Bandwidth Mismatch Correction for a Two-Channel Time-Interleaved A/D Converter

Patrick Satarzadeh, Bernard C. Levy, Paul J. Hurst
Department of Electrical and Computer Engineering
University of California, Davis

May 29, 2007

Outline

- **Motivation**
- Time-Interleaved S/H Model
- Correction Method
- Simulations
- Conclusion

Motivation

- Applications requiring ADCs operating at high data rates $\Rightarrow$ **Time-interleaved ADCs**.
- Constituent ADCs have gain, offset, timing, and bandwidth mismatches that need to be estimated and corrected in the digital domain.
- Correction achieved by digital filters operating on ADCs outputs.

Motivation (cont'd)

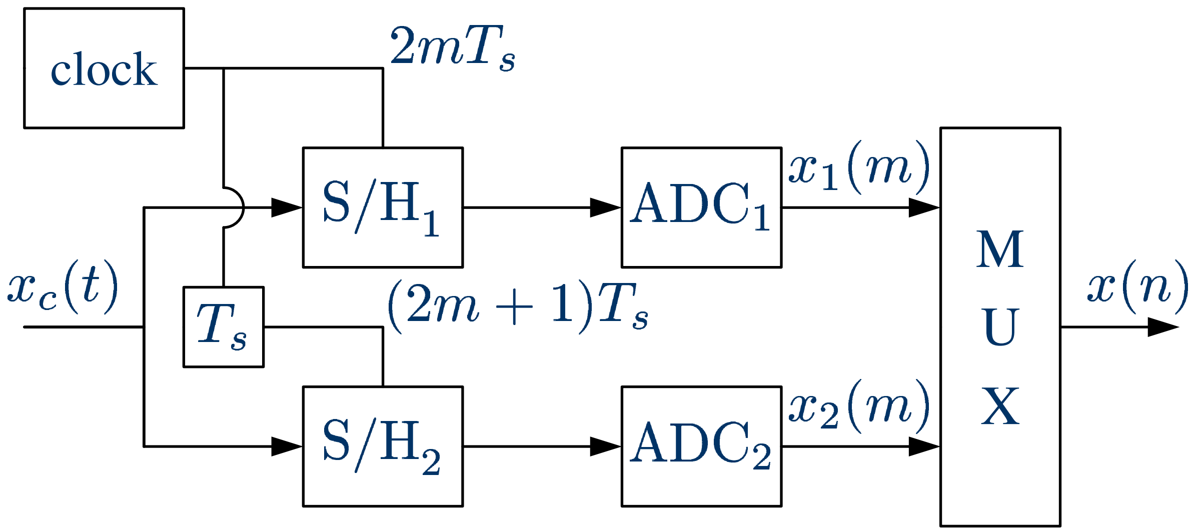
- Much attention has been given to the gain and timing mismatch (Jamal et al. 2004, Elbornsson et al., 2004, Huang and Levy, 2006).
- Attention only recently given to bandwidth mismatch (Tsai, Hurst, and Lewis 2006), where analysis relied on assuming input signal was a sinusoid, correction method required two correction filters
- Our method makes no such assumption regarding input signal other than it being bandlimited
- Philosophy of correction method, correct the output of one channel so the two channel outputs appear to have encountered matched S/Hs, requires only one correction filter

Outline

- Motivation
- **Time-Interleaved S/H Model**
- Correction Method
- Simulations
- Conclusion

Model

- A CT bandlimited signal $x_c(t)$ with bandwidth B, can be recovered from its samples $x(n) = x_c(nT_s)$ if $f_s = 1/T_s \geq f_N = B/\pi$.
- For 2-channel time-interleaved ADC, instead of one fast ADC, can use 2 slow ADCs operating at $f_s/2$.



Model (cont'd)

- Assuming a S/H may be modeled by a RC circuit, where R denotes on-resistance and C denotes hold capacitance. Then operation of first channel given by

$$y_1(t) + \tau_1 \frac{dy_1(t)}{dt} = x_c(t)\ ,\ (2k-1)T_s < t \leq 2kT_s$$
$$y_1(t) = y_1(2kT_s)\ ,\ 2kT_s \leq t \leq (2k+1)T_s$$

yielding the sample $s_1(k) = y_1(2kT_s)$. Similarly for second channel

$$y_2(t) + \tau_2 \frac{dy_2(t)}{dt} = x_c(t)\ ,\ 2kT_s < t \leq (2k+1)T_s$$
$$y_2(t) = y_2\big((2k-1)T_s\big)\ ,\ (2k-1)T_s \leq t \leq 2kT_s$$

yielding sample $s_2(k) = y_2((2k+1)T_s)$, where $\tau_i, i = 1, 2$ are the respective time constants.

Model (cont'd)

- Solving the differential equation for channel one and setting $t = 2kT_s$ yields

$$s_1(k) = \alpha_1 s_1(k-1) + a_1(k) \tag{1}$$

where $\alpha_1 = \exp(-T_s/\tau_1)$ and

$$a_1(k) = \frac{1}{\tau_1}\int_{(2k-1)T_s}^{2kT_s} \exp\left(\frac{-(2kT_s - v)}{\tau_1}\right) x_c(v)dv \, . \tag{2}$$

- Defining $b_1(n) = b_{1,c}(nT_s)$ and

$$b_{1,c}(t) = h_1(t) * x_c(t) \, , \tag{3}$$

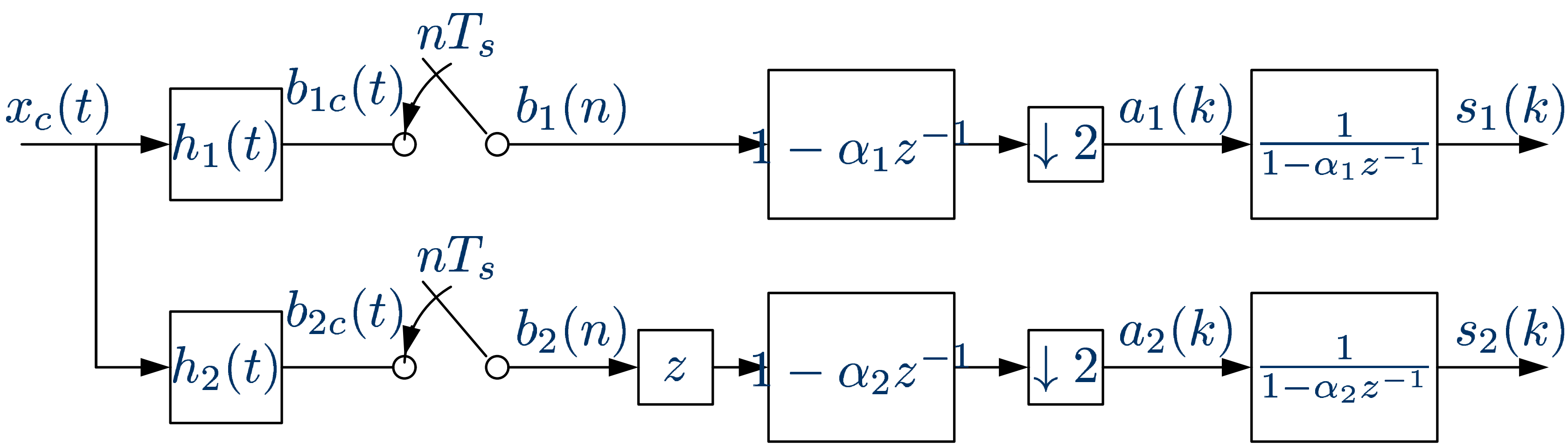
where $h_1(t) = \tau_1^{-1}\exp(-t/\tau_1)$ is the continuous time impulse response of the RC sampling circuit, $a_1(k)$ in may be rewritten as

$$a_1(k) = b_1(2k) - \alpha_1 b_1(2k-1) \, . \tag{4}$$

Department of Electrical and Computer Engineering
University of California at Davis

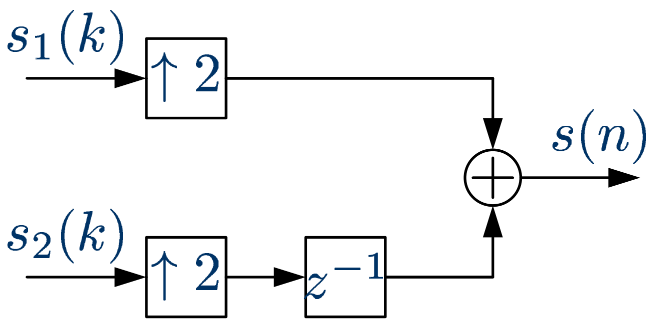
Model (cont'd)

Repeating the calculation for the second channel yields the following model.



Model (cont'd)

The signals $s_1(k)$ and $s_2(k)$ are then recombined using



Outline

- Motivation
- Time-Interleaved S/H Model
- **Correction Method**
- Simulations
- Conclusion

Correction Method

- Seek to correct $s_2(k)$ such that $s_2^c(k)$ is would be produced if $\mathrm{S/H}_2$ had the same time constant as $\mathrm{S/H}_1$
- Assume knowledge of time constants τ_i and attenuation coefficients α_i
- Observe that the filter $1 - \alpha_i z^{-1}$ can be applied to sequence $s_i(k)$ to produce $a_i(k)$, which reduces to the the correction of $a_2(k)$.
- From model, the DTFTs of $a_i(k)$ can be expressed as

$$A_1(e^{j\omega}) = \frac{1}{2}\left[(1 - \alpha_1 e^{-j\omega/2}) B_1(e^{j\omega/2}) + (1 - \alpha_1 e^{-j(\omega/2-\pi)}) B_1(e^{j(\omega/2-\pi)})\right]$$

$$A_2(e^{j\omega}) = \frac{1}{2}\left[(e^{-j\omega/2} - \alpha_2) B_2(e^{j\omega/2}) + (e^{-j(\omega/2-\pi)} - \alpha_2) B_2(e^{j(\omega/2-\pi)})\right]$$

Correction (cont'd)

- Let $R(j\Omega) \triangleq \frac{H_2(j\Omega)}{H_1(j\Omega)}$ represent the relative mismatch between the S/Hs, where $H_i(j\Omega) = (1 + j\Omega\tau_i)^{-1}$, $i = 1,\, 2$ denotes CTFT of the RC sampling circuit.

- Then we can express

$$\begin{bmatrix} A_1(e^{j\omega}) \\ A_2(e^{j\omega}) \end{bmatrix} = \mathbf{M}(e^{j\omega}) \begin{bmatrix} B_1(e^{j\frac{\omega}{2}}) \\ B_1(e^{j(\frac{\omega}{2}-\pi)}) \end{bmatrix}, \qquad (5)$$

- The entries of $\mathbf{M}(e^{j\omega})$ are defined by

$$\mathbf{M}(e^{j\omega}) = \frac{1}{2} \begin{bmatrix} 1-\alpha_1 e^{-j\frac{\omega}{2}} & 1+\alpha_1 e^{-j\frac{\omega}{2}} \\ (e^{\frac{j\omega}{2}}-\alpha_2)R(\frac{j}{T_s}\frac{\omega}{2}) & -(e^{\frac{j\omega}{2}}+\alpha_2)R\left(\frac{j}{T_s}(\frac{\omega}{2}-\pi)\right) \end{bmatrix}. \qquad (6)$$

Correction (cont'd)

- If two channels matched, we have $\alpha_1 = \alpha_2$ and $R(j\Omega) = 1$, so letting $a_2^c(k)$ denotes the corrected version yields

$$\begin{bmatrix} A_1(e^{j\omega}) \\ A_2^c(e^{j\omega}) \end{bmatrix} = \mathbf{M}_c(e^{j\omega}) \begin{bmatrix} B_1(e^{j\frac{\omega}{2}}) \\ B_1(e^{j(\frac{\omega}{2}-\pi)}) \end{bmatrix} \tag{7}$$

- The entries of $\mathbf{M}_c(e^{j\omega})$ are given by

$$\mathbf{M}_c(e^{j\omega}) = \frac{1}{2} \begin{bmatrix} 1 - \alpha_1 e^{-j\frac{\omega}{2}} & 1 + \alpha_1 e^{-j\frac{\omega}{2}} \\ (e^{\frac{j\omega}{2}} - \alpha_1) & -(e^{\frac{j\omega}{2}} + \alpha_1) \end{bmatrix}. \tag{8}$$

Correction (cont'd)

- Using (5) and (8) digital filtering operation to produce $a_2^c(k)$ is given by

$$\begin{bmatrix} A_1(e^{j\omega}) \\ A_2^c(e^{j\omega}) \end{bmatrix} = \mathbf{M}_c(e^{j\omega})\mathbf{M}^{-1}(e^{j\omega}) \begin{bmatrix} A_1(e^{j\omega}) \\ A_2(e^{j\omega}) \end{bmatrix} . \tag{9}$$

- Only difficulty with (9) is that it requires the inversion of $\mathbf{M}(e^{j\omega})$. To avoid this, perform a first order approximation

$$\mathbf{M}^{-1}(e^{j\omega}) \simeq \mathbf{M}_c^{-1}(e^{j\omega})\big(\mathbf{I}_2 - \mathbf{\Delta}(e^{j\omega})\mathbf{M}_c^{-1}(e^{j\omega})\big) , \tag{10}$$

  where $\mathbf{\Delta}(e^{j\omega})$ represents mismatch terms and $\mathbf{I}_2$ denotes the identity matrix of size 2.

Correction (cont'd)

Due to space limitations we simply present

$$\big(\mathbf{I}_2 - \mathbf{\Delta}(e^{j\omega})\mathbf{M}_c^{-1}(e^{j\omega})\big) = \left[\begin{array}{cc} 1 & 0 \\ G_1(e^{j\omega}) & G_2(e^{j\omega}) \end{array}\right], \tag{11}$$

with

$$G_1(e^{j\omega}) = \frac{1}{2}\big[e^{j\frac{\omega}{2}}D(e^{j\frac{\omega}{2}}) + e^{j(\frac{\omega}{2}-\pi)}D(e^{j(\frac{\omega}{2}-\pi)})\big]$$
$$G_2(e^{j\omega}) = 1 + \frac{1}{2}\big[D(e^{j\frac{\omega}{2}}) + D(e^{j(\frac{\omega}{2}-\pi)})\big]$$

$$D(e^{j\omega}) \triangleq 1 - \frac{(1-\alpha_2 e^{-j\omega})}{(1-\alpha_1 e^{-j\omega})}R(\frac{j\omega}{T_s})\,. \tag{12}$$

where $D(e^{j\omega})$ characterizes mismatch effects.

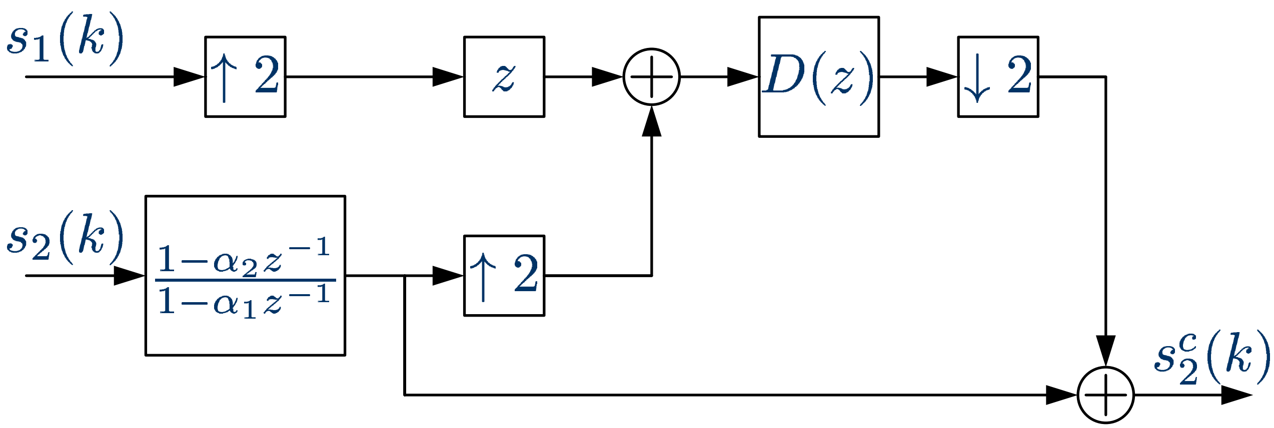
Correction (cont'd)

- Now correct $s_2(k)$ instead, which can be obtained by noting $S_2^c(e^{j\omega}) = (1 - \alpha_1 e^{-j\omega})^{-1} A_2^c(e^{j\omega})$,
- Thus using (9), (10), and (11) we obtain

$$
\begin{aligned}
S_2^c(e^{j\omega}) &= \frac{1}{1 - \alpha_1 e^{-j\omega}} \left[G_1(e^{j\omega}) A_1(e^{j\omega}) + G_2(e^{j\omega}) A_2(e^{j\omega}) \right] \\
&= G_1(e^{j\omega}) S_1(e^{j\omega}) + G_2(e^{j\omega}) \frac{1 - \alpha_2 e^{-j\omega}}{1 - \alpha_1 e^{-j\omega}} S_2(e^{j\omega}) \, . \qquad (13)
\end{aligned}
$$

Correction (cont'd)

This suggest the following multirate DSP architecture



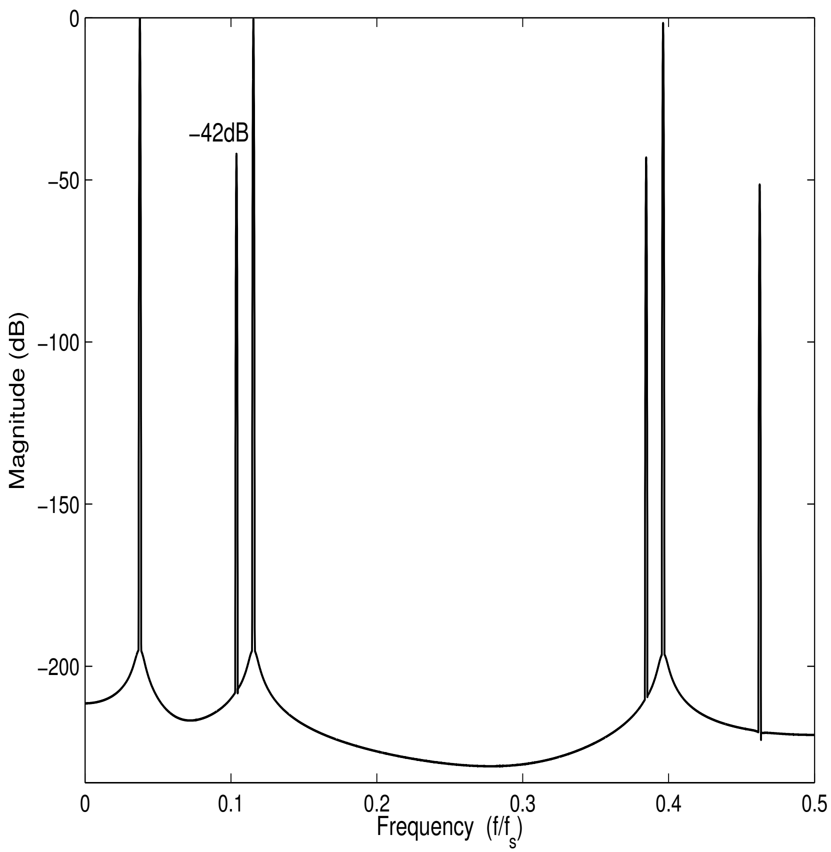
Outline

- Motivation
- Time-Interleaved S/H Model
- Correction Method
- **Simulations**
- Conclusion

Simulations

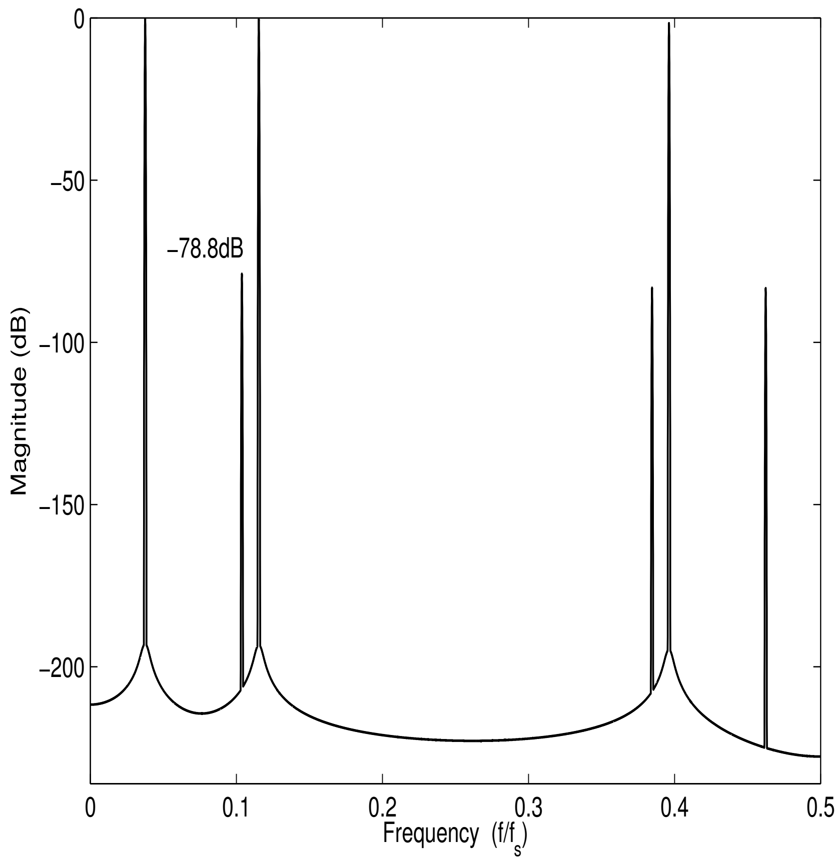
- Let input $x_c(t)$ be the sum of three sinusoids with unit amplitude and operating at frequencies $\Omega_1 = .0376\Omega_s$, $\Omega_2 = .1154\Omega_s$, and $\Omega_3 = .3962\Omega_s$.
- Let bandwidths of the two S/H devices be $\Omega_{c1} = 1/\tau_1 = \Omega_s/2$ and $\Omega_{c2} = 1/\tau_2 = .95\Omega_{c1}$, which represents a five percent bandwidth mismatch.
- For simplicity assume no quantization noise from ADC
- Use FIR approximation of $D(e^{j\omega})$ obtained by using a second order cone program (SOCP), which minimizes the ℓ_1 error between approximate and actual frequency responses.

Simulations (cont'd)



- Uncorrected Spectrum
- In addition to the three desired tones at frequencies Ω_i with $i = 1,\ 2,\ 3$, the output contains three undesired tones at frequencies $\Omega_s/2 - \Omega_i$,
- Peak undesired tone occurring at -42.4 dB

Simulations (cont'd)



- Corrected Spectrum
- $D(e^{j\omega})$ is approximated using a 21-tap FIR filter,
- Peak undesired tone attenuated to -78.8 dB
- Results in overall gain of 36.8 dB

Simulations (cont'd)

- Table shows the attenuation of the largest undesired tone as a function of filter length.
- Correction scheme has a performance ceiling of about 43 dB.
- Ceiling is due to the first-order power series expansion used in the approximation of $\mathbf{M}^{-1}(e^{j\omega})$.

Number of Taps	Attenuation of Largest Undesired Tone (dB)
5	9.49
11	19.75
21	36.84
41	42.93
101	43.14

Outline

- Motivation
- Blind Calibration Method
- Large Sample CRLB
- Simulations
- **Conclusion**

Conclusion

- Gave a hybrid/analog filterbank model of a time-interleaved S/H ADC.
- Correction method requires only one relatively short FIR filter.
- Simulation shows attenuation of 36.8 dB with 21-tap filter
- Method has ceiling since it only compensates first order mismatch effects.

Thank you!!