EEC 212
Problem Set 7

Professor Hurst

1

Approximations for second-order z-domain transfer functions: Let

$$H(z) = \frac{N(z)}{D(z)} = \frac{N(z)}{1 + \alpha z^{-1} + \beta z^{-2}}$$

If the poles are $p_1 = re^{j\theta}$ and $p_2 = re^{-j\theta}$, then

$$D(z) = 1 - 2r\cos\theta z^{-1} + r^2 z^{-2}.$$

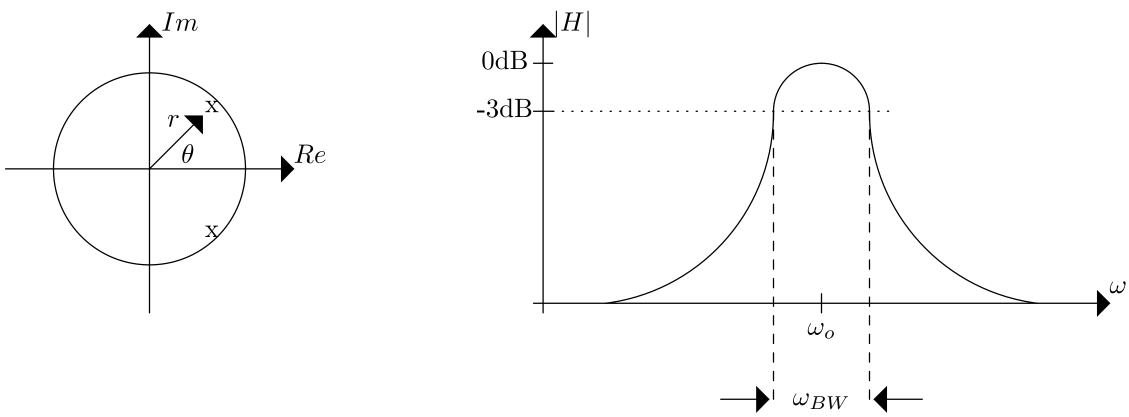


Figure 1: Poles and Unit Circle on z-plane (left), Bandpass filter frequency Response (right)

We can make the following approximation for $r \approx 1$:

$$\begin{aligned} \theta &\approx 2\pi\frac{\omega_o}{\omega_s} = \omega_o T_s \\ \text{and,} \quad 2(1-r) &\approx \omega_{BW} T_s \\ \Longrightarrow \quad Q &\approx \frac{\omega_o}{\omega_{BW}} = \frac{\theta}{2(1-r)} \end{aligned}$$

where, ω_o is the resonant frequency, ω_{BW} is the 3dB bandwidth, and Q is the quality factor.

(a) Show that the approximation above for ω_o ($\theta \approx \omega_o T_s$) is correct.

(b) Using the approximations, find α, β, as a function of ω_o, T_s, and Q. Assume that $\omega_o T_s \ll 1$.

(c) Using the expressions derived in (b), find α and β for $\omega_o = 2\pi(1\text{kHz})$, $T_s = 1/40\text{kHz}$, and $Q = 10$.

Use $N(z) = -K(1 - z^{-1})$. (This gives a bandpass transfer function). Find the capacitor values for Fig 5.10 or 5.13 in Gregorian & Temes (see the SC biquad handout) or a biquad in Section 14.5 of our text. Pick K so that $|H(\omega_o)| \approx 1$.

(d) Voltage scale to equalize the peak magnitudes of the transfer functions from the input to the outputs of the two op amps. (Using SWITCAP to determine the voltage-scale factor.) Also, impedance scale, use $C_{min} = 0.25$pF.

(e) Verify your design using SWITCAP. Plot $|V_{out}/V_{in}|(f)$ and $|V_{out-opamp1}/V_{in}|(f)$

(f) Estimate the maximum ΔV_{out} during a half-clock cycle, either by hand calculations or by using the circuit in Figure 2 and SWITCAP. From this, give a slew rate specification for the op amp in the filter in (d). (Your answer should be in terms of the peak input voltage, which isn't specified.)

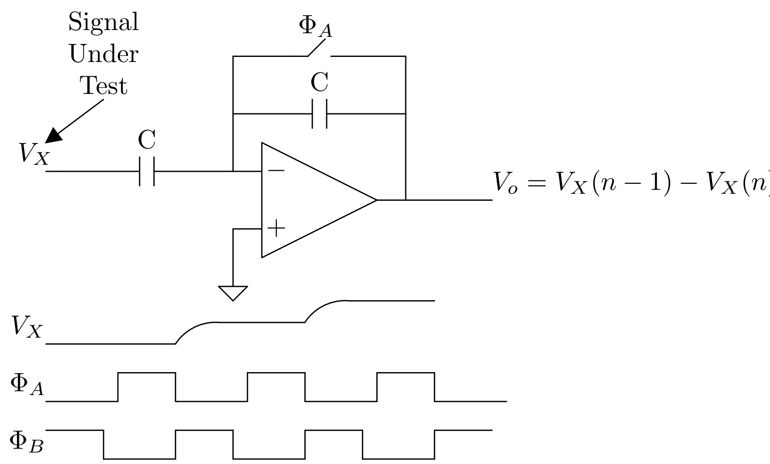
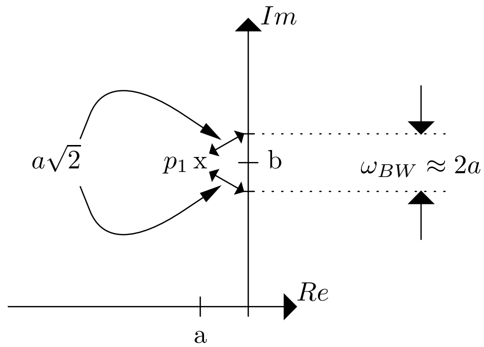


Figure 2: Test circuit for finding maximum ΔV_{out} (Connect 'signal under test, V_x,' to the filter output V_{out})

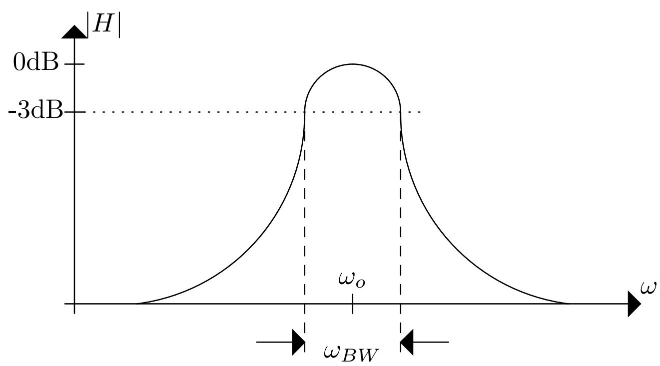
(HINT: Problem 1) In the s-plane, if a pair of poles are "close" to the jω-axis ($p_{1,2} = a \pm jb,\ a \ll b$), we have a bandpass filter with $Q \gg 1$. The center frequency will be $\omega_o \approx b$ and the 3dB bandwidth $\omega_{BW} \approx 2a$:



Vector lengths:

$$
\begin{aligned}
a &= |j\omega_o - p_1| \\
a\sqrt{2} &= \left| j\left(\omega_o \pm \frac{\omega_{BW}}{2}\right) - p_1 \right|
\end{aligned}
$$

By geometric analysis $\omega_{BW} = 2a$.

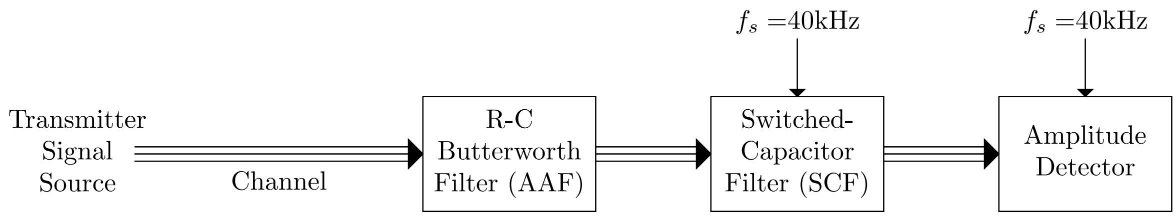


$$
\begin{aligned}
|H(s)| &= \frac{|N|}{|s-p_1|\,|s-p_2|} \\
&\approx \frac{|N|}{|s-p_1|\,|2p_2|} \quad \text{for s near p}_1
\end{aligned}
$$

2

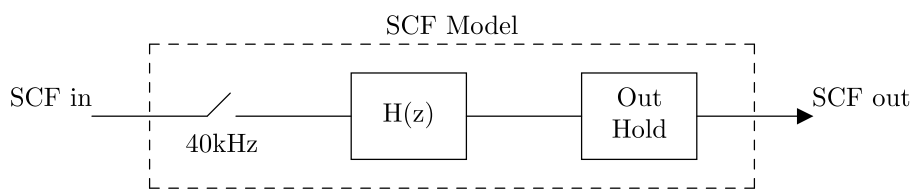
Sampling:

(a) The second-order switched-capacitor filter (SCF) of problem 1 is to be used to pick-off 1kHz tones in a communication system which may send any sinusoid with frequency from 0 Hz to 150 kHz at 1V$_{rms}$. If the detector following the SCF has a threshold set at 10mV$_{rms}$, what order Butterworth anti-alias filter (AAF) is required to ensure no false detects? (Use f_{-3dB} =2kHz.)



$$|\text{Butterworth}(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_{-3dB}}\right)^{2n}}}$$

(b) For your AAF, sketch the sampled spectrum at the input of $H(z)$ in the SCF model below for a 1kHz, 1V$_{rms}$ input signal and for a 81kHz input signal.



(c) Sketch the output spectrum of the SCF for the two input frequencies in (b) assuming the SCF output is held each clock cycle as shown in Figure 3.

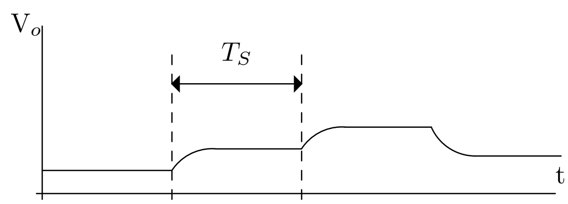


Figure 3: Output versus time.