Effect of Nonlinearity in the CMFB Circuit that Uses the Differential-Difference Amplifier*

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***Abstract*—This paper considers one type of common-mode feedback (CMFB) circuit that uses two differential pairs to sense the common-mode output voltage (V_{oc}) of a fully differential op amp. Analysis of the circuit and simulation show that even powers of the differential-mode output voltage (V_{od}) affect the common-mode control signal generated by the CMFB circuit. As a result, undesired tones appear in V_{oc} and V_{od} due to transistor nonlinearity in the CMFB circuit. SPICE simulation results show that adding degeneration resistors to the differential pairs reduces the undesirable tones significantly.**

I. Introduction

Fully differential (FD) operational amplifiers (op amps) are widely used in integrated circuits because they provide larger output voltage swing than single-ended circuits, they reject common-mode noise, and they have no even-order nonlinearities in the output if the circuit is balanced [1]. However, the drawback of FD circuits is that common-mode feedback (CMFB) circuitry is needed to set the common-mode voltage at the op-amp output. The CMFB circuitry senses the CM output voltage and uses negative feedback to set the CM output voltage to a value that maximizes the op-amp output voltage swing.

This paper focuses on one particular type of CMFB circuit that uses two differential pairs. It is sometimes referred to as the Differential-Difference Amplifier (DDA) [2][3]. Advantages of this CMFB circuit include that it exhibits large transconductance [1] and it does not resistively load the op-amp output [2]. However, large signal analysis and simulation results presented in this paper demonstrate problems with nonlinearity in the CMFB circuit and the presence of undesired tones in the op-amp common-mode (CM) and differential-mode (DM) output voltages stemming from this nonlinearity. Finally, degeneration resistors are shown to improve the performance of the CMFB circuit.

II. Analysis of CMFB Circuit

A. Small Signal Analysis

The schematic of a CMFB circuit using DDA is shown in

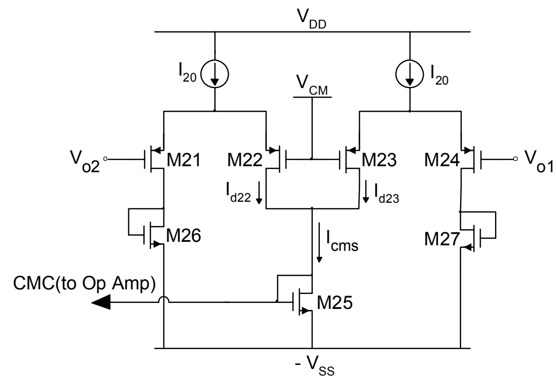

Fig. 1a. *CMFB circuit implemented with DDA.*

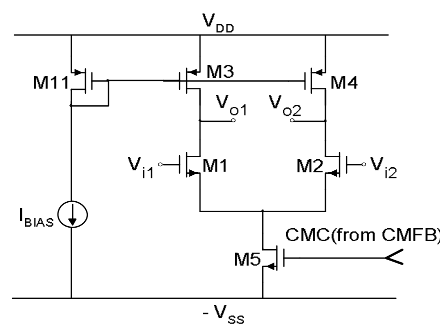

Fig. 1b. *Simple op amp that uses the CMFB in Fig. 1a.*

Fig. 1a. The matched differential pairs M21-22 and M23-24 connect to the op-amp outputs V_{o1} and V_{o2}, sense the CM output voltage [$V_{oc} = (V_{o1}+V_{o2})/2$] of the op amp, and compare V_{oc} with the desired CM output voltage, V_{CM}. A negative feedback loop sets V_{oc} to be about equal to V_{CM} by adjusting a bias current in the op amp that is related to the CM control-signal output of the CMFB circuit, which is I_{cms} in Fig. 1a.

The basic operation of the CMFB circuit can be illustrated by modeling M21-24 as ideal, linear transconducting elements. Since the drain current in M25 comes from the drain currents of M22 and M23, we get:

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$$I_{d25} = I_{cms} = I_{20} + g_m (V_{oc} - V_{CM}) = I_{20} + g_m V_{err} \quad (1)$$

where g_m is the small signal transconductance of M21-24 and $V_{err} = V_{oc} - V_{CM}$. A simple op amp that uses this CMFB circuit is shown in Fig. 1b. It is composed of NMOS input differential pair M1-2, PMOS active loads M3-4, and NMOS current source M5. In this op amp, $I_{d5} = 2I_{cms}$. Using negative feedback, the CMFB circuit adjusts I_{d5} so that $V_{oc} \approx V_{CM}$.

B. Large Signal Analysis

Assuming transistors M21-24 in Fig. 1a are matched and operate in the saturation region with drain current given by the widely used square-law expression [4], large-signal analysis in [5] shows that the drain currents of transistors M22 and M23 are given by

$$-I_{d22} = \frac{I_{20}}{2} + \frac{K}{2}(V_{o2} - V_{CM})\sqrt{4V_{ov}^2 - (V_{o2} - V_{CM})^2} \quad (2a)$$

$$-I_{d23} = \frac{I_{20}}{2} + \frac{K}{2}(V_{o1} - V_{CM})\sqrt{4V_{ov}^2 - (V_{o1} - V_{CM})^2} \quad (2b)$$

where $K = 0.5\mu C_{ox}W/L$ and $V_{ov} = \sqrt{I_{20}/2K}$. Note that (2a) and (2b) are only valid when M21-24 operate in the saturation region, which requires:

$$|V_{o1} - V_{CM}|, |V_{o2} - V_{CM}| \le \sqrt{2}V_{ov} \quad (3)$$

Using $V_{o1} = V_{oc} + V_{od}/2$, $V_{o2} = V_{oc} - V_{od}/2$, $V_{err} = V_{oc} - V_{CM}$, and assuming $|V_{err}| << |V_{od}|$, (2a) and (2b) become

$$-I_{d22} \approx \frac{I_{20}}{2} + \frac{K}{2}(V_{err} - \frac{V_{od}}{2})(2V_{ov})\sqrt{1 - \frac{\left(\left(\frac{V_{od}}{2}\right)^2 - V_{err}V_{CM}\right)}{4V_{ov}^2}} \quad (4a)$$

$$-I_{d23} \approx \frac{I_{20}}{2} + \frac{K}{2}(V_{err} + \frac{V_{od}}{2})(2V_{ov})\sqrt{1 - \frac{\left(\left(\frac{V_{od}}{2}\right)^2 + V_{err}V_{CM}\right)}{4V_{ov}^2}} \quad (4b)$$

Carrying out Taylor series expansions of (4a) and (4b) using $\sqrt{1-x} \approx 1 - x/2 - x^2/8 - x^3/16 - 5x^4/128 - \ldots$, with $x = [(V_{od}/2)^2 - V_{err}V_{CM}]/4V_{ov}^2$ in (4a) and $x = [(V_{od}/2)^2 + V_{err}V_{CM}]/4V_{ov}^2$ in (4b), keeping the first five terms in the expansions, and summing the resulting series; $I_{cms} = I_{d25}$ can be expressed as

$$I_{d25} = I_{cms} = -(I_{d22} + I_{d23}) \approx I_{20} + 2KV_{ov}V_{err} \times \left[1 - a_2V_{od}^2 - a_4V_{od}^4 - a_6V_{od}^6 - a_8V_{od}^8 - \ldots\right] \quad (5)$$

where

$$a_2 \approx \frac{3}{32V_{ov}^2} + \frac{V_{err}^2}{128V_{ov}^4} \quad (6a)$$

$$a_4 \approx \frac{5}{2048V_{ov}^4} + \frac{5V_{err}^2}{4096V_{ov}^6} + \frac{5V_{err}^4}{32768V_{ov}^8} \quad (6b)$$

$$a_6 \approx \frac{7}{65536V_{ov}^6} + \frac{35V_{err}^2}{262144V_{ov}^8} \quad (6c)$$

$$a_8 \approx \frac{45}{8388608V_{ov}^8} \quad (6d)$$

If $|V_{od}| << |V_{ov}|$, then the even-order V_{od} terms in (5) become insignificant and (5) agrees with (1) [$g_m = 2KV_{ov}$ for transistors operating in saturation]. However, having V_{od} with significantly large magnitude so that (1) is no longer valid is quite reasonable, since op amps usually produce large differential output voltages. In this case, (5) clearly indicates that the DM output voltage V_{od} affects the current I_{cms} produced by the CMFB circuit. This is undesirable since ideally the CMFB circuit would generate a current I_{cms} that is related to V_{err} but not to V_{od} and the CMFB loop would drive V_{err} to zero and hence V_{oc} to V_{CM}. As shown in (5), I_{cms} here is a nonlinear function of the DM output voltage (V_{od}), and thus a time-varying V_{od} can cause the op amp to have a time-varying bias current $I_{d5} = 2I_{cms}$. A time-varying bias current will affect the op-amp outputs, particularly when V_{od} is large. Also, a small DC level shift occurs due to the contribution of all the even nonlinearity terms so that $V_{oc}(DC) \neq V_{CM}$.

III. Simulation Results

A. Comparing (5) with (2) and SPICE

Due to finite CMFB loop gain, V_{oc} is not exactly V_{CM} as desired. Therefore, according to (5), the op-amp DM output, V_{od}, affects the current I_{cms} and causes V_{oc} not to be constant if V_{od} is not constant. First, to verify its accuracy, (5) was simulated in MATLAB and the results were compared with I_{cms} given by the sum of (2a) and (2b) for which no approximation was applied. The simulation parameters were I_{20} = 100μA, $(W/L)_{21\text{-}24}$ = 6μm/0.8μm, and $k_p = \mu_p C_{ox}$ = 59.2μA/V^2. It was found that for the V_{od} range that satisfies (3), (5) agrees with the sum of (2a) and (2b) with an error less than 0.03% for V_{err} = 0.01V and 1.00% for V_{err} = 0.1V.

To further examine the validity of (5), the CMFB circuit in Fig. 1a was simulated in SPICE using level 3 models for a 0.35μm CMOS process with $V_{DD} = V_{SS}$ = 1.65V, V_{CM} = 0V, V_{od} was a 10kHz sine wave with amplitude in the range of 0V to 1.16V (to keep all transistors in the saturation region), I_{20} = 100μA, $(W/L)_{21\text{-}24}$ = 6μm/0.8μm, $(W/L)_{25}$ = 16μm/0.8μm and $(W/L)_{26,27}$ = 8μm/0.8μm. A Fast-Fourier Transformation (FFT) was performed on the CM sense current, I_{cms}, obtained from both SPICE and MATLAB simulations, and the FFT spectra are shown in Fig. 2 for $(V_{od})_{pk}$ = 1.16V. The two plots are virtually identical; they coincide almost everywhere except for the low-amplitude noise in the spectrum from SPICE at 50, 70, 80 and 90 kHz, which we believe is due to 'numerical noise' in the SPICE

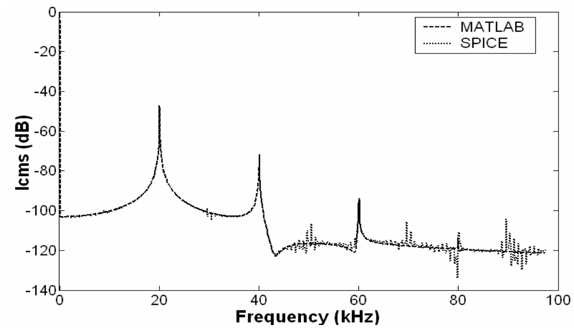

Fig. 2. FFT spectra of I_{cms} from MATLAB and SPICE Simulations

simulation. This close agreement implies that (5) accurately predicts the CMFB circuit performance for the signal range that keeps the CMFB transistors in the saturation region.

B. Closed CMFB Loop Simulation

To form a CMFB loop, the gates of the transistors M25 and M5 in Fig. 1a and 1b, respectively, are connected to form the current mirror that controls the bias current in the simple op amp. The simulation parameters for the CMFB circuit and the SPICE models given in the previous section were used. In addition, for the op amp in Fig. 1b, $V_{DD} = V_{SS} = 1.65V$, $V_{ic} = 0V$, $I_{BIAS} = 100\mu A$, $(W/L)_{1,2} = 19\mu m/1.4\mu m$, $(W/L)_{3,4,11} = 3.2\mu m/0.8\mu m$ and $(W/L)_5 = 32\mu m/0.8\mu m$.

Fig. 3 shows the FFT spectra of V_{oc} and V_{od} with an op-amp DM sinusoidal input at 10kHz with $(V_{id})_{pk} = 11mV$ that gives $(V_{od})_{pk} = 1.16V$, which causes the transistors in the CMFB circuit to operate on the edge of saturation at the output swing limits. Ideally, V_{oc} would be constant and V_{od} would be a 10kHz signal. Additional simulations showed that the undesired tones in Fig. 3 are more than 10dB smaller when an ideal CMFB circuit is used. Therefore, nonlinearity in the CMFB circuit in Fig. 1a is the dominant cause of the undesired tones in the V_{oc} and V_{od} spectra in Fig. 3. Noticeable undesired tones appear here since the magnitude of V_{od} is greater than $(V_{ov})_{21-24} = 0.475V$. The presence of odd harmonics in the V_{od} spectrum can be explained as follows. The DM output of the op amp is given by

$$V_{od} = A_{dm} V_{id} = -(g_m R_o) V_{id} \tag{7}$$

where g_m is the small-signal transconductance of the op amp, and R_o is the op-amp output resistance. Based on small-signal analysis [5], g_m is proportional to $(I_{d5})^{1/2}$ and R_o is inversely proportional to I_{d5}. Hence, the DM gain, A_{dm}, is inversely proportional to $(I_{d5})^{1/2}$. Since (5) shows that the op-amp bias current, I_{d5}, contains even-order terms of V_{od}, A_{dm} is a function of even-order terms of V_{od}. Using this result in (7), it can be shown that V_{od} is an odd function of V_{id}, i.e.,

$$V_{od} = b_1 V_{id} + b_3 V_{id}^3 + b_5 V_{id}^5 + \cdots \tag{8}$$

Fig. 3b shows that V_{od} contains V_{id} amplified (i.e., the tone at 10kHz) and odd harmonics of V_{id}, in agreement with (8). So based on analysis and simulation, nonlinearity in the CMFB circuit can cause undesired tones to appear in the op-amp DM and CM outputs.

C. Device Mismatch

When transistors M21-24 are matched, only even harmonics of V_{od} appear in I_{cms} and V_{oc} as shown in (5) and Fig. 3a, respectively. However, with device mismatch, both odd and even powers of V_{od} affect I_{cms} and therefore V_{oc}. Figure 4 shows the FFT spectra of V_{oc} and V_{od} with worst case mismatch of 2% introduced in each differential pair in the CMFB circuit. Transistor mismatch in the CMFB circuit does not significantly affect the spectrum of V_{od} or that of the even harmonics of 10kHz in V_{oc}. However, comparing with Fig. 3, simulation results reveal that odd harmonics of the 10kHz op-amp DM input signal with noticeable magnitudes appear in V_{oc}.

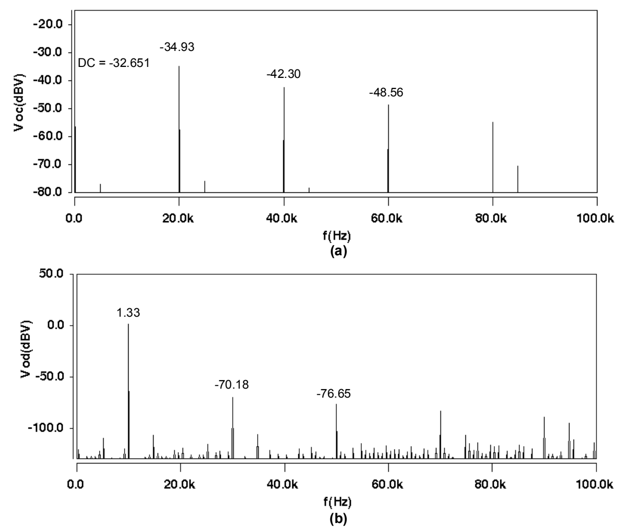

Fig. 3. *FFT spectra of (a) V_{oc} and (b) V_{od} with closed CMFB loop*

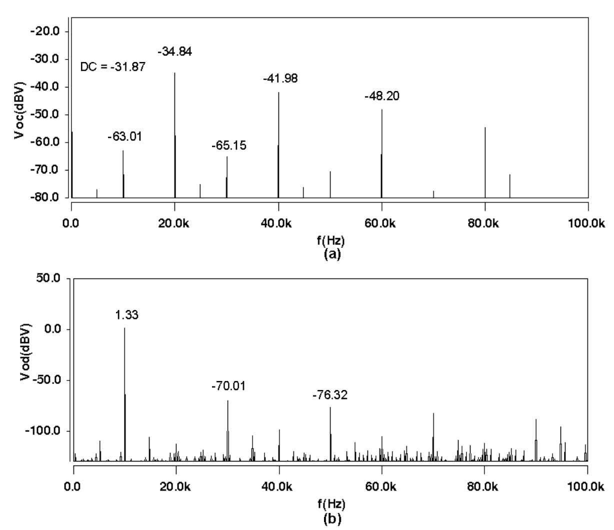

Fig. 4. *FFT spectra of (a) V_{oc} and (b) V_{od} with 2% device mismatch in the CMFB circuit with closed CMFB loop*

IV. CMFB Circuit with Degeneration Resistors

A. Adding Resistors

Using long channel devices for the differential pair transistors M21-24 in the CMFB will provide better linearity in the CMFB circuit. However, to avoid large capacitance from the resulting large devices, degeneration resistors can be used instead to improve the linearity of the differential pairs as shown in Fig. 5. With degeneration resistors, the loop gain of the CMFB circuit is reduced by $1+g_mR/2$, where R is the degeneration resistance in Fig. 5 and g_m is the small signal transconductance of M21-24. To maintain the same CMFB loop gain with the addition of degeneration resistors, the current gain of mirror M25-M5 can be increased by increasing the W/L ratio of transistor M5. In addition, the resulting extra bias current can be removed in the op amp by a DC current source I_{DRAW} as shown in Fig. 5.

B. Simulation Results

The circuit in Fig. 5 was simulated in SPICE with the parameters described above. Degeneration resistance R = 70kΩ and I_{DRAW} = 890μA were used. The M5-M25 current gain was increased from 2 to 11. Fig. 6 shows the spectra of V_{oc} and V_{od}. Compared to Fig. 4, the magnitudes of the undesired tones in the V_{oc} and V_{od} spectra are reduced. Based on additional simulations, the harmonics in V_{od} in Fig. 6b are now dominated by distortion caused by the transistors in the op amp. The CMFB circuit has been linearized by the degeneration resistors, and the undesired tones caused by nonlinearity in the CMFB circuit have been reduced.

V. Conclusion

Large signal analysis of the CMFB circuit that uses two differential pairs has been presented. Ideally, the CMFB loop gain is infinite, the CMFB drives V_{oc} to V_{CM}, and V_{err} is zero. However, in practice V_{err} is not zero. In that case, both analysis and simulation illustrate that even powers of V_{od} can affect the output of the CMFB circuit in Fig. 1a and can introduce undesired tones in the CM and DM outputs of an op amp that uses this CMFB circuit. If mismatch is present in the CMFB circuit, simulations show that additional undesired tones will be generated. SPICE simulations verified that degeneration resistors added in the CMFB circuit improve the linearity of the CMFB circuit and reduce the magnitude of the undesirable tones in the op-amp outputs.

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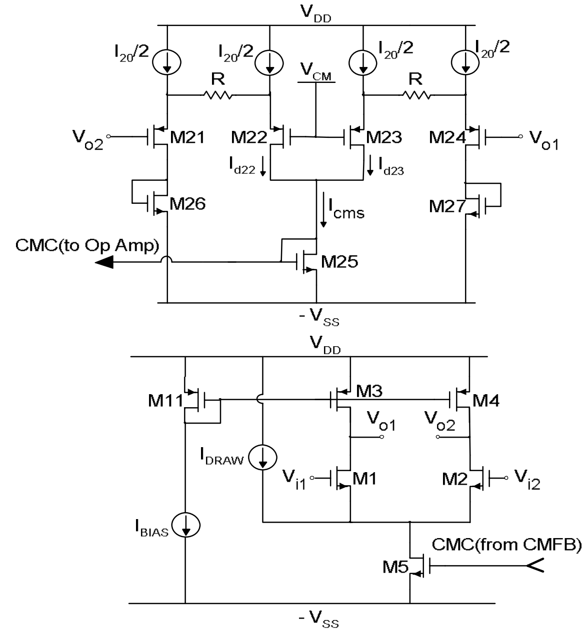

Fig. 5. *Modified CMFB circuit with degeneration resistors (top) and op amp (bottom) with I_{DRAW} added.*

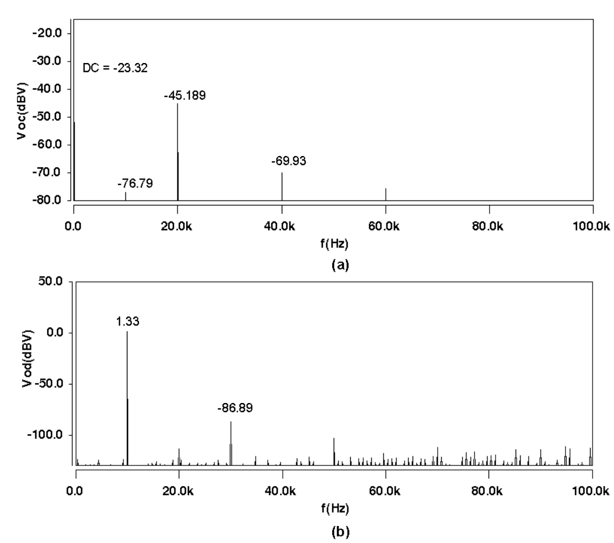

Fig. 6. *FFT spectra of (a) V_{oc} and (b) V_{od} with degeneration resistors added to CMFB circuit*