\Lambda}(t), t), f_a^0(\mathbf{\Lambda}(t), t))$ for different given sensor operating points δ.

Theorem 1: The optimal access policy is time-invariant and belief-independent. Specifically, the optimal transmission probabilities are solely determined by the sensor operating point δ and the maximum allowed probability of collision ζ, i.e., for any chosen channel a, belief state $\mathbf{\Lambda}(t)$, and slot t, we have

$$(f_a^1(\mathbf{\Lambda}(t), t), f_a^0(\mathbf{\Lambda}(t), t)) = \begin{cases} (1, \frac{\zeta-\delta}{1-\delta}), & \delta < \zeta, \\ (1, 0), & \delta = \zeta, \\ (\frac{\zeta}{\delta}, 0), & \delta > \zeta. \end{cases} \quad (3)$$

Proof: See [10] for details. □

Theorem 1 enables us to study the impact of sensor operating characteristics on the optimal access policy. As illustrated in Fig. 2, the ROC curve can be partitioned into two regions: the "conservative" region ($\delta > \zeta$) and the "aggressive" region ($\delta < \zeta$). When $\delta > \zeta$, the spectrum sensor is more likely to misidentify an opportunity (*i.e.,* a busy channel is sensed to be idle). Hence, the access policy should be conservative to ensure that the probability of collision is bounded below ζ. Specifically, even when the sensing outcome $\Theta_a = 1$ indicates that the channel is available, the user should only transmit with probability $\frac{\zeta}{\delta} < 1$. When the channel is sensed to be busy: $\Theta_a = 0$, the user should trust the sensing outcome and refrain from transmission. On the other hand, when $\delta < \zeta$, the spectrum sensor is more likely to overlook an opportunity (*i.e.,* an idle channel is sensed to be busy). Hence, the user should adopt an aggressive access policy: always transmit when the channel is sensed to be available and transmit with probability $\frac{\zeta-\delta}{1-\delta} > 0$ even when the channel is sensed to be busy. When $\delta = \zeta$, the optimal access policy is deterministic: always trust the sensing outcome.

B. The Separation Principle

Given belief state $\mathbf{\Lambda}(t)$ at the beginning of slot t, we can rewrite the design constraint in (2) as

$$P_a(t) = \sum_{\theta=0}^{1} \Pr\{\Phi_a = 1 \,|\, \Theta_a = \theta\} \Pr\{\Theta_a = \theta \,|\, S_a(t) = 0\}$$
$$= \delta f_a^1(\mathbf{\Lambda}(t), t) + (1-\delta) f_a^0(\mathbf{\Lambda}(t), t). \quad (4)$$

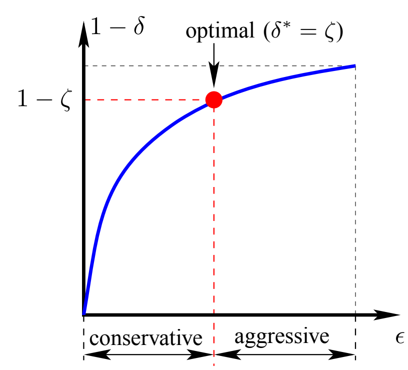

Fig. 2. Partition of an ROC curve.

Careful inspection of (3) and (4) reveals that the constraint given in (2) is satisfied regardless of the chosen channel. We thus have a separation principle (Theorem 2) for the optimal joint OSA design, which decouples the design of spectrum sensor and access policy from that of sensing policy. Following this separation principle, we obtain closed-form optimal sensor operating point δ^* and access policy π_c^* in Theorem 3.

Theorem 2: **Separation Principle**

The joint design of OSA formulated in (2) *can be obtained in two steps without losing optimality. First, choose sensor operating point δ and access policy π_c according to* (3) *to maximize the expected immediate reward. Second, choose sensing policy π_s to maximize the expected total reward.*

Proof: See [10] for details. □

Theorem 3: The optimal sensor operating point is $\delta^* = \zeta$. The optimal access policy π_c^* is given by $\Phi_a^* = \Theta_a$.

Proof: See [10] for details. □

Theorem 3 reveals the existence of deterministic optimal access policy for the constrained POMDP given in (2). Specifically, the optimal access policy π_c^* is to simply trust the sensing outcome: $\Phi_a^* = \Theta_a$, *i.e.,* access if and only if the channel is detected to be available.

C. The Optimal Sensing Policy

In Theorem 3, we have obtain the optimal sensor operating point δ^* and the optimal access policy π_c^*. Since δ^* and π_c^* have been chosen to ensure the constraint regardless of the chosen channel, we are free to search for the optimal sensing policy π_s^* over the whole design space. The design of the sensing policy thus becomes an unconstrained POMDP, where optimality can be achieved by deterministic policies.

Let $V_t(\mathbf{\Lambda}(t))$ denote the maximum total expected reward obtained from slot $1 \le t \le T$ given the belief state $\mathbf{\Lambda}(t)$ at the beginning of slot t. Given sensor operating point δ^* and access policy π_c^*, we obtain $V_t(\mathbf{\Lambda}(t))$ recursively by

$$\begin{aligned} V_t(\mathbf{\Lambda}(t)) &= \max_a \sum_{\mathbf{s}\in\mathcal{S}} \sum_{\mathbf{s}'\in\mathcal{S}} \Lambda_{\mathbf{s}'}(t) P_{\mathbf{s}',\mathbf{s}} \sum_{k_a=0}^{1} Q_{\mathbf{s}}(k_a) \\ &\qquad \times [k_a B_a + V_{t+1}(\mathcal{T}(\mathbf{\Lambda}(t) \,|\, a, k_a))], \quad 1 \le t < T, \\ V_T(\mathbf{\Lambda}(T)) &= \max_a \sum_{\mathbf{s}\in\mathcal{S}} \sum_{\mathbf{s}'\in\mathcal{S}} \Lambda_{\mathbf{s}'}(t) P_{\mathbf{s}',\mathbf{s}} Q_{\mathbf{s}}(1) B_a, \end{aligned} \tag{5}$$

where $Q_{\mathbf{s}}(0) = 1 - Q_{\mathbf{s}}(1)$, $Q_{\mathbf{s}}(1) \stackrel{\Delta}{=} \Pr\{K_a = 1 \,|\, \mathbf{S}(t) = \mathbf{s}\} = 1_{[s_a=1]}(1-\epsilon^*)$ is the probability of successful transmission under current spectrum occupancy state $\mathbf{S}(t) = \mathbf{s} = [s_1, \ldots, s_N]$. Note that $1_{[s_a=1]}$ indicates whether channel a is idle given $\mathbf{S}(t) = \mathbf{s}$ and ϵ^* is the probability of false alarm that can be achieved when the spectrum sensor operates at δ^*. The updated belief state $\mathbf{\Lambda}(t+1) = \mathcal{T}(\mathbf{\Lambda}(t) \,|\, a, k_a)$ can be obtained via Bayes rule as

$$\Lambda_{\mathbf{s}}(t+1) = \frac{\sum_{\mathbf{s}'\in\mathcal{S}} \Lambda_{\mathbf{s}'}(t) P_{\mathbf{s}',\mathbf{s}} Q_{\mathbf{s}}(k_a)}{\sum_{\mathbf{s}''\in\mathcal{S}} \sum_{\mathbf{s}'\in\mathcal{S}} \Lambda_{\mathbf{s}'}(t) P_{\mathbf{s}',\mathbf{s}''} Q_{\mathbf{s}''}(k_a)}. \tag{6}$$

The optimal sensing policy π_s^* can be obtained by solving the optimality equation given in (5). It is shown in [11] that $V_t(\mathbf{\Lambda}(t))$ is piecewise linear and convex, leading to a linear programming procedure for calculating π_s^*.

V. Simulation Examples

In this section, we provide two simulation examples to study the impacts of sensor operating point δ and mismatched Markov model on the performance of the optimal OSA. We consider $N = 3$ independently evolving channels with the same bandwidth $B_n = 1$. As illustrated in Fig. 3, the state transition of spectrum occupancy can be characterized by $\boldsymbol{\alpha} \stackrel{\Delta}{=} [\alpha_1, \alpha_2, \alpha_3]$ and $\boldsymbol{\beta} \stackrel{\Delta}{=} [\beta_1, \beta_2, \beta_3]$, where α_n denotes the probability that channel n transits from state 0 (busy) to state 1 (idle) and β_n denotes the probability that it stays in state 1. We assume that the spectrum occupancy dynamics remain unchanged during $T = 10$ slots. The throughput of the secondary user is measured by the expected total reward per slot, *i.e.,* $V_1(\mathbf{\Lambda}(1))/T$, where $\mathbf{\Lambda}(1)$ is given by the stationary distribution of the underlying Markov process.

At the beginning of each slot, the spectrum sensor takes M measurements $\{Y_i\}_{i=1}^M$ of the chosen channel. We assume that both the channel noise and the signal of primary users can be modeled as white Gaussian processes $\mathcal{N}$. Then, the spectrum sensor performs the following hypotheses test:

$$\begin{cases} \mathcal{H}_0 \text{ (idle channel)}: & Y_i \sim \mathcal{N}(0, \sigma_0^2), \; i = 1, \cdots, M, \\ \mathcal{H}_1 \text{ (busy channel)}: & Y_i \sim \mathcal{N}(0, \sigma_1^2), \; i = 1, \cdots, M, \end{cases}$$

where σ_0^2 is the noise power and σ_1^2 is the primary signal power. It can be readily shown that the energy detector is optimal under Neyman-Pearson (NP) criterion [13, Sec. 2.6.2]:

$$\sum_{i=1}^{M} Y_i^2 \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \eta, \tag{7}$$

where the threshold η determines the false alarm and miss detection rates of the detector. The ROC curve of the energy detector is given by [13, Sec. 2.6.2]

$$1 - \delta = 1 - \gamma\left(\frac{M}{2}, \eta\frac{\sigma_0^2}{\sigma_1^2}\right), \quad \epsilon = 1 - \gamma\left(\frac{M}{2}, \eta\right) \tag{8}$$

where $(\sigma_1^2 - \sigma_0^2)/\sigma_0^2$ is the SNR and $\gamma(n, a) = \frac{1}{\Gamma(n)} \int_0^a t^{n-1} e^{-t}\, dt$ is the incomplete gamma function. In all the figures, we assume $M = 10$ and SNR = 5dB.

Fig. 4 studies the impact of sensor operating point δ on the throughput and the optimal access policy of the secondary user. The upper figure plots the maximum throughput of the secondary user for each given sensor operating point δ. The optimal access policy is specified by the transmission probabilities (f_a^0, f_a^1), which are shown in the middle and

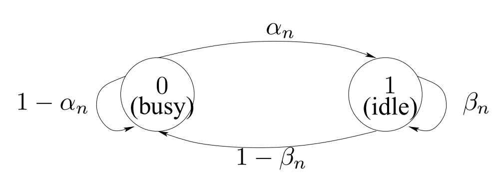

Fig. 3. The Markov channel model

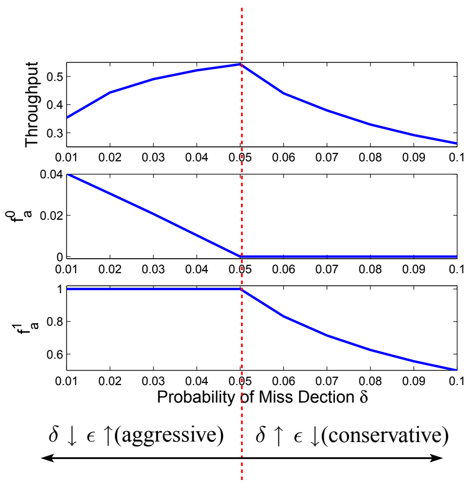

Fig. 4. The impact of spectrum sensor operating point on the throughput of the secondary user. $\boldsymbol{\alpha} = [0.2, 0.4, 0.6]$, $\boldsymbol{\beta} = [0.8, 0.6, 0.4]$, $\zeta = 0.05$.

the lower figures, respectively. We can see that the maximum throughput is achieved at $\delta^* = \zeta = 0.05$ and the transmission probabilities change with δ as given by Theorem 1. Interestingly, the throughput curve is concave with respect to δ in the "aggressive" region ($\delta < \zeta$) and convex in the "conservative" region ($\delta > \zeta$). The performance thus degrades at a faster rate when the sensor operating point drifts toward the "conservative" region. This suggests that miss detections (which lead to collisions) are more harmful to the performance of OSA than false alarms (which represent missed opportunities).

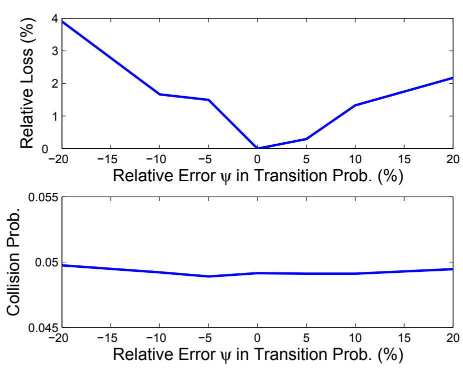

Fig. 5. The impact of inaccurate transition probabilities on the throughput of the secondary user. $\boldsymbol{\alpha} = [0.2, 0.4, 0.6]$, $\boldsymbol{\beta} = [0.8, 0.6, 0.4]$, $\zeta = 0.05$.

In Fig. 5, we study the impact of mismatched Markov model on the performance of the optimal OSA. We assume that the spectrum occupancy evolves according to the transition probabilities given by $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ while the secondary user employs the optimal OSA policy based on inaccurate transition probabilities $\boldsymbol{\alpha}'$ and $\boldsymbol{\beta}'$. In the upper figure, we plot the relative throughput loss of the secondary user as a function of the relative error ψ in transition probabilities which is given by $\psi = \frac{\alpha'_n - \alpha_n}{\alpha_n} \times 100\% = \frac{\beta'_n - \beta_n}{\beta_n} \times 100\%$. Clearly, the maximum throughput is achieved when the relative error is zero (*i.e.,* the secondary user has accurate information on transition probabilities). Inaccurate transition probabilities can cause performance loss. We find that the relative performance loss is below 4% even when the absolute relative error is up to 20%. In the lower figure, we examine the probability of collision perceived by the primary network. We find that the probability of collision is not affected by mismatched transition probabilities. The reason behind this observation is the separation principle: the optimal sensor operating point and the optimal access policy, which determine the probability of collision, are independent of the spectrum occupancy dynamics.

VI. Conclusion

In this paper, we took a cross-layer approach to OSA design. By jointly optimizing the spectrum sensor at the physical layer and the sensing/access policies at the MAC layer, we developed optimal OSA strategy that maximizes the throughput of the secondary user under a constraint on the collision probability perceived by the primary network. By exploiting the rich structure of the problem, we established a separation principle for the optimal joint design, which decouples the design of spectrum sensor and access policy from that of sensing policy. We studied the impact of sensor operating characteristics on the access policy and the tradeoff between false alarm and miss detection in spectrum opportunity identification. We observed that miss detections are more harmful to the performance of OSA than false alarms.

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