1) a) For the circuit below, find the DC output voltage and the DC currents flowing in the transistors. For this part of the problem, take \( \lambda = 0 \) and use \( V_{IN(DC)} = 3 \) V.

\[ V_{O(DC)} = 1.5V \quad I_{D1} = 1.15mA \quad I_{D2} = 1.15mA \]

\[
\begin{align*}
V_{IN} + \sqrt{V_{IN}} & \quad m_1 \quad 5V \quad m_2 \quad V_O \\
\left( \frac{W}{L} \right)_1 & \quad \left( \frac{W}{L} \right)_2 = 20
\end{align*}
\]

\[
I_{D1} = I_{D1} \quad \text{and} \quad V_{IN} = V_{GS1} + V_{GS2} \\
\Rightarrow \frac{k'(W)}{2L_1}(V_{GS1} - V_t)^2 = \frac{k'(W)}{2L_2}(V_{GS2} - V_t)^2 \\
\Rightarrow V_{GS1} = V_{GS2} = \frac{V_{IN}}{2} = 1.5V \\
\Rightarrow I_{D1} = I_{D2} = \frac{k'(W)}{2L_1}(V_{GS1} - V_t)^2 \\
= \frac{180\mu A}{2}(20)(1.5 - 0.7)^2 \\
= 1.15mA
\]

b) What is the region of operation for each transistor?

\( M_1 \) is \textit{active (or sat)} \quad \textit{M_2 is active (or sat)}

\( V_{g1} > V_t \quad \text{and} \quad V_{g2} > V_t \quad \text{and} \quad V_{g1} = V_{g2} \)

\( V_{g1} = 2V < V_t, \quad V_{g2} > V_t \text{ and } V_{g1} = V_{g2} = 0 < V_t \)

c) For this part of the problem, assume \( V_{IN(DC)} \) is adjusted to set \( I_{D1} = I_{D2} = 100 \mu A \) and both transistor are in the active region. Find the small signal voltage gain, \( \frac{V_o}{V_{in}} \).

\[ \frac{V_o}{V_{in}} = \frac{1}{2} \]

\[ N_0 = \frac{g_m_1 R_L}{1 + g_m_1 R_L} \]

\[ = \frac{g_m_1 g_m_2}{1 + g_m_1 g_m_2} = \frac{1}{1 + 1} = \frac{1}{2} \]
c) Alternate (using our amp models):

\[ N_0 = \frac{R_L}{R_0 + R_L} \]

\[ = 1 \cdot \frac{1}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}} = \frac{1}{2} \]

\[ R_0 = \frac{1}{g_{m1}} \quad g_{m1} = g_{m2} \]
2) a) Draw the differential mode and common mode half-circuits for the circuit below. Give values for all passive elements.

\[ \omega \frac{L}{Z} = 10 \text{ for all } xtr\text{s.} \]

\[ R_Q = 100\, \text{k} \Omega \]

b) Compute \( A_{DM} \) and \( A_{CM} \) using these values. Assume \( I_{D1} = I_{D2} = 1 \, \text{mA} \) and all transistors are active, and ignore the body effect.

\[ A_{DM} = -63.3 \quad \text{and} \quad A_{CM} = -0.5 \]

\[ A_{DM} = -g_{m1} \left( \frac{r_{01} \| r_{03}}{2} \right) = -63.3 \]

\[ A_{CM} = -\frac{g_{m1}}{1 + g_{m1}(2R_Q)} \left( \frac{r_{01} \| r_{03}}{2} \right) \]

\[ = -4.99 \times 10^{-6} \left( \frac{r_{01} \| r_{03}}{2} \right) \]

\[ = -4.99 \times 10^{-6} \left( \frac{19\, \text{mA} \| 100\, \text{k} \Omega}{2} \right) \]

\[ = -0.5 \]
2) continued:

c) Compute $v_{o1}(t)$ if $v_{i1}(t) = 12 \text{ mV} \cdot \sin(20t)$ and $v_{i2}(t) = 10 \text{ mV} \cdot \sin(20t)$.

\[ v_{o1}(t) = \frac{v_{i1} + v_{i2}}{2} \]

\[ N_{i1} = 2 \text{ mV} \sin(20t) = \frac{v_{i1} - v_{i2}}{2} \]

\[ N_{i2} = 11 \text{ mV} \sin(20t) = \frac{v_{i1} + v_{i2}}{2} \]

\[ N_{o1} = \frac{N_{i1} + N_{i2}}{2} \]

\[ = \frac{2 \text{ mV} \sin(20t)}{2} + \frac{11 \text{ mV} \sin(20t)}{2} \]

\[ = -6.73 \text{ (2 mV} \sin(20t)) + 0.5 \text{ (11 mV} \sin(20t)) \]

\[ = -6.8 \text{ mV} \sin(20t) \]

3) What is the output resistance of the circuit below? Assume all transistors are in the active region, and $(W/L) = 10$ for all transistors.

\[ R_{\text{out}} = \] 

\[ R_{\text{out}} = r_{o3} \cdot \left( g_{m3} \cdot (r_{o1} g_{m2} r_{o2}) \right) \]

\[ = \left( \frac{1}{2} \frac{V}{A} \right)^3 \left( \frac{600 \mu A}{V} \right)^2 \]

\[ = 4.5 \times 10^{10} \Omega \]

\[ \text{all } R_o = \frac{1}{\lambda n 100 \mu A} = \frac{1}{2} \text{ M} \Omega \]

\[ g_m = \sqrt{2 k' \frac{W}{L} I_p} = \frac{600 \mu A}{V} \]
4) A bias circuit that generates an output current $I_O$ is shown below. Assume that all transistors are active and that the drain currents are all nonzero. Also assume that the drain of M6 connects to circuitry that is not shown, so that nonzero drain current can flow in M6. Estimate the currents $I_O$, $I_A$, and $I_B$. Use $(W/L) = 10$ and $\lambda = 0$ for all transistors.

$I_O =$ _________  $I_A =$ _________  $I_B =$ _________

\(V\)

\[ \begin{align*}
\text{(1)} & \quad m_3 \rightarrow I_B = 2I_A \\
\text{(2)} & \quad m_1 = m_6 \Rightarrow I_o = I_A \quad \& \quad I_{d1} = I_A = I_{d2} \quad (m_1 = m_2) \\
\text{(3)} & \quad k_{CL} \Rightarrow I_B = I_{d2} + \frac{V_{GS2}}{10k\Omega} \quad (V_{GS1} = V_{GS2} = V_{GS3}) \\
\text{(4)} & \quad V_{GS2} = V_{GS1} \approx V_{tn} + \sqrt{\frac{2I_A}{k_n' W/L}} \\
\end{align*} \]

1, 3, 4 allow calculation of $I_A = 104\mu A$

\[ \begin{align*}
\text{(2)} & \quad \Rightarrow I_o = I_A = 104\mu A \\
\text{(1)} & \quad \Rightarrow I_B = 2I_A = 208\mu A \\
\end{align*} \]
5) A current mirror is shown below. Assume that the drain of M2 connects to circuitry that is not shown, so that nonzero drain current can flow in M2. Also assume M2 is active.

For this current mirror, take $I_{IN} = 1\, mA$ (a DC current). All transistors have $W/L = 10$.

a) Find $I_O$ and $R_O$ for the model of the output port of the current mirror shown below. (You can use $\lambda = 0$ when calculating $I_O$.)

\[
I_O = \frac{1}{mA}, \quad R_O = \frac{50k\Omega}{\text{v}}.
\]

\[
I_{p_1} = I_{IN}, \quad V_{GS_1} = V_{GS_2}
\]

\[
\Rightarrow I_{p_2} = I_O = I_{IN} = 1mA
\]

\[
R_O = R_{O_2} = \sqrt{\frac{1}{\lambda_n(1mA)}} = 50k\Omega
\]

b) What is $V_{IN}$? $V_{IN} = 2.8V$.

\[
V_{IN} = V_{GS_1} + V_{GS_2} = 2.8V
\]

\[
V_{GS_1} = V_{ss} + \frac{2I_{p_1}}{k'_{nL}} = 1.75V
\]

\[
V_{GS_2} = V_{ss} + \frac{2(I_{IN} - I_{p_1})}{k'_{nL}} = 1.05V
\]

c) What is $V_{O(min)}$, the minimum output voltage that keeps the output transistor active?

\[
V_{O(min)} = 1.05V,
\]

\[
V_{OV_2} = \left(\frac{2I_{p_2}}{k'_{nL}}\right)^{\frac{1}{2}} = 1.05V
\]
6) Estimate the input offset voltage $V_{OS}$ for the differential circuit below. Recall that $V_{OS}$ is the DC value of the differential input voltage $V_{ID}$ that gives zero DC differential output voltage $V_{OD}$.

Assume M1 and M2 are active and identical with $W/L = 100$.

$$V_{OS} = \quad$$

![Circuit Diagram]

$$V_{OS_{out}} = V_{out} (V_{ID} = 0) = 5V - (9k\Omega)100\mu A$$

$$- (5V - (11k\Omega)100\mu A) = 0.2V$$

$$V_{OS} \approx \frac{V_{OS_{out}}}{A_{dm}} \approx \frac{0.2V}{-(-19)} = 10mV$$

$$A_{dm} = -g_mR_L = \sqrt{2k_{m}I_{D}(100\mu A)}(10k\Omega)$$

$$= -1.9 \frac{mA}{V} (10k\Omega) = -19$$
6) \text{ alternate:}

\[ I_{D_1} + I_{D_2} = 200 \mu A \]

\[ \text{if } V_{dd} = 0: \quad I_{D_1} qk = I_{D_2} \frac{qk}{k} \]

\[ \Rightarrow I_{D_1} + \frac{q}{11} I_{D_1} = 200 \mu A \]

\[ I_{D_1} = 110 \mu A \]

\[ \Rightarrow I_{D_2} = 90 \mu A \]

\[ V_{ID} = (V_{GS_1} - V_{GS_2}) = \]

\[ = (V_t + V_{OV_1} - (V_t + V_{OV_2})) \]

\[ = (V_{OV_1} - V_{OV_2}) \]

\[ = \sqrt{\frac{2 I_{D_1}}{k' L}} - \sqrt{\frac{2 I_{D_2}}{k' L}} \]

\[ = 0.1106 - 0.1 \]

\[ = 10.6 \text{ mV} \]