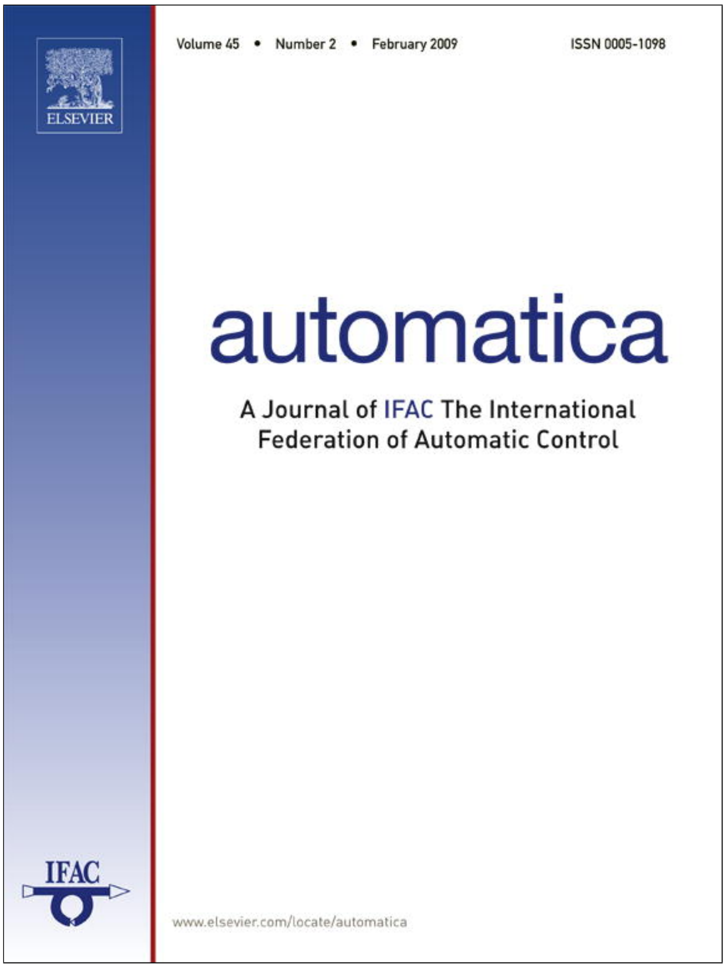
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Reliable decentralized PID controller synthesis for two-channel MIMO processes ☆

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ABSTRACT

Reliable stabilization and regulation of two-channel decentralized multi-input multi-output (MIMO) control systems is considered. The system has integral-action due to using proportional + integral + derivative (PID) controllers. Closed-loop stability and asymptotic tracking of step-input references are achieved at each output channel when all controllers are operational. Stability is maintained when one of the controllers fails completely and is set to zero. Controller synthesis procedures are proposed for stable MIMO plants and for several unstable MIMO plant classes that admit PID controllers. These synthesis procedures are applied to various examples of process systems to illustrate the design methodology.

1. Introduction

A stabilizing controller synthesis method is developed for linear, time-invariant (LTI), multi-input multi-output (MIMO) systems using a two-channel decentralized controller structure, with the objectives of decentralized closed-loop stabilization, reliable stability in case of complete failure of one of the two channels, and integral-action achieved with low-order simple PID controllers.

The decentralized controller structure has advantages although it restricts the stabilizing controller class. Fully decentralized control designs can be difficult also because of the interaction between the control loops. This introduces the problem of input–output pairing, to be decided in the first stage of the design before controller tuning. A method used to measure interactions and assess appropriate pairing is the relative gain array (RGA) (Campo & Morari, 1994). It is assumed here that the input–output pairing of the decentralized structure is already completed and the given plant model is partitioned into two MIMO channels *a priori*. An important control design requirement is reliability of the system's closed-loop stability against complete failure of certain channels. Reliable designs were considered under full-feedback and decentralized controller structures in Braatz, Morari, and Skogestad (1994), Gündeş and Kabuli (2001), Siljak (1980) and Tan, Siljak, and Ikeda (1992). In reliable control systems, when sensor and/or actuator failures occur and controllers in failed channels are taken out of service, the remaining controllers maintain closed-loop stability of the entire system. The completely different approach of fault tolerant control, based on first defining and storing all feasible controllers, guarantees stability by using a switching strategy among these controllers depending on failures (Seron, Zhuo, De Dona, & Martinez, 2008). Reliable stabilization requires no switching or re-tuning of controllers. An important performance objective is asymptotic tracking of constant reference inputs with zero steady-state error, achieved by designing controllers with integral-action. The simplest integral-action controllers are in proportional + integral + derivative (PID) form (Goodwin, Graebe, & Salgado, 2001). Although PID controllers are desirable due to easy implementation and tuning, their simplicity presents a major restriction that they can control only certain plants.

The problem studied here has several layers of difficulty due to the restricted decentralized structure of the controller, the requirement of closed-loop stability when one controller is taken out, and the restrictions in the class of (unstable) processes that can be stabilized using PID controllers. Achieving reliable closed-loop stability with either one of the controllers subject to failure is more demanding on the design than expecting that stability is maintained when a pre-specified one of the two controllers may fail. It is assumed that the failure of the controller C_j, $j = 1, 2$,

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is recognized and the failed controller is taken out of service (with its states reset to zero). This catastrophic failure is modeled by setting $C_j = 0$. A design that is reliable against either failure is called *fully reliable*; if only one specific controller may fail, then the control design is called *partially reliable*. These and other definitions are provided in Section 2, where the problem considered is formally stated. Section 3 is devoted to MIMO processes that are open-loop stable. Although both partially and fully reliable decentralized controllers exist for stable plants, the requirement of PID controllers and integral-action imposes additional conditions on the plants. These conditions and a reliable decentralized PID controller synthesis procedure are given in Section 3.1. For the stable plant case in Section 3, any other PID tuning method can be used to design C_2 that stabilizes one sub-block of the plant; for a fully reliable design, C_1 simultaneously stabilizes two systems related to the plant. Each of these blocks to be stabilized are MIMO systems if each channel has multiple inputs and outputs. There are no established PID tuning methods achieving simultaneous stabilization of two systems applicable in the MIMO setting. The synthesis methods proposed in this section are illustrated by examples. For certain PID stabilizable unstable plant classes (Gündeş & Özgüler, 2007), Section 4 investigates existence conditions for reliable decentralized controllers and proposes controller synthesis procedures. Partially reliable PID controller designs are illustrated for two examples.

The designs proposed here achieve closed-loop stability and asymptotic tracking of step-input references with zero steady-state error when all channels are operational, and maintain closed-loop stability of the overall system when either channel fails, with integral-action still present in the channel that remains active. The proposed designs also achieve asymptotic rejection of output disturbances since this is mathematically equivalent to the tracking problem. The proposed controllers also achieve robust closed-loop stability under sufficiently small additive or multiplicative plant uncertainty. The synthesis procedure for each plant class considered here allows freedom in choosing many of the design parameters. These parameters may be chosen to optimize the response in case of other performance specifications. Since the only goal here is reliable regulation, only stability and asymptotic tracking of constant inputs are emphasized and other performance objectives are not specified.

Notation. Let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}_+$ denote complex, real, positive real numbers; $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{Re}(s) \geq 0\} \cup \{\infty\}$ is the extended closed right-half plane; I_n is the $n \times n$ identity matrix; $\mathbf{R_p}$ denotes real proper rational functions of s; $\mathbf{S}$ is the stable subset with no $\mathcal{U}$-poles; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$. A square matrix $M \in \mathcal{M}(\mathbf{S})$ is called unimodular iff $M^{-1} \in \mathcal{M}(\mathbf{S})$. The H_∞-norm of $M(s) \in \mathcal{M}(\mathbf{S})$ is $\|M\| := \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$; $\bar{\sigma}$ is the maximum singular value and $\partial\mathcal{U}$ is the boundary of $\mathcal{U}$. Wherever this causes no confusion, (s) in transfer functions such as $G(s)$ is dropped. We use coprime factorizations over $\mathbf{S}$. We abbreviate right-coprime (RC) and left-coprime (LC).

2. Problem statement and preliminaries

Consider the LTI decentralized feedback system $Sys(G, C_D)$ with two MIMO channels as in Fig. 1. The feedback system is well-posed; the plant and controller have no hidden-modes associated with eigenvalues in the unstable region $\mathcal{U}$. The plant $G \in \mathbf{R_p}^{r\times m}$ and the decentralized controller $C_D \in \mathbf{R_p}^{m\times r}$ are partitioned as:

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}, \quad G_{ij} \in \mathbf{R_p}^{r_i\times m_j}, i, j = 1, 2, \tag{1}$$

$$C_D = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \mathrm{diag}\,[\,C_1\,,\,C_2\,], \quad C_j \in \mathbf{R_p}^{m_j\times r_j}. \tag{2}$$

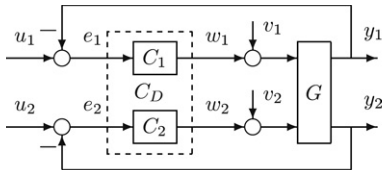

Fig. 1. The two-channel decentralized system $Sys(G, C_D)$.

Let $m := m_1 + m_2$, $r := r_1 + r_2$. We assume throughout that $\mathrm{rank}\, G(s) = r$ and $\mathrm{rank}\, G_{jj}(s) = r_j$ for $j = 1, 2$. Let

$$u := \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \qquad v := \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \qquad e := \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$
$$w := \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \qquad y := \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

denote the input and output vectors. Then the closed-loop transfer function H_{cl} from (u, v) to (w, y) is:

$$H_{cl} = \begin{bmatrix} C_D(I + GC_D)^{-1} & -C_D(I + GC_D)^{-1}G \\ GC_D(I + GC_D)^{-1} & (I + GC_D)^{-1}G \end{bmatrix}. \tag{3}$$

We have three main goals in developing a systematic synthesis method: **(1)** Design the decentralized stabilizing controller C_D so that the closed-loop system $Sys(G, C_D)$ achieves asymptotic tracking of step-input references with zero steady-state error. **(2)** The design should be reliable so that closed-loop stability is achieved if both channels are operational, and is still maintained even if one of the controllers C_1 or C_2 fails completely. The failure of C_j is recognized and the failed controller is taken out of service (with its states reset to zero). This catastrophic failure is modeled by setting the failed controller's transfer-matrix equal to zero. With $C_1 = 0$, the system is called $Sys(G, 0, C_2)$; with $C_2 = 0$, the system is called $Sys(G, C_1, 0)$. The failed channel does not achieve asymptotic tracking with zero steady-state error. **(3)** The controller order should be restricted; reliable decentralized stabilization should be achieved using PID controllers, where $C_j \in \mathbf{R_p}^{m_j\times r_j}$, $j = 1, 2$, should be in the proper PID controller form (Goodwin et al., 2001):

$$C_j = K_{Pj} + \frac{1}{s}K_{Ij} + \frac{s}{\tau_j s + 1}K_{Dj}, \quad j = 1, 2 \tag{4}$$

where $K_{Pj}, K_{Ij}, K_{Dj} \in \mathbb{R}^{m_j\times r_j}$ are called the proportional, integral, and derivative constants, respectively, and $\tau_j \in \mathbb{R}_+$, $j = 1, 2$. The integral-action in C_j is present when $K_{Ij} \neq 0$. Subsets of PID controllers are obtained by setting one or two of the three constants equal to zero; (4) becomes a PI-controller when $K_{Dj} = 0$ and an I-controller when $K_{Pj} = K_{Dj} = 0$.

Let $G = Y^{-1}X$ be a left-coprime-factorization (LCF) of $G \in \mathbf{R_p}^{r\times m}$, where $X \in \mathbf{S}^{r\times m}$, $Y \in \mathbf{S}^{r\times r}$, $\det Y(\infty) \neq 0$. Let $C_D = N_c D_c^{-1}$ be a right-coprime-factorization (RCF) of $C_D \in \mathbf{R_p}^{m\times r}$, where $N_c \in \mathbf{S}^{m\times r}$, $D_c \in \mathbf{S}^{r\times r}$, $\det D_c(\infty) \neq 0$. Let the (input-error) transfer-function from u to e be denoted by H_{eu} and let the (input–output) transfer-function from u to y be denoted by H_{yu}; then $H_{eu} = (I_r + GC_D)^{-1} = I_r - GC_D(I_r + GC_D)^{-1} =: I_r - GH_{wu} =: I_r - H_{yu}$.

Definition 1. **(a)** The system $Sys(G, C_1, C_2)$ is *stable* if the closed-loop transfer-function H_{cl} from (u, v) to (w, y) is stable. **(b)** The system $Sys(G, 0, C_2)$ is *stable* if with $C_1 = 0$, the closed-loop transfer-function H_2 from (u_2, v) to (w_2, y) is stable. **(c)** The system $Sys(G, C_1, 0)$ is *stable* if with $C_2 = 0$, the closed-loop transfer-function H_1 from (u_1, v) to (w_1, y) is stable. **(d)** The stable system $Sys(G, C_1, C_2)$ has *integral-action* if $H_{eu}(0) = 0$, i.e., H_{eu} has blocking-zeros at $s = 0$. **(e)** The decentralized stabilizing controller $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is an *integral-action controller* if

C_D stabilizes G, and D_c of any RCF $C_D = N_c D_c^{-1}$ has blocking-zeros at $s = 0$, i.e., $D_c(0) = 0$. **(f)** The decentralized stabilizing controller $C_D = \text{diag}\,[\,C_1, C_2\,]$ is a *partially reliable controller* if the systems $Sys(G, C_1, C_2)$ and $Sys(G, 0, C_2)$ are both stable. **(g)** The decentralized stabilizing controller $C_D = \text{diag}\,[\,C_1, C_2\,]$ is a *fully reliable controller* if all three systems $Sys(G, C_1, C_2)$, $Sys(G, 0, C_2)$ and $Sys(G, C_1, 0)$ are stable. $\square$

With $G = Y^{-1}X$, and $C_D = N_c D_c^{-1}$, the controller C_D stabilizes G if and only if

$$M_{cl} := YD_c + XN_c \tag{5}$$

is unimodular (Gündeş & Desoer, 1990; Vidyasagar, 1985). Suppose that $Sys(G, C_1, C_2)$ is stable and that step input references are applied to the system. Then the steady-state error $e(t)$ due to step inputs at $u(t)$ goes to zero if and only if $H_{eu}(0) = 0$. By Definition 1, the stable system $Sys(G, C_1, C_2)$ achieves asymptotic tracking of constant reference inputs with zero steady-state error if and only if it has integral-action. Write H_{eu} as $H_{eu} = (I_r + GC_D)^{-1} = D_c M_D^{-1} Y$. By Definition 1, if $C_D = N_c D_c^{-1}$ is an integral-action controller, then the system $Sys(G, C_1, C_2)$ has integral-action. In the special case that the plant G is stable, $Sys(G, C_1, C_2)$ has integral-action if *and only if* $C_D = \text{diag}\,[\,C_1, C_2\,]$ is an integral-action controller.

Lemma 1 (*Necessary Conditions for Integral-Action*). *Let $G \in \mathbf{R_p}^{r\times m}$, rank $G(s) = r$.* **(i)** *There exists an integral-action controller stabilizing G only if G has no transmission-zeros at $s = 0$.* **(ii)** *There exists a PID controller stabilizing G only if G is strongly stabilizable.* $\square$

In Section 3, we consider reliable decentralized controller design for stable plants. When G is stable, it is possible to achieve fully reliable control. In Section 4, we investigate conditions for existence of reliable decentralized controllers for unstable plants.

3. Reliable stabilization of stable plants

Let the plant G partitioned as in (1) be stable, i.e., $G_{ij} \in \mathbf{S}^{r_i\times m_j}$, $i, j = 1, 2$. Then G can be factorized as $G = Y^{-1}X = I^{-1}G$. Lemma 2 gives necessary and sufficient conditions for decentralized stabilization.

Lemma 2. *Let $G \in \mathbf{S}^{r\times m}$ and $C_D \in \mathbf{R_p}^{m\times r}$ be as in (1) and (2), respectively. Let $N_{cj}D_{cj}^{-1}$ be an RCF of C_j, $j = 1, 2$.*

(i) *$Sys(G, C_1, C_2)$ is stable if and only if*

$$M_{cl} := D_c + GN_c = \begin{bmatrix} D_{c1} + G_{11}N_{c1} & G_{12}N_{c2} \\ G_{21}N_{c1} & D_{c2} + G_{22}N_{c2} \end{bmatrix}$$

$$\text{is unimodular}. \tag{6}$$

(ii) *$Sys(G, 0, C_2)$ is stable if and only if C_2 stabilizes $G_{22} \in \mathcal{M}(\mathbf{S})$; equivalently,*

$$M_2 := D_{c2} + G_{22}N_{c2} \quad \text{is unimodular}. \tag{7}$$

(iii) *$Sys(G, C_1, 0)$ is stable if and only if C_1 stabilizes $G_{11} \in \mathcal{M}(\mathbf{S})$; equivalently,*

$$M_1 := D_{c1} + G_{11}N_{c1} \quad \text{is unimodular}. \tag{8}$$

(iv) *$Sys(G, C_1, C_2)$ and $Sys(G, 0, C_2)$ are both stable if and only if (7) holds and*

$$M_W := D_{c1} + (G_{11} - G_{12}N_{c2}M_2^{-1}G_{21})N_{c1}$$

$$\text{is unimodular}; \tag{9}$$

equivalently, C_2 stabilizes $G_{22} \in \mathbf{S}^{r_2\times m_2}$ and C_1 stabilizes $W \in \mathbf{S}^{r_1\times m_1}$ defined as

$$\begin{aligned} W &:= G_{11} - G_{12}N_{c2}M_2^{-1}G_{21} \\ &= G_{11} - G_{12}C_2(I + G_{22}C_2)^{-1}G_{21}. \end{aligned} \tag{10}$$

(v) *$Sys(G, C_1, C_2)$, $Sys(G, 0, C_2)$ and $Sys(G, C_1, 0)$ are all stable if and only if (6), (7), and (8) hold; equivalently, C_2 stabilizes $G_{22} \in \mathcal{M}(\mathbf{S})$, and C_1 stabilizes both $G_{11} \in \mathcal{M}(\mathbf{S})$ and $W \in \mathcal{M}(\mathbf{S})$ simultaneously.* $\square$

Remark. The transfer-function $W = G_{11} - G_{12}C_2(I + G_{22}C_2)^{-1}G_{21}$ in (10) is the "system" that the controller C_1 "sees", with C_2 already connected. Therefore, with C_2 stabilizing G_{22}, the system $Sys(G, C_1, C_2)$ is stabilized if the controller C_1 stabilizes W from w_1 to y_1. $\square$

Theorem 1. *Let $G \in \mathbf{S}^{r\times m}$ and $C_D \in \mathbf{R_p}^{m\times r}$ be as in (1) and (2), respectively.* **(i)** Partially reliable decentralized controller*: The controller $C_D = \text{diag}\,[C_1, C_2]$ is a partially reliable controller for G if and only if*

$$C_2 = Q_2(I - G_{22}Q_2)^{-1} = (I - Q_2G_{22})^{-1}Q_2, \tag{11}$$

$$C_1 = Q_1(I - WQ_1)^{-1} = (I - Q_1W)^{-1}Q_1, \tag{12}$$

where

$$W = G_{11} - G_{12}Q_2G_{21}, \tag{13}$$

for any $Q_2 \in \mathbf{S}^{m_2\times r_2}$ such that $\det(I - G_{22}Q_2)(\infty) \neq 0$, $Q_1 \in \mathbf{S}^{m_1\times r_1}$ such that $\det(I - WQ_1)(\infty) \neq 0$.

(ii) Fully reliable decentralized controller*: The controller $C_D = \text{diag}\,[\,C_1, C_2\,]$ is a fully reliable controller for G if and only if C_2 is given by (11), W is given by (13), and C_1 is given by (12), where $Q_1 \in \mathbf{S}^{m_1\times r_1}$ also satisfies*

$$I + (G_{11} - W)Q_1 = I + G_{12}Q_2G_{21}Q_1 \quad \text{is unimodular.} \;\square \tag{14}$$

By Theorem 1, stable plants are partially and also fully reliably stabilizable. For fully reliable stabilization, C_1 simultaneously stabilizes the two stable systems W and G_{11} (equivalently, Q_1 strongly stabilizes $(G_{11} - W) = I + G_{12}Q_2G_{21}$) by choosing Q_1 so that (14) holds. A sufficient condition to satisfy this unimodularity constraint is to choose $Q_1 \in \mathcal{M}(\mathbf{S})$ such that $\|Q_1\| < \|G_{12}Q_2G_{21}\|^{-1}$, which implies $\|G_{12}Q_2G_{21}Q_1\| < 1$; hence, $(I + G_{12}Q_2G_{21}Q_1)$ is unimodular (Vidyasagar, 1985).

3.1. Reliable PID controllers for stable plants

Although partially and fully reliable decentralized controllers of unconstrained order can be designed for stable plants, the requirement of reliable decentralized stabilization using PID controllers of the form (4) imposes additional constraints. By Lemma 1, there exist a decentralized PID controller with nonzero integral constant $K_I \in \mathbb{R}^{m\times r}$ only if G has no transmission-zeros at $s = 0$. Similarly, by Lemma 2(iv), there exist partially reliable decentralized PID controllers $C_D = \text{diag}[\,C_1, C_2\,]$ with nonzero integral constants $K_{Ij} \in \mathbb{R}^{m_j\times r_j}$ only if G and G_{22} have no transmission-zeros at $s = 0$; by Lemma 2(v), there exist fully reliable decentralized PID controllers $C_D = \text{diag}[\,C_1, C_2\,]$ with nonzero integral constants $K_{Ij} \in \mathbb{R}^{m_j\times r_j}$ only if G, G_{11}, G_{22} have no transmission-zeros at $s = 0$. For existence of partially reliable decentralized PID controllers, the necessary condition that G and G_{22} have no transmission-zeros at $s = 0$ is also sufficient; Proposition 1(a) presents a partially reliable synthesis procedure under this assumption. However, for existence of fully reliable decentralized PID controllers, the necessary condition is not sufficient. For fully reliable stabilization, the PID controller C_1 has to simultaneously stabilize the two stable systems W and G_{11}. Lemma 3 presents a necessary condition for existence of simultaneous integral-action controllers:

Lemma 3 (*Simultaneous Integral-Action Design for Stable Systems*). *Let $G_{11}, W \in \mathbf{S}^{r_1 \times m_1}$. Let rank $G_{11}(0) = r$, rank $W(0) = r$. Let $G_{11}(0)^I \in \mathbb{R}^{m_1 \times r_1}$ denote a right-inverse of $G_{11}(0) \in \mathbb{R}^{r_1 \times m_1}$. Suppose that $(G_{11} - W)$ has at least one blocking-zero at some $z \in \mathcal{U}$ (including infinity). Then there exists an integral-action controller that simultaneously stabilizes G_{11} and W only if*

$$\det [\, W(0)G_{11}(0)^I \,] > 0. \quad \square \tag{15}$$

Applying Lemma 3 to the simultaneous stabilization of G_{11} and W given in (10), whenever $G_{12}C_2(I + G_{22}C_2)^{-1}G_{21}$ has at least one blocking-zero at some $z \in \mathcal{U}$ (including infinity), there exists a PID controller (or any integral-action controller) C_1 that simultaneously stabilizes G_{11} and W only if

$$\begin{aligned} &\det [\, W(0)G_{11}(0)^I \,] \\ &\quad = \det[\, I - G_{12}(0)G_{22}(0)^I G_{21}(0)G_{11}(0)^I \,] > 0. \end{aligned} \tag{16}$$

In some cases, (16) is necessary: (1) when either (or both) of the off-diagonal blocks G_{12} or G_{21} in G have blocking-zeros in $\mathcal{U}$or are strictly-proper; (2) when the controller C_2 has blocking-zeros in $\mathcal{U}$, or is strictly-proper.

Remark. Condition (15) of Lemma 3 is still not a sufficient condition for the general MIMO plant case. A sufficient *but not necessary* condition for existence of fully reliable decentralized PID controllers when G, G_{11}, G_{22} have no transmission-zeros at $s = 0$ is that $W(0)G_{11}(0)^I = [\, I - G_{12}(0)G_{22}(0)^I G_{21}(0)G_{11}(0)^I \,]$ is symmetric, positive-definite. Proposition 1(b) presents a fully reliable synthesis procedure under this assumption, which facilitates the design of PID controllers that achieve fully reliable decentralized stabilization. $\square$

Proposition 1 (*Reliable Decentralized PID Controller Synthesis for Stable Plants*). *Let $G \in \mathbf{S}^{r \times m}$ be as in (1), with rank $G(0) = r$.*

(a) (Partially reliable decentralized PID controller synthesis) *:Let rank $G_{22}(0) = r_2$ and let $G_{22}(0)^I$ denote a right-inverse of $G_{22}(0) \in \mathbb{R}^{r_2 \times m_2}$*. Step 1 : Design C_2: *Choose any $\hat{K}_{p2}$, $\hat{K}_{d2} \in \mathbb{R}^{m_2 \times r_2}$, $\tau_2 \in \mathbb{R}_+$ and define*

$$\hat{C}_2 := \hat{K}_{p2} + \frac{s}{\tau_2 s + 1}\hat{K}_{d2} + \frac{1}{s}G_{22}(0)^I. \tag{17}$$

Then a PID controller C_2 that stabilizes G_{22} is

$$C_2 = \gamma_2 \hat{C}_2, \quad \gamma_2 < \left\| \frac{1}{s}\,(s\,G_{22}\,\hat{C}_2 - I\,) \right\|^{-1}. \tag{18}$$

Define W as in (10). Let $W(0)^I \in \mathbb{R}^{m_1 \times r_1}$ denote a right-inverse of $W(0) = G_{11}(0) - G_{12}(0)G_{22}(0)^I G_{21}(0) \in \mathbb{R}^{r_1 \times m_1}$. Step 2 : Design C_1: *Choose any $\hat{K}_{p1}, \hat{K}_{d1} \in \mathbb{R}^{m_1 \times r_1}$, $\tau_1 \in \mathbb{R}_+$ and define*

$$\hat{C}_1 := \hat{K}_{p1} + \frac{s}{\tau_1 s + 1}\,\hat{K}_{d1} + \frac{1}{s}\,W(0)^I. \tag{19}$$

Then a PID controller C_1 that stabilizes W is

$$C_1 = \gamma_1 \hat{C}_1, \quad \gamma_1 < \left\| \frac{1}{s}\,(s\,W\,\hat{C}_1 - I\,) \right\|^{-1}. \tag{20}$$

Then $C_D = \operatorname{diag}[\, C_1,\, C_2 \,]$ is a partially reliable decentralized PID controller. For $\hat{K}_{dj} = 0$, (18) and (20) are PI-controllers (or I-controllers if also $\hat{K}_{pj} = 0$).

(b) (Fully reliable decentralized PID controller synthesis) : *Let rank $G_{jj}(0) = r_j$ and let $G_{jj}(0)^I$ denote a right-inverse of $G_{jj}(0) \in \mathbb{R}^{r_j \times m_j}$, $j = 1, 2$. Let $W(0)G_{11}(0)^I = I - G_{12}(0)G_{22}(0)^I G_{21}(0)G_{11}(0)^I$ be symmetric, positive-definite. Design C_2 as in (18). Then define W as in (10). Choose any $\hat{K}_{p1}, \hat{K}_{d1} \in \mathbb{R}^{m_1 \times r_1}$, $\tau_1 \in \mathbb{R}_+$ and define*

$$\tilde{C}_1 := \hat{K}_{p1} + \frac{s}{\tau_1 s + 1}\,\hat{K}_{d1} + \frac{1}{s}\,G_{11}(0)^I. \tag{21}$$

Then a PID controller C_1 that simultaneously stabilizes G_{11} and W is given by

$$\begin{aligned} C_1 = \beta_1\,\tilde{C}_1, \quad \beta_1 < \min \Bigg\{ &\left\| \frac{1}{s}\,(s\,G_{11}\,\tilde{C}_1 - I\,) \right\|^{-1}, \\ &\left\| \frac{1}{s}\,(s\,W\,\tilde{C}_1 - W(0)\,G_{11}(0)^I\,) \right\|^{-1} \Bigg\}. \end{aligned} \tag{22}$$

Then $C_D = \operatorname{diag}[C_1, C_2]$ is a fully reliable decentralized PID controller. For $\hat{K}_{dj} = 0$, (18) and (22) are PI-controllers (or I-controllers if also $\hat{K}_{pj} = 0$). $\square$

Remark (*Robustness of the Fully Reliable Decentralized Stabilizing Controllers*). The decentralized controllers in Proposition 1 achieve robust reliable stability under 'sufficiently small' plant uncertainty. Let $\Delta \in \mathbf{S}^{r \times m}$ be a stable additive perturbation partitioned into two channels as G in (1). Then $C_D = \operatorname{diag}[C_1, C_2]$ is a fully reliable decentralized PID controller stabilizing $G + \Delta$ for all $\Delta \in \mathbf{S}^{r \times m}$ such that $\|\Delta\| < \|C_D(I + GC_D)^{-1}\|^{-1}$, and $\|\Delta_{jj}\| < \|C_j(I + G_{jj}C_j)^{-1}\|^{-1}$, $j = 1, 2$. For block-diagonal structured multiplicative perturbations $\Delta = \operatorname{diag}[\Delta_{11}\,,\ \Delta_{22}\,]$, C_D is a fully reliable decentralized PID controller stabilizing the plant $G(I + \Delta)$ under pre-multiplicative perturbations for all $\Delta \in \mathbf{S}^{m \times m}$, $\Delta_{jj} \in \mathbf{S}^{m_j \times m_j}$ such that $\|\Delta\| < \|C_D G(I + C_D G)^{-1}\|^{-1}$, $\|\Delta_{jj}\| < \|C_j G_{jj}(I + C_j G_{jj})^{-1}\|^{-1}$, $j = 1, 2$. Similarly, C_D is a fully reliable decentralized PID controller stabilizing the plant $(I + \Delta)G$ under post-multiplicative perturbations for all $\Delta \in \mathbf{S}^{r \times r}$, $\Delta_{jj} \in \mathbf{S}^{r_j \times r_j}$ such that $\|\Delta\| < \|GC_D(I + GC_D)^{-1}\|^{-1}$, and $\|\Delta_{jj}\| < \|G_{jj}C_j(I + G_{jj}C_j)^{-1}\|^{-1}$, $j = 1, 2$. The free parameter choices in the proposed controller synthesis method may be used to maximize the allowable perturbation magnitudes. $\square$

Remark (*Special Case of Single Output in Channel One*). If the first channel of G has a single output, i.e., $r_1 = 1$, then (16) is equivalent to $W(0)G_{11}(0)^I \in \mathbb{R}$ is symmetric, positive-definite. If $(G_{11} - W)$ has any blocking-zeros in $\mathcal{U}$, then it follows from Proposition 1 and Lemma 3 that the condition $W(0)G_{11}(0)^I > 0$ in (16) becomes both *necessary and sufficient* for existence of fully reliable decentralized PID controllers as in Corollary 1. $\square$

Corollary 1 (*Synthesis for Stable Plants with Single Output in First Channel*). *Under the assumptions of Proposition 1, let $r_1 = 1$. Suppose that $(G_{11} - W) = G_{12}C_2(I + G_{22}C_2)^{-1}G_{21}$ has some blocking-zero in $\mathcal{U}$. Then there exist fully reliable decentralized PID controllers for $G \in \mathcal{M}(\mathbf{S})$ if and only if $W(0)G_{11}(0)^I = [\,1 - G_{12}(0)G_{22}(0)^I G_{21}(0)G_{11}(0)^I] > 0$.* $\square$

Example 1 illustrates the fully reliable decentralized PID controller design using the synthesis procedure of Proposition 1. Example 2 considers a stable process where a fully reliable decentralized PID controller does not exist.

Example 1. The plant is a simplified delay-free model of a control system that manipulates the flow rate of two drugs (dopamine and sodium nitroprusside) to a specific critical care patient (Bequette, 2003). The anesthesiologist infuses several drugs to the patient during surgery to maintain the outputs (the main arterial pressure

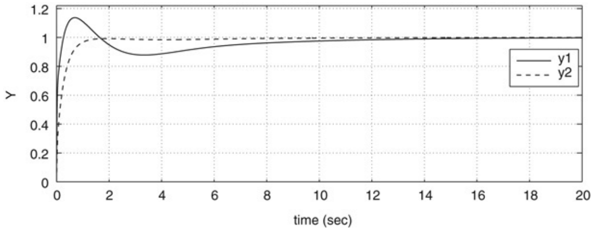

Fig. 2. Example 1 step-responses with $C_D = \text{diag}[C_1, C_2]$.

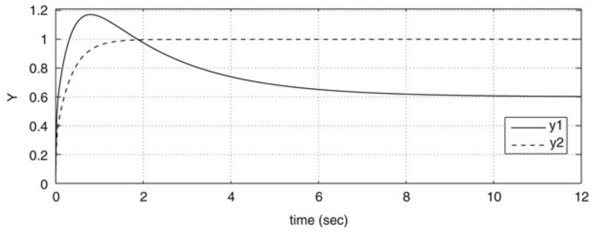

Fig. 3. Example 1 step-responses with $C_D = \text{diag}[0, C_2]$.

and cardiac output) close to their desired set-points. Let the plant's transfer-function be

$$G = \begin{bmatrix} \frac{-6}{0.67s+1} & \frac{3}{2s+1} \\ \frac{12}{0.67s+1} & \frac{5}{5s+1} \end{bmatrix} \in \mathbf{S}^{2\times 2},$$

where $r_1 = 1$ and G_{12}, G_{21} are strictly-proper; hence, $W(0)G_{11}(0)^I > 0$ is necessary and sufficient for achieving fully reliable decentralized PID design in this case. Then rank $G(0) = 2$, $G_{11}(0) \neq 0$, $G_{22}(0) \neq 0$, $W(0)G_{11}(0)^{-1} = I - G_{12}(0)G_{22}^{-1}G_{21}(0)G_{11}^{-1}(0) = 2.2 > 0$. Design C_2 that stabilizes G_{22}: The proposed design allows complete freedom in these parameters, so choose $\hat{K}_{p2} = 1.05$, $\hat{K}_{d2} = 0.1$, $\tau_2 = 0.02$. These choices affect the system's response and can be changed if satisfactory response is not obtained. We varied these parameters until the closed-loop poles were sufficiently damped. With $\gamma_2 = 3.9 < 4$ satisfying (18), we obtain the PID controller C_2 in (18) as $C_2 = \frac{0.4719s^2+4.111s+0.78}{s(0.02s+1)}$. Then $W \in \mathbf{S}$ in (10) is

$$W = \frac{-8.955s^4 - 1299s^3 - 7909s^2 - 3466s - 384.2}{s^5 + 75.79s^4 + 363.3s^3 + 523.5s^2 + 238.6s + 29.1}.$$

Design C_1 that simultaneously stabilizes G_{11} and W: Choose $\hat{K}_{p1} = -0.1$, $\hat{K}_{d1} = -0.05$, $\tau_1 = 0.02$. With $\beta_1 = 0.3 < \min\{2.3099, 0.3186\}$ satisfying (22), we obtain the PID controller C_1 in (22) as

$$C_1 = \frac{-0.0156s^2 - 0.031s - 0.05}{s(0.02s+1)}.$$

Fig. 2 shows the step responses of $Sys(G, C_D)$ (with both channels of $C_D = \text{diag}[C_1, C_2]$ active) for the two outputs y_1 (dashed), y_2 (solid), with unit-steps applied at both references u_1, u_2. Due to the integral-action in each PID controller, both channels achieve asymptotic tracking with zero steady-state error. Fig. 3 shows the step responses of $Sys(G, 0, C_2)$ when $C_1 = 0$, i.e., $C_D = \text{diag}[0, C_2]$. Since only the second control channel is operational, the output y_1 does not track the step reference due to lack of integral action in the first channel. □

Example 2. The system is the quadruple-tank apparatus (Johansson, 2000), which consists of four interconnected water tanks and two pumps. The output variables are the water levels of the

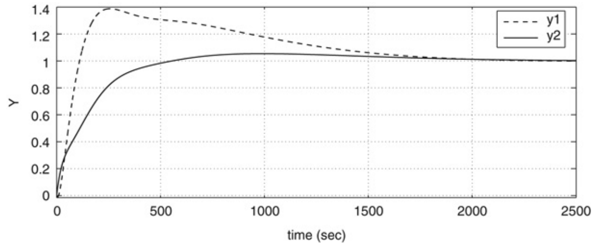

Fig. 4. Example 2 step-responses with $C_D = \text{diag}[C_1, C_2]$.

two lower tanks, and they are controlled by the currents that are manipulating two pumps. One of the two transmission-zeros of the linearized system dynamics can be moved between the positive and negative real axis by changing a valve. Let the plant be $G = \begin{bmatrix} \frac{3.7c_1}{62s+1} & \frac{3.7(1-c_2)}{(23s+1)(62s+1)} \\ \frac{4.7(1-c_1)}{(30s+1)(90s+1)} & \frac{4.7c_2}{90s+1} \end{bmatrix} \in \mathbf{S}^{2\times 2}$ for the linearized model at some operating point, where $r_1 = 1$ and G_{12}, G_{21} are strictly-proper; hence, $W(0)G_{11}(0)^I > 0$ is necessary and sufficient for achieving fully reliable decentralized PID design in this case. The adjustable transmission-zeros of G depend on the parameters c_1, c_2 (the proportions of water flow into the tanks adjusted by two valves). For $c_1 = 0.43$, $c_2 = 0.34$, the transmission-zeros of G are at $z_1 = 0.0229 > 0$, $z_2 = -0.0997$. Since rank $G(0) = 2$ and $G_{22}(0) \neq 0$, a partially reliable decentralized PID controller for G can be designed following Proposition 1(a). Although $G_{11}(0) \neq 0$, G does not admit a fully reliable decentralized PID controller since $W(0)G_{11}(0)^{-1} = I - G_{12}(0)G_{22}^{-1}G_{21}(0)G_{11}^{-1}(0) = -4.1464 < 0$, i.e., condition (15) of Lemma 3 is not satisfied. Design a partially reliable decentralized controller $C_D = \text{diag}[C_1, C_2]$ following Proposition 1(a): As in the case of Example 1, we choose the initial parameters of C_2 completely arbitrarily; they could be varied to improve the response if other performance constraints were present. Let $\hat{K}_{p2} = 150$, $\hat{K}_{d2} = 20$, $\tau_2 = 0.01$. Choose $\gamma_2 = 0.005 < 0.0067$ satisfying (18). A PID controller as in (18) is

$$C_2 = \frac{0.1075s^2 + 0.75s + 0.003129}{s(0.01s+1)}.$$

Then $W \in \mathbf{S}$ in (10) is

$$W = \frac{0.02566s^4 + 0.002597s^3 + 8.651\times 10^{-5}s^2 - 2.559\times 10^{-7}s - 3.245\times 10^{-9}}{s^5 + 0.1173s^4 + 0.00501s^3 + 9.408\times 10^{-5}s^2 + 7.191\times 10^{-7}s + 1.296\times 10^{-5}}.$$

Design C_1 that stabilizes W: Let $\hat{K}_{p1} = -100$, $\hat{K}_{d1} = -1$, $\tau_1 = 0.01$. Choose $\gamma_1 = 0.002 < 0.0044$. A PID controller as in (20) is

$$C_1 = \frac{-0.004s^2 - 0.2s - 0.0007991}{s(0.01s+1)}.$$

Fig. 4 shows the step responses of $Sys(G, C_D)$ (both channels of $C_D = \text{diag}[C_1, C_2]$ active) for outputs y_1 (dashed), y_2 (solid), with unit-steps applied at u_1, u_2. Fig. 5 shows the step responses of $Sys(G, 0, C_2)$ when C_1 is taken out, i.e., $C_D = \text{diag}[0, C_2]$. The output y_1 does not track the step reference due to lack of integral action in the first channel. □

4. Reliable stabilization of unstable plants

Let the plant $G \in \mathbf{R_p}^{r\times m}$ be partitioned as in (1). Let $G = Y^{-1}X$ be an LCF of G and $G_{22} = Y_{22}^{-1}X_{22}$ be an LCF of G_{22}. It is assumed that the denominator matrix Y is in upper-block-triangular form as in (23) (Gündeş & Desoer, 1990; Vidyasagar, 1985):

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = Y^{-1}X = \begin{bmatrix} Y_{11} & Y_{12} \\ 0 & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}. \quad (23)$$

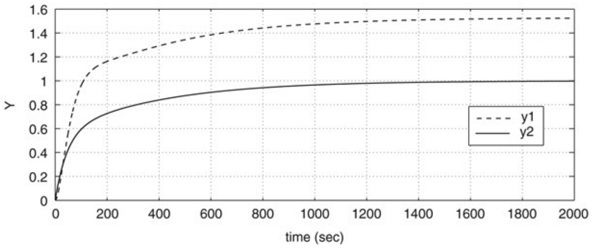

Fig. 5. Example 2 step-responses with $C_D = \text{diag}[0, C_2]$.

Lemma 4 gives necessary and sufficient conditions for the stability of the two channel decentralized system for normal and failure modes.

Lemma 4. *Let $G \in \mathbf{R_p}^{r\times m}$ and $C_D \in \mathbf{R_p}^{m\times r}$ be as in (23) and (2), respectively. Let $N_{cj}D_{cj}^{-1}$ be an RCF of C_j, $j = 1, 2$.* **(i)** *$Sys(G, C_1, C_2)$ is stable if and only if*

$$M_{cl} := YD_c + XN_c = \begin{bmatrix} Y_{11}D_{c1} + X_{11}N_{c1} & Y_{12}D_{c2} + X_{12}N_{c2} \\ X_{21}N_{c1} & Y_{22}D_{c2} + X_{22}N_{c2} \end{bmatrix} \text{ is unimodular}. \tag{24}$$

(ii) *$Sys(G, 0, C_2)$ is stable if and only if*

$$M_{cl2} = \begin{bmatrix} Y_{11} & Y_{12}D_{c2} + X_{12}N_{c2} \\ 0 & Y_{22}D_{c2} + X_{22}N_{c2} \end{bmatrix} \text{ is unimodular;} \tag{25}$$

equivalently, Y_{11} is unimodular and C_2 stabilizes G_{22}, i.e.,

$$M_2 := Y_{22}D_{c2} + X_{22}N_{c2} \quad \text{is unimodular}. \tag{26}$$

(iii) *$Sys(G, C_1, 0)$ is stable if and only if*

$$M_{cl1} := \begin{bmatrix} Y_{11}D_{c1} + X_{11}N_{c1} & Y_{12} \\ X_{21}N_{c1} & Y_{22} \end{bmatrix} \text{ is unimodular;} \tag{27}$$

equivalently, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ is LC, (Y_{12}, Y_{22}) is RC, and C_1 stabilizes G_{11}.

(iv) *$Sys(G, C_1, C_2)$ and $Sys(G, 0, C_2)$ are both stable if and only if Y_{11} is unimodular, (26) holds and*

$$M_W := Y_{11}D_{c1} + (X_{11} - (Y_{12}D_{c2} + X_{12}N_{c2})M_2^{-1}X_{21})N_{c1} \text{ is unimodular;} \tag{28}$$

equivalently, with Y_{11} unimodular, C_2 stabilizes $G_{22} \in \mathbf{S}^{r_2\times m_2}$ and C_1 stabilizes $W \in \mathbf{S}^{r_1\times m_1}$ defined as

$$W := Y_{11}^{-1}[X_{11} - (Y_{12}D_{c2} + X_{12}N_{c2})M_2^{-1}X_{21}]; \tag{29}$$

i.e., $W \in \mathcal{M}(\mathbf{S})$ is the same as (10).

(v) *$Sys(G, C_1, C_2)$, $Sys(G, 0, C_2)$ and $Sys(G, C_1, 0)$ are all stable if and only if (24), (25) and (27) hold; equivalently, with Y_{11} unimodular, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ LC, (Y_{12}, Y_{22}) RC, the controller C_2 stabilizes $G_{22} \in \mathcal{M}(\mathbf{R_p})$, and the controller C_1 stabilizes $G_{11} \in \mathcal{M}(\mathbf{R_p})$ and $W \in \mathcal{M}(\mathbf{S})$ simultaneously.* □

The necessary and sufficient conditions for existence of partially reliable and fully reliable decentralized controllers are formally stated in Theorem 2:

Theorem 2. *Let $G \in \mathbf{R_p}^{r\times m}$ and $C_D \in \mathbf{R_p}^{m\times r}$ be as in (23) and (2), respectively. Let $G_{22} = \tilde{X}_{22}\tilde{Y}_{22}^{-1}$ be an RCF of G_{22}.* **(i)** Partially reliable decentralized controller: **(a)** *There exists a partially reliable decentralized controller for G if and only if Y_{11} is unimodular.* **(b)** *Let Y_{11} be unimodular. The controller $C_D = \text{diag}\,[C_1, C_2]$ is a partially reliable decentralized controller for G if and only if C_2 is*

$$C_2 = N_{c2}D_{c2}^{-1} = (\widetilde{U} + \tilde{Y}_{22}Q_2)(\widetilde{V} - \tilde{X}_{22}Q_2)^{-1} \tag{30}$$

for any $\widetilde{V}, \widetilde{U} \in \mathcal{M}(\mathbf{S})$ such that $Y_{22}\widetilde{V} + X_{22}\widetilde{U} = I$, and $Q_2 \in \mathbf{S}^{m_2\times r_2}$ such that $\det(\widetilde{V} - \tilde{X}_{22}Q_2)(\infty) \neq 0$. For

$$\begin{aligned} W &= G_{11} - G_{12}C_2(I + G_{22}C_2)^{-1}G_{21} \\ &= G_{11} - G_{12}N_{c2}Y_{22}G_{21} \in \mathcal{M}(\mathbf{S}) \end{aligned} \tag{31}$$

defined as in (29), C_1 is given by

$$C_1 = Q_1(I - WQ_1)^{-1} = (I - Q_1W)^{-1}Q_1 \tag{32}$$

for any $Q_1 \in \mathbf{S}^{m_1\times r_1}$ such that $\det(I - WQ_1)(\infty) \neq 0$.

(ii) Fully reliable decentralized controller: **(a)** *There exists a fully reliable decentralized controller for G if and only if Y_{11} is unimodular, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ is LC, (Y_{12}, Y_{22}) is RC, G_{12} and G_{21} are strongly stabilizable and $G_{11} - W = G_{12}C_2(I + G_{22}C_2)^{-1}G_{21}$ for some C_2 that stabilizes G_{22}.* **(b)** *Let Y_{11} be unimodular, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ be LC, (Y_{12}, Y_{22}) be RC; let $G_{11} - W = G_{12}C_2(I + G_{22}C_2)^{-1}G_{21}$ be strongly stabilizable for some C_2 that stabilizes G_{22}. The controller C_D is a fully reliable decentralized controller for G if and only if C_2 is given by (30) for some $Q_2 \in \mathbf{S}^{m_2\times r_2}$ such that $G_{11} - W = G_{12}C_2(I + G_{22}C_2)^{-1}G_{21} = G_{12}(\widetilde{U} + \tilde{Y}_{22}Q_2)Y_{22}G_{21}$ is strongly stabilizable, and C_1 is given by (32) for some $Q_1 \in \mathbf{S}^{m_1\times r_1}$ such that*

$$M_{W1} := Y_{22} + X_{21}Q_1G_{12}C_2(I + G_{22}C_2)^{-1} \text{ is unimodular}. \quad \square \tag{33}$$

By Theorem 2, the only necessary condition on the unstable plant G for existence of partially reliable decentralized controllers of unconstrained order is that Y_{11} is unimodular in the denominator matrix Y of $G = Y^{-1}X$. Additional constraints are imposed on G when C_1, C_2 are restricted to be PID controllers of the form (4).

4.1. Reliable PID controllers for unstable G with no $\mathcal{U}$-zeros in G_{22}

We investigate partially reliable decentralized PID stabilization of a class of unstable plants, where the sub-block G_{22} of G has no transmission-zeros in the region of instability $\mathcal{U}$ (including infinity). Let the plant $G \in \mathbf{R_p}^{r\times m}$ be as in (23) and satisfy the following assumptions:

Assumptions 1. **(i)** rank $G(s) = r$ and G has no transmission-zeros at $s = 0$. **(ii)** Y_{11} is unimodular; we take $Y_{11} = I_{r_1}$. **(iii)** $G_{22} \in \mathbf{R_p}^{r_2\times r_2}$, i.e., G_{22} is square ($m_2 = r_2$); rank $G_{22} = r_2$ and G_{22} has no transmission-zeros or blocking-zeros in the region of instability $\mathcal{U}$ (including infinity). Therefore, with $Y_{22} = G_{22}^{-1} \in \mathbf{S}^{r_2\times r_2}$, an LCF is

$$G_{22} = (G_{22}^{-1})^{-1}I_{r_2} =: Y_{22}^{-1}X_{22}. \tag{34}$$

Assumption 1(i) is a necessary condition for existence of any integral-action controller for the plant G. Assumption 1(ii) is a necessary condition for existence of partially reliable decentralized controllers by Theorem 2. Assumption 1(iii) is a sufficient condition for existence of PID controllers. Although G_{22} does not have any zeros in $\mathcal{U}$, and all of its zeros are in the stable region $\mathbb{C} \setminus \mathcal{U}$, the other sub-blocks of the plant G may have $\mathcal{U}$-zeros. If channel two is SISO, i.e., $r_2 = 1$, then these assumptions imply that G_{22} is minimum-phase and has relative-degree equal to zero. There are no restrictions on the poles of G_{22}. With these assumptions, (23) becomes

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} I & Y_{12} \\ 0 & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & I \end{bmatrix} = \begin{bmatrix} X_{11} - Y_{12}G_{22}X_{21} & X_{12} - Y_{12}G_{22} \\ G_{22}X_{21} & G_{22} \end{bmatrix}.$$

Proposition 2 gives a systematic synthesis method for partially reliable decentralized PID controllers:

Proposition 2 (*Partially Reliable Decentralized PID Controller Synthesis When G_{22} has no $\mathcal{U}$-Zeros*). *Let the plant $G \in \mathbf{R_p}^{r\times m}$ be as in (23) and satisfy Assumption 1, with G_{22} as in (34).* Step 1 : Design C_2: *Choose any $K_{D2} \in \mathbb{R}^{r_2\times r_2}$, $\tau_2 \in \mathbb{R}_+$, $g \in \mathbb{R}_+$. Choose any nonsingular $\hat{K}_{p2} \in \mathbb{R}^{r_2\times r_2}$. Choose $\gamma_2 \in \mathbb{R}_+$ satisfying*

$$\gamma_2 > \left\| \left[G_{22}^{-1} + \frac{s}{\tau_2 s+1} K_{D2} \right] \hat{K}_{p2}^{-1} \right\|. \tag{35}$$

Let $K_{P2} = \gamma_2 \hat{K}_{p2}$, $K_{I2} = \gamma_2 g \hat{K}_{p2}$. Then a PID controller C_2 that stabilizes G_{22} is given by

$$C_2 = \gamma_2 \hat{K}_{p2} + \frac{\gamma_2 g}{s} \hat{K}_{p2} + \frac{s}{\tau_2 s+1} K_{D2}. \tag{36}$$

For $K_{D2} = 0$, (36) is a PI-controller; for $g = 0$, (36) is a PD-controller.
With C_2 as in (36), define $W := G_{11} - G_{12} C_2 (I + G_{22} C_2)^{-1} G_{21}$ as in (29), equivalently (10), where $W(0) = X_{11}(0) - X_{12}(0)X_{21}(0) \in \mathbb{R}^{r_1\times m_1}$. Let $W(0)^I \in \mathbb{R}^{m_1\times r_1}$ denote a right-inverse of $W(0)$. Step 2 : Design C_1: *Follow* Step 2 *of Proposition 1 to design C_1 as in (20) that stabilizes $W \in \mathcal{M}(\mathbf{S})$. Then $C_D = \mathrm{diag}\,[\, C_1,\ C_2\,]$ is a partially reliable decentralized PID controller.* □

4.2. Reliable PID controllers for unstable G with one $\mathcal{U}$-zero in G_{22}

We investigate partially reliable decentralized PID stabilization of a class of unstable plants, where the sub-block G_{22} has one blocking-zero in the region of instability $\mathcal{U}$(including infinity). Let the plant $G \in \mathbf{R_p}^{r\times m}$ be as in (23) and satisfy the following assumptions:

Assumptions 2. **(i)** rank $G(s) = r$ and G has no transmission-zeros at $s = 0$. **(ii)** Y_{11} is unimodular; we take $Y_{11} = I_{r_1}$. **(iii)** $G_{22} \in \mathbf{R_p}^{r_2\times r_2}$, i.e., G_{22} is square ($m_2 = r_2$); rank $G_{22} = r_2$ and G_{22} has no transmission-zeros at $s = 0$; G_{22} has one blocking-zero $z \in \mathcal{U} \cap \{\mathbb{R} \setminus 0\}$ (including infinity). For any $a \in \mathbb{R}_+$, an LCF is

$$G_{22} = \left[\frac{(1-s/z)}{s+a} G_{22}^{-1} \right]^{-1} \left[\frac{(1-s/z)}{s+a} I \right] =: Y_{22}^{-1} X_{22}, \tag{37}$$

where $z \in \mathbb{R}_+$, $0 < z \leq \infty$.

Assumption 2(i–ii) are the same necessary conditions as Assumption 1(i–ii); Assumption 2(iii) is another sufficient condition for existence of PID controllers. Although G_{22} is assumed to have only one zero in $\mathcal{U}$, and all of its other zeros are in the stable region $\mathbb{C}\setminus\mathcal{U}$, the other sub-blocks of the plant G may have any number of $\mathcal{U}$-zeros. There are no restrictions on the poles of G_{22}.

With these assumptions, (23) becomes

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} I & Y_{12} \\ 0 & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & \dfrac{(1-s/z)}{s+a} I \end{bmatrix} = \begin{bmatrix} X_{11} - Y_{12}Y_{22}^{-1}X_{21} & X_{12} - Y_{12}Y_{22}^{-1} \\ Y_{22}^{-1}X_{21} & X_{22} \end{bmatrix}. \tag{38}$$

Proposition 3 gives a systematic synthesis method for partially reliable decentralized PID controllers:

Proposition 3 (*Partially Reliable Decentralized PID Controller Synthesis When G_{22} has one $\mathcal{U}$-Zero*). *Let the plant $G \in \mathbf{R_p}^{r\times m}$ be as in (23) and satisfy Assumption 2, with G_{22} as in (37). Let $Y_{22}(\infty)^{-1} = (1-s/z)^{-1} s\, G_{22}(s)|_{s\to\infty}$.* Step 1 : Design C_2: *Choose $K_{D2} \in \mathbb{R}^{r_2\times r_2}$, $\tau_2 \in \mathbb{R}_+$ such that $\hat{K}_{p2} := Y_{22}(\infty) - \frac{1}{z\tau_2} K_{D2}$ is nonsingular. Choose any $g \in \mathbb{R}_+$. Define $\Psi \in \mathcal{M}(\mathbf{S})$ as*

$$\Psi := \frac{s}{s+g}(1-s/z) \left[G_{22}^{-1}(s) + \frac{s}{\tau_2 s+1} K_{D2} \right] \hat{K}_{p2}^{-1} - sI. \tag{39}$$

If $z > \|\Psi\|$, then choose $\gamma_2 \in \mathbb{R}_+$ satisfying

$$\gamma_2 > \frac{\|\Psi\|}{1 - \|\Psi\|/z}. \tag{40}$$

Let $K_{P2} = \frac{\gamma_2}{1+\gamma_2/z} \hat{K}_{p2}$, $K_{I2} = g K_{P2}$. Then a PID controller C_2 that stabilizes G_{22} is given by

$$C_2 = \frac{\gamma_2}{1+\gamma_2/z} \hat{K}_{p2} + \frac{\gamma_2 g}{(1+\gamma_2/z)s} \hat{K}_{p2} + \frac{s}{\tau_2 s+1} K_{D2}. \tag{41}$$

For $K_{D2} = 0$, (41) is a PI-controller; for $g = 0$, (41) is a PD-controller. With C_2 as in (41), define $W := G_{11} - G_{12} C_2 (I + G_{22} C_2)^{-1} G_{21}$ as in (29), equivalently (10), where $W(0) = X_{11}(0) - a X_{12}(0) X_{21}(0) \in \mathbb{R}^{r_1\times m_1}$. Let $W(0)^I \in \mathbb{R}^{m_1\times r_1}$ denote a right-inverse of $W(0)$. Step 2 : Design C_1: *Follow* Step 2 *of Proposition 1 to design C_1 as in (20) that stabilizes $W \in \mathcal{M}(\mathbf{S})$. Then $C_D = \mathrm{diag}\,[\, C_1,\ C_2\,]$ is a partially reliable decentralized PID controller.* □

Remark. In Proposition 3, if z is at infinity, i.e., $Y_{22} = \frac{1}{s+a} G_{22}^{-1} \in \mathcal{M}(\mathbf{S})$, and $Y_{22}(\infty)^{-1} = s\, G_{22}(s)|_{s\to\infty}$, then $\hat{K}_{p2} = Y_{22}(\infty)$ is nonsingular for any K_{D2}, τ_2. Obviously, $z = \infty > \|\Psi\|$ holds and γ_2 in (40) is chosen to satisfy $\gamma_2 > \|\Psi\|$. □

Remark (*Robustness of the Partially Reliable Decentralized Stabilizing Controllers*). The decentralized controllers in Propositions 2 and 3 achieve robust reliable stability under 'sufficiently small' plant uncertainty. Let $\Delta \in \mathbf{S}^{r\times m}$ be a stable additive perturbation partitioned into two channels as G in (1). Since $G = Y^{-1}X$ is stabilized by the partially reliable decentralized controller $C_D = \mathrm{diag}\,[\,C_1, C_2\,] = N_c D_c^{-1}$ in Proposition 2 (similarly in Proposition 3), we have M_{cl} and M_2 are unimodular. Under additive plant uncertainty, C_D stabilizes $G + \Delta = Y^{-1}(X + Y\Delta)$ if and only if $M_\Delta := YD_c + (X + Y\Delta)N_c = (I + Y\Delta N_c M_{cl}^{-1})M_{cl}$ is unimodular; similarly, C_2 stabilizes $G_{22} + \Delta_{22} = Y_{22}^{-1}(X_{22} + Y_{22}\Delta_{22})$ if and only if $M_{22\Delta} = (I + Y_{22}\Delta_{22}N_{c2}M_2^{-1})M_2$ is unimodular. With M_{cl} and M_2 unimodular, M_Δ is unimodular if and only if $(I + Y\Delta N_c M_{cl}^{-1})$, equivalently $(I + \Delta N_c M_{cl}^{-1} Y)$ is unimodular; similarly, $M_{22\Delta}$ is unimodular if and only if $(I + \Delta_{22}N_{c2}M_2^{-1}Y_{22})$ is unimodular. A sufficient condition for unimodularity is $\|\Delta N_c M_{cl}^{-1} Y\| < 1$; similarly, $\|\Delta_{22}N_{c2}M_2^{-1}Y_{22}\| < 1$. The proposed $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is a partially reliable decentralized PID controller stabilizing $G + \Delta$ for all $\Delta \in \mathbf{S}^{r\times m}$ such that $\|\Delta\| < \|\,C_D(I + GC_D)^{-1}\|^{-1}$, and $\|\Delta_{22}\| < \|\,C_2(I + G_{22}C_2)^{-1}\|^{-1}$. For block-diagonal structured multiplicative perturbations $\Delta = \mathrm{diag}\,[\Delta_{11}\,,\ \Delta_{22}\,]$, it can be shown similarly that C_D is a partially reliable decentralized PID controller stabilizing the plant $G(I + \Delta) = Y^{-1}X(I + \Delta)$ under pre-multiplicative perturbations for all $\Delta \in \mathbf{S}^{m\times m}$, $\Delta_{22} \in \mathbf{S}^{m_2\times m_2}$ such that $\|\Delta\| < \|\,C_D G(I + C_D G)^{-1}\|^{-1}$, and $\|\Delta_{22}\| < \|\,C_2 G_{22}(I + C_2 G_{22})^{-1}\|^{-1}$. We can also consider coprime factor perturbations $\Delta_Y \in \mathbf{S}^{r\times r}$, $\Delta_X \in \mathbf{S}^{r\times m}$, partitioned as $G = Y^{-1}X$. Then C_D is a partially reliable decentralized PID controller stabilizing the plant $(Y + \Delta_Y)^{-1}(X + \Delta_X)$ for all $\Delta_Y, \Delta_X, \Delta_{Y22}, \Delta_{X22} \in \mathcal{M}(\mathbf{S})$ such that $\|\Delta_Y \quad \Delta_X\| < \|\begin{bmatrix} D_c \\ N_c \end{bmatrix} M^{-1}\|^{-1}$, and $\|\Delta_{Y22} \quad \Delta_{X22}\| < \|\begin{bmatrix} D_{c2} \\ N_{c2} \end{bmatrix} M_2^{-1}\|^{-1}$. The free parameter choices affect the numerator matrices N_{c1}, N_{c2}; for example, the norms can be minimized over all $\hat{K}_{p1}$ or $\hat{K}_{d1}$ to find the corresponding maximum allowable perturbation magnitudes $\|\Delta_Y \quad \Delta_X\|$. $\square$

Examples 3 and 4 illustrate the partially reliable decentralized PID controller synthesis procedure of Proposition 3.

Example 3. The unstable plant in this example is obtained from a linearized model of a sugar mill process (Goodwin et al., 2001). Let the plant's transfer-function G be

$$G = Y^{-1}X = \begin{bmatrix} \dfrac{-5}{25s+1} & \dfrac{s^2-0.005(s+1)}{s(s+1)} \\ \dfrac{1}{25s+1} & \dfrac{-0.0023}{s} \end{bmatrix},$$

where G_{22} has a pole at $s=0$ and has only one zero at infinity, i.e., has relative-degree equal to one. With $Y_{22} = \frac{-s}{0.0023(s+a)}$, for any $a \in \mathbb{R}_+$, an LCF of G in the form of (38) is

$$G = \begin{bmatrix} 1 & \dfrac{-50}{23} \\ 0 & \dfrac{-s}{0.0023(s+a)} \end{bmatrix}^{-1} \times \begin{bmatrix} \dfrac{-165}{23(25s+1)} & \dfrac{s}{(s+1)} \\ \dfrac{-s}{0.0023(25s+1)(s+a)} & \dfrac{1}{(s+a)} \end{bmatrix}.$$

The only $\mathcal{U}$-pole of G_{22} is at $s=0$. The plant G and also G_{22} have no transmission-zeros at $s=0$ (G has a transmission-zeros at $s=0.137 \in \mathcal{U}$ and $s=-0.1205$). The only zero of G_{22} is at infinity. Therefore, Assumption 2 hold. Following Proposition 3, we first design C_2: As in the examples of Section 3, we choose the initial parameters of C_2 completely arbitrarily since no other performance criteria are specified and the only goal is reliable stabilization with asymptotic tracking of step references. Let $K_{D2} = -0.0348$, $\tau_2 = 0.01$, $g = 0.01$; then $\hat{K}_{p2} = Y_{22}(\infty) = -1/0.0023$. Choose $\gamma_2 = 0.02 > 0.01 = \|\Psi\|$ satisfying (40). Then the PID controller C_2 in (41) is

$$C_2 = \frac{-0.02}{0.0023}\left(1+\frac{0.01}{s}\right) - \frac{0.0348s}{0.01s+1}.$$

Proceeding to Step 2, we follow Proposition 1 to design C_1 stabilizing $W = G_{11} - G_{12}C_2(I+G_{22}C_2)^{-1}G_{21} \in \mathbf{S}$, where

$$W = \frac{0.2869s^4 + 14.5781s^3 - 20.23s^2 - 0.5797s - 0.005739}{(s+1)(s+1/25)(s^3+100.0280s^2+2.0002s+0.02)}.$$

Choose $\hat{K}_{d1} = -1$, $\hat{K}_{p1} = -5$, $\tau_1 = 0.01$. With $\gamma_1 = 0.0882 < 0.0892 = \|\frac{sW\hat{C}_1 - I}{s}\|^{-1}$, the PID controller C_1 in (20) is $C_1 = -0.4410 - \frac{0.0123}{s} - \frac{0.0882s}{0.01s+1}$. With $C_D = \text{diag}[C_1, C_2]$, Fig. 6 shows the step responses for the two outputs y_1, y_2, with unit-steps applied at both references u_1, u_2. Both channels of $C_D = \text{diag}\,[C_1, C_2]$ are operational. Both channels display undershoot due to the transmission-zero at $s = 0.137 \in \mathcal{U}$ of G. Fig. 7 shows the step responses when C_1 fails, i.e., $C_D = \text{diag}[0, C_2]$, with only the second channel operational. The partially reliable design guarantees closed-loop stability when $C_1 = 0$ but asymptotic tracking with zero steady-state error is achieved only in the second channel. $\square$

Example 4. Consider a chemical reactor model adopted from El-Farra, Mhaskar, and Christofides (2004), where the concentration of the inlet reactant and the rate of heat input are manipulated to regulate the outlet reactant concentration and the reactor

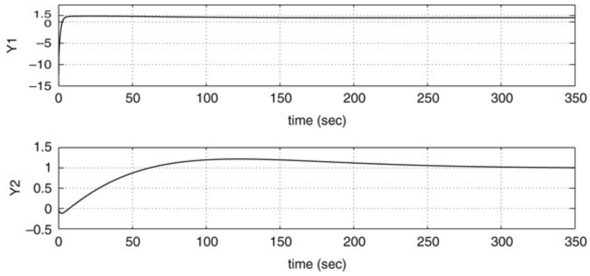

Fig. 6. Example 3 step-responses with $C_D = \text{diag}[C_1, C_2]$.

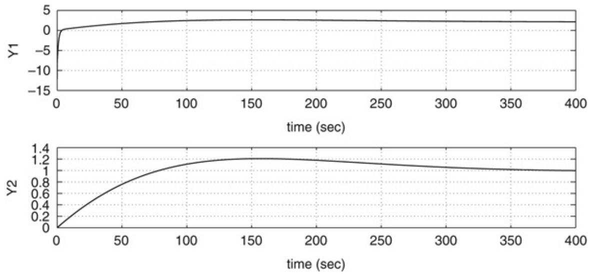

Fig. 7. Example 3 step-responses with $C_D = \text{diag}[0, C_2]$.

temperature. The linearization around one of the operating points gives the unstable plant transfer-matrix

$$G = \frac{1}{d}\begin{bmatrix} 1.67s - 0.1232 & -0.0018934 \\ 4.143 & 4.184s + 0.1218 \end{bmatrix},$$

$d = 100(s-0.0614)(s+0.167)$, where G_{22} has poles at $s = 0.0614 \in \mathcal{U}$, $s = -0.0167$, and a zero at infinity. With $Y_{22} = \frac{d}{w}$, where $w = (4.184s+0.1218)(s+a)$ for any $a \in \mathbb{R}_+$, an LCF of G in the form of (38) is

$$Y^{-1}X = \begin{bmatrix} 1 & 0.005 \\ 0 & \dfrac{d}{w} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{1.67}{100(s+0.167)} & \dfrac{0.02092}{100(s+0.167)} \\ \dfrac{4.143}{w} & \dfrac{1}{(s+a)} \end{bmatrix}.$$

The plant satisfies Assumption 2 since G_{22} has only one $\mathcal{U}$-zero (at infinity) and hence, partially reliable decentralized PID controllers exist. The other sub-blocks of G may have any number of $\mathcal{U}$-zeros: G_{11} has two $\mathcal{U}$-zeros (at $s = 0.1232/1.67$ and infinity); G_{12} and G_{21} each have two $\mathcal{U}$-zeros (both at infinity). We apply the synthesis in Proposition 3 with $z = \infty$. Let $K_{D2} = 10$, $\tau_2 = 0.02$, $g = 20$; then $\hat{K}_{p2} = Y_{22}(\infty) = 23.9006$. Choose $\gamma_2 = 20 > 14.2384 = \|\Psi\|$ satisfying (40). Then a PID controller stabilizing G_{22} as in (41) is

$$C_2 = 478.012 + \frac{9560.2}{s} + \frac{10s}{0.02s+1},$$

and $W \in \mathbf{S}$ as in (29) becomes

$$W = \frac{0.0167s^4 + 1.5199s^3 + 23.4691s^2 + 334.3807s + 5.5800}{(s+0.167)(s^4+91.03s^3+1406.46s^2+20040.26s+582.22)}.$$

To design C_1 that stabilizes W, let $\hat{K}_{p1} = -10$, $\hat{K}_{d1} = 0.1$, $\tau_1 = 0.02$. With $W(0) = 0.0574$, $\hat{C}_1 = -10 + \frac{0.1s}{0.02s+1} + \frac{17.4247}{s}$ as in (19). Choose $\gamma_1 = 0.05 < 0.0605$ satisfying the norm in (20). Then a PID controller stabilizing W as in (20) is

$$C_1 = -0.5 + \frac{0.8712}{s} + \frac{0.005s}{0.02s+1}.$$

Fig. 8 shows the step responses for the outputs y_1, y_2, with unit-steps applied at both u_1, u_2. Both channels of $C_D = [C_1, C_2]$ are operational. Fig. 9 shows the step responses when C_1 fails, i.e., $C_D = \text{diag}[0, C_2]$, with only the second channel operational. The partially

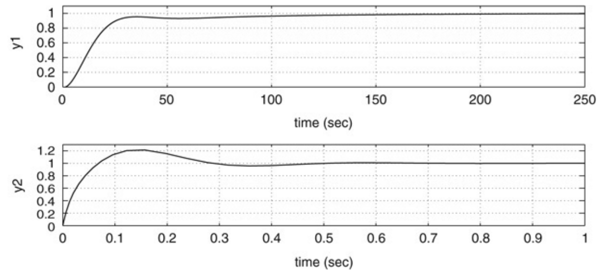

Fig. 8. Example 4 step-responses with $C_D = \mathrm{diag}[C_1, C_2]$.

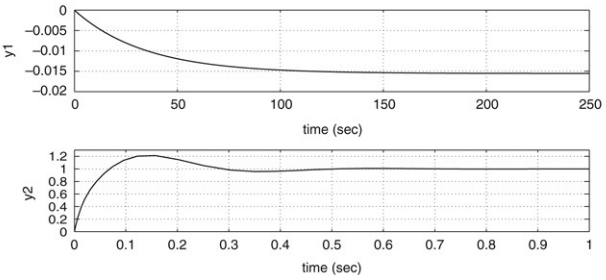

Fig. 9. Example 4 step-responses with $C_D = \mathrm{diag}[0, C_2]$.

reliable design guarantees closed-loop stability when $C_1 = 0$ but asymptotic tracking with zero steady-state error is achieved only in the second channel that has integral-action. □

5. Conclusions

We proposed systematic synthesis of fully reliable or partially reliable decentralized PID controllers that achieve closed-loop stability and asymptotic tracking of step-input references at each output channel when both channels are operational, and maintain closed-loop stability even when one of the controllers is turned off. The proposed PID controllers provide robust stability for small plant uncertainty. For stable processes, we gave illustrative examples for a fully reliable PID design and also for a partially reliable design, where a fully reliable PID controller did not exist. In the case of unstable processes, due to the restrictive low order of the PID controllers, only partially reliable PID designs were explored. The class of unstable plants had no restrictions on the unstable poles but only a single blocking-zero in the region of instability was allowed in the main plant channel. Our continued study will expand the synthesis methods to unstable plants with more freedom on these zeros and will identify unstable plant classes where fully reliable designs are achievable. The synthesis method can be modified to apply to decentralized structures with more than two MIMO control channels.

Appendix. Proofs

Proof of Lemma 1. (i) Let $G = \tilde{X}\tilde{Y}^{-1}$ be an RCF of G. By Definition 1, if C_D is a stabilizing integral-action controller, then the system $Sys(G, C_D)$ has integral-action. Since $Sys(G, C_D)$ is stable, $H_{wu} = C_D(I_r + GC_D)^{-1} \in \mathcal{M}(\mathbf{S})$ and $H_{yu} = GH_{wu} = \tilde{X}\tilde{Y}^{-1}H_{wu} \in \mathcal{M}(\mathbf{S})$ implies $\tilde{Y}^{-1}H_{wu} =: R \in \mathcal{M}(\mathbf{S})$. Since $Sys(G, C_D)$ has integral-action, $H_{eu}(0) = 0$ implies $H_{yu}(0) = I_r = \tilde{X}(0)R(0)$. Therefore, $\mathrm{rank}\,\tilde{X}(0) = r$, equivalently, G has no transmission-zeros at $s = 0$. **(ii)** An RCF of the PID controller $C_{pid} = K_P + \frac{1}{s}K_I + \frac{s}{\tau s+1}K_D$ when $K_I \neq 0$ is $C_{pid} = N_c D_c^{-1} = (C_{pid}\,\frac{s}{s+e})(\frac{s}{s+e}I_r)^{-1} = (\frac{s}{s+e}[K_P + \frac{s}{\tau s+1}K_D] + \frac{1}{s+e}K_I)(\frac{s}{s+e}I_r)^{-1}$ for any $e \in \mathbb{R}_+$. For all $z > 0$, $\det D_c(z) = \det \frac{z}{z+e}I > 0$. If C_{pid} stabilizes G, by (5) M_{cl} unimodular implies $\det M_{cl}(z) = \det Y(z) \det D_c(z)$ has the same sign for all $z \in \mathcal{U}$ such that $X(z) = 0$; i.e., $\det Y(z)$ has the same sign at all blocking-zeros of G, i.e., G has the parity-interlacing-property Vidyasagar (1985); therefore G is strongly stabilizable. □

Proof of Lemma 2. Writing $G \in \mathcal{M}(\mathbf{S})$ as $G = Y^{-1}X = I^{-1}G$, M_{cl} in (5) is $M_{cl} = D_c + GN_c$ as in (6). **(i)** By Definition 1(a) and (3), the closed-loop transfer function $H_{cl} = \begin{bmatrix} N_c \\ -D_c \end{bmatrix} M_{cl}^{-1} \begin{bmatrix} I & -G \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}$ from (u, v) to (w, y) is stable if and only if $M_{cl}^{-1} = (D_c + GN_c)^{-1} \in \mathcal{M}(\mathbf{S})$, equivalently, M_{cl} in (6) is unimodular. **(ii)** By Definition 1(b) and (6), if $C_1 = 0 = N_{c1}D_{c1}^{-1}$, then with $N_{c1} = I$, $D_{c1} = 0$, $M_{cl} = \begin{bmatrix} I & G_{12}N_{c2} \\ 0 & M_2 \end{bmatrix}$ is unimodular if and only if M_2 in (7) is unimodular. **(iii)** Following similar steps as in (ii), M_{cl} is unimodular with $C_2 = 0 = N_{c2}D_{c2}^{-1}$ if and only if M_1 in (8) is unimodular. **(iv)** By (i), (ii), (iii), $Sys(G, C_1, C_2)$ and $Sys(G, 0, C_2)$ are stable if and only if M_{cl} and M_2 are both unimodular. With M_2 unimodular, $M_{cl} = \begin{bmatrix} I & G_{12}N_{c2}M_2^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} M_W & 0 \\ G_{21}N_{c1} & M_2 \end{bmatrix}$ is unimodular if and only if M_W is unimodular, i.e., C_1 stabilizes W. **(v)** By (i)–(iv), $Sys(G, C_1, C_2)$, $Sys(G, 0, C_2)$, $Sys(G, C_1, 0)$ are all stable if and only if M_2 is unimodular (C_2 stabilizes G_{22}), M_W is unimodular (C_1 stabilizes W), and M_1 is unimodular (C_1 stabilizes G_{11}). □

Proof of Theorem 1. (i) By Definition 1(f) and Lemma 2(iv), C_D is partially reliable if and only if C_2 stabilizes $G_{22} \in \mathcal{M}(\mathbf{S})$ and C_1 stabilizes $W \in \mathcal{M}(\mathbf{S})$. All controllers stabilizing these stable systems are given by (11)–(12), where $Q_2, Q_1 \in \mathcal{M}(\mathbf{S})$ are arbitrary but chosen so that C_2, C_1 are proper, i.e., $\det(I - G_{22}Q_2)(\infty) \neq 0$ and $\det(I - WQ_2)(\infty) \neq 0$ (Gündeş & Desoer, 1990; Vidyasagar, 1985). With C_2 as in (11), $M_2 = I$; hence, W is as in (13). **(ii)** By Definition 1-(g) and Lemma 2(v), the partially reliable controller in part (i) becomes fully reliable if $C_1 = Q_1(I - WQ_1)^{-1}$ also stabilizes $G_{11} \in \mathcal{M}(\mathbf{S})$, i.e., by (8), $M_1 = D_{c1} + G_{11}N_{c1} = (I - WQ_1) + G_{11}Q_1$ is unimodular, equivalently, (14) holds. □

Proof of Lemma 3. Let $C = N_c D_c^{-1}$ be an integral-action controller simultaneously stabilizing G_{11} and W. Then for $e \in \mathbb{R}_+$, $\hat{D}_c \in \mathcal{M}(\mathbf{S})$, $D_c =: \frac{s}{s+e}\hat{D}_c$. Therefore, $M_1 = \frac{s}{s+e}\hat{D}_c + G_{11}N_c$, $M_W = \frac{s}{s+e}\hat{D}_c + WN_c$ are unimodular. By assumption, $G_{11}(z_o) = W(z_o)$ for some $z_o \in \mathbb{R}_+ \cup \{\infty\}$ implies $M_1(z_o) - M_W(z_o) = [G_{11}(z_o) - W(z_o)]N_c(z_o) = 0$, i.e., $M_1(z_o) = M_W(z_o)$. Since $\det M_1(z_o) = \det M_W(z_o)$ for $z_o \in \mathcal{U}$, $\det M_1(s)$, $\det M_W(s)$ have the same sign for all $s \in \mathcal{U} \cap \mathbb{R}$. At $s = 0$, $M_1(0) = G_{11}(0)N_c(0)$ implies $N_c(0) = G_{11}(0)^I M_1(0)$; hence, $M_W(0) = W(0)N_c(0) = W(0)G_{11}(0)^I M_1(0)$. With same sign for $\det M_W(0)$ and $\det M_1(0)$, $\det M_W(0) = \det[W(0)G_{11}(0)^I] \det M_1(0)$ implies (15). □

Proof of Proposition 1. (a) Write (18) as $C_2 = N_{c2}D_{c2}^{-1} = (C_2 D_{c2})(\frac{s}{s+e}I)^{-1}$, $(e > 0)$. Write M_2 in (7) as $M_2 = D_{c2} + G_{22}N_{c2} = \frac{s}{s+e}I + \frac{s}{s+e}G_{22}C_2 = \frac{(s+\gamma_2)}{(s+e)}\,(\,\frac{s}{s+\gamma_2}I + \frac{\gamma_2 s}{s+\gamma_2}G_{22}\hat{C}_2\,) = \frac{(s+\gamma_2)}{(s+e)}(\,I + \frac{\gamma_2 s}{s+\gamma_2}\,[\,\frac{1}{s}(s\,G_{22}\,\hat{C}_2 - I)\,]) = \frac{(s+\gamma_2)}{(s+e)}(\,I + \frac{\gamma_2 s}{s+\gamma_2}\,[\,G_{22}\,(\hat{K}_{p2} + \frac{s}{\tau_2 s+1}\,\hat{K}_{d2}) + \frac{1}{s}(G_{22}(s)G_{22}(0)^I - I)\,])$. If $\gamma_2 < \|\frac{1}{s}(s\,G_{22}\,\hat{C}_2 - I)\|^{-1}$ as in (18), then M_2 is unimodular; hence, C_2 stabilizes $G_{22} \in \mathcal{M}(\mathbf{S})$. Therefore, $C_2(I + G_{22}C_2)^{-1} \in \mathcal{M}(\mathbf{S})$ and $W = G_{11} - G_{12}C_2(I + G_{22}C_2)^{-1}G_{21} \in \mathcal{M}(\mathbf{S})$. Repeat the same steps for C_1 in (20) to show that $M_W := D_{c1} + WN_{c1}$ is unimodular; hence, C_1 stabilizes $W \in \mathcal{M}(\mathbf{S})$. By Lemma 2(iv), $C_D = \mathrm{diag}\,[C_1, C_2]$ is a partially reliable controller, with the PID controllers C_2, C_1 as in (18) and (20). **(b)** As shown in (a), C_2 in (18) stabilizes G_{22}. Similarly, $C_1 = N_{c1}D_{c1}^{-1} = (C_1 D_{c1})(\frac{s}{s+e}I)^{-1}$ in (22) stabilizes $G_{11} \in \mathcal{M}(\mathbf{S})$ because $\beta_1 < \|\frac{1}{s}(s\,G_{11}\tilde{C}_1 - I)\|^{-1}$ implies $M_1 = D_{c1} + G_{11}N_{c1} = \frac{s}{s+a}I + \frac{s}{s+e}G_{11}C_1 = \frac{(s+\beta_1)}{(s+e)}\,(\,\frac{s}{s+\beta_1}I + \frac{\beta_1 s}{s+\beta_1}G_{11}\tilde{C}_1\,) = \frac{(s+\beta_1)}{(s+e)}(\,I + \frac{\beta_1 s}{s+\beta_1}\,[\,\frac{1}{s}(s\,G_{11}\,\tilde{C}_1 - I)\,])$ is unimodular, where $[\,\frac{1}{s}(s\,G_{11}\,\tilde{C}_1 - I)\,] = [\,G_{11}\,(\hat{K}_{p1} + \frac{s}{\tau_1 s+1}\hat{K}_{d1}) + \frac{1}{s}(G_{11}(s)G_{11}(0)^I - I)\,] \in \mathcal{M}(\mathbf{S})$. By assumption, $R_o := W(0)G_{11}(0)^I$ is symmetric, positive-definite implies $(sI + \beta_1 R_o)^{-1} \in \mathcal{M}(\mathbf{S})$ and $\|(sI + \beta_1 R_o)^{-1} sI\| = 1$. Then $\beta_1 < \|\frac{1}{s}(sW\tilde{C}_1 - R_o)\|^{-1}$ implies $M_W := D_{c1} + WN_{c1} = \frac{s}{s+e}I + WC_1\frac{s}{s+e} = \frac{1}{(s+e)}(sI + \beta_1 R_o)((sI + \beta_1 R_o)^{-1}sI + \beta_1(sI + \beta_1 R_o)^{-1}sW\tilde{C}_1) = \frac{1}{(s+e)}(sI+\beta_1 R_o)(I+\beta_1 s(sI+\beta_1 R_o)^{-1}[\frac{1}{s}(sW\tilde{C}_1 - R_o)])$ is unimodular, where $[\frac{1}{s}(sW\tilde{C}_1 - R_o)] = [W(\hat{K}_{p1} + \frac{s}{\tau_1 s+1}\hat{K}_{d1}) + \frac{1}{s}(W(s)G_{11}(0)^I - W(0)G_{11}(0)^I)] \in \mathcal{M}(\mathbf{S})$. Therefore, C_1 stabilizes also $W \in \mathcal{M}(\mathbf{S})$. By Lemma 2(v), $C_D = \mathrm{diag}[C_1, C_2]$ is a fully reliable controller, with the PID controllers C_2, C_1 as in (18) and (22). In (a) and (b), since $\hat{K}_{pj}$, $\hat{K}_{dj}$ are arbitrary, they can be zero. $\square$

Proof of Lemma 4. **(i)** By Definition 1(a) and (5), the closed-loop transfer function from (u, v) to (w, y),

$$H_{cl} = \begin{bmatrix} N_c \\ -D_c \end{bmatrix} M_{cl}^{-1} \begin{bmatrix} Y & -X \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}$$

is stable if and only if $M_{cl}^{-1} = (YD_c + XN_c)^{-1} \in \mathcal{M}(\mathbf{S})$, i.e., M_{cl} in (24) is unimodular. **(ii)** By Definition 1(b) and (24), if $C_1 = 0 = N_{c1}D_{c1}^{-1}$, then the closed-loop transfer-function H_2 from (u_2, v_1, v_2) to (w_2, y_1, y_2) is stable if and only if M_{cl2} in (25) is unimodular. But M_{cl2} is unimodular if and only if Y_{11} and M_2 are both unimodular. **(iii)** By Definition 1(c) and (24), if $C_2 = 0 = N_{c2}D_{c2}^{-1}$, then the closed-loop transfer-function H_1 from (u_1, v_1, v_2) to (w_1, y_1, y_2) is stable if and only if M_{cl1} in (27) is unimodular. The controller C_1 stabilizes G_{11} if and only if

$$H_{G1} = \begin{bmatrix} C_1(I+G_{11}C_1)^{-1} & -C_1(I+G_{11}C_1)^{-1}G_{11} \\ G_{11}C_1(I+G_{11}C_1)^{-1} & (I+G_{11}C_1)^{-1}G_{11} \end{bmatrix}$$

$$= \begin{bmatrix} N_{c1} & 0 \\ -D_{c1} & 0 \end{bmatrix} M_{cl1}^{-1} \begin{bmatrix} Y_{11} & -X_{11} \\ 0 & -X_{21} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} \in \mathcal{M}(\mathbf{S}).$$

If $Sys(G, C_1, 0)$ is stable, i.e., $M_{cl1}^{-1} \in \mathcal{M}(\mathbf{S})$, then $H_{G1} \in \mathcal{M}(\mathbf{S})$, i.e., C_1 stabilizes G_{11}. Furthermore, M_{cl1} unimodular implies

$$\mathrm{rank}\, M_{cl1} = \mathrm{rank} \begin{bmatrix} X_{11} & Y_{11} & Y_{12} \\ X_{21} & 0 & Y_{22} \end{bmatrix} \begin{bmatrix} N_{c1} & 0 \\ D_{c1} & 0 \\ 0 & I_{r_2} \end{bmatrix} = r;$$

hence, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$

is LC. Also $\mathrm{rank}\, M_{cl1} = r$ implies $\mathrm{rank} \begin{bmatrix} Y_{12} \\ Y_{22} \end{bmatrix} = r_2$, i.e., (Y_{12}, Y_{22}) is RC. To prove the converse, suppose $H_{G1} =: N_R M_{cl1}^{-1} N_L + F \in \mathcal{M}(\mathbf{S})$, with $(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix})$ LC and (Y_{12}, Y_{22}) RC. Then (N_{c1}, D_{c1}) RC and (Y_{12}, Y_{22}) RC imply rank $\begin{bmatrix} M_{cl1} \\ N_R \end{bmatrix} = r$; hence (N_R, M_{cl1}) is RC. Similarly, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ LC implies rank $\begin{bmatrix} M_{cl1} & N_L \end{bmatrix} = r$; hence, (M_{cl1}, N_L) is LC. Therefore, $H_{G1} = N_R M_{cl1}^{1} N_L + F \in \mathcal{M}(\mathbf{S})$ implies $M_{cl1}^{-1} \in \mathcal{M}(\mathbf{S})$; hence, $Sys(G, C_1, 0)$ is stable. **(iv)** By (i), (ii), (iii), $Sys(G, C_1, C_2)$ and $Sys(G, 0, C_2)$ are stable if and only if M_{cl} in (24), Y_{11}, and M_2 in (26) are all unimodular. With M_2 unimodular, defining $J := (Y_{12}D_{c2} + X_{12}N_{c2})$,

$$M_{cl} = \begin{bmatrix} I & JM_2^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} M_W & 0 \\ X_{21}N_{c1} & M_2 \end{bmatrix}$$

is unimodular if and only if M_W in (28) is unimodular, equivalently, C_1 stabilizes W. **(v)** By (i), (ii), (iii), (iv), $Sys(G, C_1, C_2)$, $Sys(G, 0, C_2)$, $Sys(G, C_1, 0)$ are all stable if and only if $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ is LC, (Y_{12}, Y_{22}) is RC, M_2 is unimodular (C_2 stabilizes G_{22}), M_W is unimodular (C_1 stabilizes W) and M_1 is unimodular (C_1 stabilizes G_{11}). $\square$

Proof of Theorem 2. **(i)** **(a)** By Definition 1(f) and Lemma 4(iv), there exists a partially reliable C_D if and only if Y_{11} is unimodular since C_2 such that M_2 is unimodular and C_1 such that M_W is unimodular can be designed as in part (b). **(b)** All controllers stabilizing $G_{22} \in \mathcal{M}(\mathbf{R_p})$ and $W \in \mathcal{M}(\mathbf{S})$ are given by (30)–(32), where $Q_2, Q_1 \in \mathcal{M}(\mathbf{S})$ are arbitrary but chosen so that C_2, C_1 are proper, i.e., $\det(\tilde{V} - \tilde{X}_{22}Q_2)(\infty) \neq 0$ and $\det(I - WQ_2)(\infty) \neq 0$ (Gündeş & Desoer, 1990; Vidyasagar, 1985). With C_2 as in (30) $M_2 = I$; hence, W is as in (31). **(ii)** **(a)** By Definition 1-(g) and Lemma 4(v), there exists a fully reliable C_D if and only if Y_{11} is unimodular, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ is LC, (Y_{12}, Y_{22}) is RC, and there exists a controller that simultaneously stabilizes $G_{11} \in \mathcal{M}(\mathbf{R_p})$ and $W \in \mathcal{M}(\mathbf{S})$. This last condition is equivalent to simultaneous stabilizability of $G_{11} - W$. **(b)** The partially reliable controller in part (i) becomes fully reliable if C_2 in (30) is such that $G_{11} - W = G_{12}C_2(I + G_{22}C_2)^{-1}G_{21} = G_{12}(\tilde{U} + \tilde{Y}_{22}Q_2)Y_{22}G_{21}$ is strongly stabilizable, and $C_1 = N_{c1}D_{c1}^{-1} = Q_1(I - WQ_1)^{-1}$ also stabilizes $G_{11} \in \mathcal{M}(\mathbf{R_p})$. With Y_{11} unimodular, $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ LC, (Y_{12}, Y_{22}) RC, $C_1 = Q_1(I - WQ_1)^{-1}$ stabilizes G_{11} if and only if M_{cl1} is unimodular, where $Y_{11}D_{c1} + X_{11}N_{c1} = I + (X_{11} - W)Q_1$, $E := -Y_{12} + (Y_{12}D_{c2} + X_{12}N_{c2})M_2^{-1}Y_{22} = (X_{12} - Y_{12}G_{22})C_2(I + G_{22}C_2)^{-1} = Y_{11}G_{12}C_2(I + G_{22}C_2)^{-1} \in \mathcal{M}(\mathbf{S})$ imply

$$M_{cl1} = \begin{bmatrix} Y_{11} + JM_2^{-1}X_{21}Q_1 & Y_{12} \\ X_{21}Q_1 & Y_{22} \end{bmatrix}$$

$$= \begin{bmatrix} I & JM_2^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ X_{21}Q_1Y_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} Y_{11} & -E \\ 0 & Y_{22} + X_{21}Q_1Y_{11}^{-1}E \end{bmatrix}$$

(where $J := (Y_{12}D_{c2} + X_{12}N_{c2})$). Then M_{cl1} is unimodular if and only if $Y_{22} + X_{21}Q_1Y_{11}^{-1}E$ as in (33) is unimodular. By assumption, (Y_{12}, Y_{22}) RC implies $(Y_{11}^{-1}E, Y_{22})$ is RC; $\left(Y, \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix}\right)$ LC implies (Y_{22}, X_{21}) is LC. Therefore, $G_{11} - W = Y_{11}^{-1}E\, Y_{22}^{-1}X_{21}$ is a bicoprime factorization; hence, (33) is equivalent to $Q_1 \in \mathcal{M}(\mathbf{S})$ strongly stabilizes $G_{11} - W$. $\square$

Proof of Proposition 2. Since G satisfies Assumption 1, $G_{22} = Y_{22}^{-1}X_{22}$, where $X_{22} = I$, $Y_{22} = G_{22}^{-1} \in \mathcal{M}(\mathbf{S})$. Write C_2 in (36) as $C_2 = N_{c2}D_{c2}^{-1} = (C_2D_{c2})(\frac{s}{s+g}I)^{-1}$. Write M_2 in (26) as $M_2 = Y_{22}D_{c2} + N_{c2} = \frac{s}{s+g}G_{22}^{-1} + \frac{s}{s+g}C_2 = \frac{s}{s+g}(G_{22}^{-1} + (1 + \frac{g}{s})\gamma_2\hat{K}_p + \frac{s}{\tau_2 s+1}K_{D2}) = \gamma_2\hat{K}_p + \frac{s}{s+g}(G_{22}^{-1} + \frac{s}{\tau_2 s+1}K_{D2}) = \gamma_2(I + \frac{s}{\gamma_2(s+g)}[G_{22}^{-1} + \frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_p^{-1})\hat{K}_p$. If $\gamma_2 > 0$ satisfies (35), then $\|\frac{s}{\gamma_2(s+g)}[G_{22}^{-1} + \frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_p^{-1}\| \leq \frac{1}{\gamma_2}\|\frac{s}{s+g}\|\,\|[G_{22}^{-1} + \frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_p^{-1}\| \leq \frac{1}{\gamma_2}\|[G_{22}^{-1} + \frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_p^{-1}\| < 1$ implies M_2 is unimodular; hence, C_2 stabilizes $G_{22} \notin \mathcal{M}(\mathbf{S})$. Therefore, $M_2^{-1} \in \mathcal{M}(\mathbf{S})$ implies $W = Y_{11}^{-1}[X_{11} - (Y_{12}D_{c2} + X_{12}N_{c2})M_2^{-1}X_{21}] \in \mathcal{M}(\mathbf{S})$, where $X_{22} = I$, $D_{c2}(0) = 0$, $M_2(0) = X_{22}(0)N_{c2}(0) = N_{c2}(0)$ imply $W(0) = X_{11}(0) - X_{12}(0)X_{22}(0)^{-1}X_{21}(0) = X_{11}(0) - X_{12}(0)X_{21}(0)$. It follows from the proof of Proposition 1 that the controller C_1 in (20) stabilizes this stable W and hence, the conclusion follows. $\square$

Proof of Proposition 3. Since G satisfies Assumption 2, $G_{22} = Y_{22}^{-1}X_{22}$, where $X_{22} = \frac{(1-s/z)}{s+a}I$, $Y_{22} = \frac{(1-s/z)}{s+a}G_{22}^{-1} \in \mathcal{M}(\mathbf{S})$ for $a > 0$. Write C_2 in (41) as $C_2 = N_{c2}D_{c2}^{-1} = (C_2D_{c2})(\frac{s}{s+g}I)^{-1}$. Write M_2 in (26) as $M_2 = Y_{22}D_{c2} + X_{22}N_{c2} = \frac{(1-s/z)}{(s+a)}\frac{s}{(s+g)}G_{22}^{-1} + \frac{(1-s/z)}{(s+a)}\frac{s}{(s+g)}C_2 = \frac{(1-s/z)}{(s+a)}\frac{s}{(s+g)}(G_{22}^{-1} + (1 + \frac{g}{s})\frac{\gamma_2}{1+\gamma_2/z}\hat{K}_{p2} + \frac{s}{\tau_2 s+1}K_{D2}) = \frac{(s+\gamma_2)}{(s+a)(1+\gamma_2/z)}(\frac{(1-s/z)}{s+\gamma_2}\gamma_2 I + (1 + \frac{\gamma_2}{z})\frac{s}{(s+g)}\frac{(1-s/z)}{(s+\gamma_2)}[G_{22}^{-1} + \frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_{p2}^{-1})\hat{K}_{p2} = \frac{(s+\gamma_2)}{(s+a)}(I + \frac{(1+\gamma_2/z)}{s+\gamma_2}\Psi)\frac{\hat{K}_{p2}}{(1+\gamma_2/z)}$. Note that

$\Psi = \frac{s}{s+g}(1-s/z)[G_{22}^{-1}+\frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_{p2}^{-1}-sI = \frac{s}{s+g}[(s+a)Y_{22}+(1-s/z)\frac{s}{\tau_2 s+1}K_{D2}]\hat{K}_{p2}^{-1} - sI = s[\frac{s}{s+g}(Y_{22} - \frac{s}{(s+1/\tau_2)z\,\tau_2}K_{D2})(Y_{22}(\infty) - \frac{1}{z}K_{D2})^{-1} - I] + \frac{s}{s+g}(aY_{22} + \frac{s}{\tau_2 s+1}K_{D2}) \in \mathcal{M}(\mathbf{S})$. If $\|\Psi\|/z < 1$, then for $\gamma_2 > 0$ satisfying (40), $\|\frac{(1+\gamma_2/z)}{s+\gamma_2}\Psi\| \le \frac{(1+\gamma_2/z)}{\gamma_2}\|\Psi\| < 1$ implies M_2 is unimodular; hence, C_2 stabilizes $G_{22} \notin \mathcal{M}(\mathbf{S})$. Therefore, $M_2^{-1} \in \mathcal{M}(\mathbf{S})$ implies $W = Y_{11}^{-1}[X_{11}-(Y_{12}D_{c2}+X_{12}N_{c2})M_2^{-1}X_{21}] \in \mathcal{M}(\mathbf{S})$, where $D_{c2}(0) = 0$, $M_2(0) = X_{22}(0)N_{c2}(0) = \frac{1}{a}N_{c2}(0)$ imply $W(0) = X_{11}(0)-X_{12}(0)X_{22}(0)^{-1}X_{21}(0) = X_{11}(0)-a\,X_{12}(0)X_{21}(0)$. It follows from the proof of Proposition 1 that the controller C_1 in (20) stabilizes this stable W and hence, the conclusion follows. □

References

Bequette, B. W. (2003). *Process control modeling, design and simulation*. NJ: Prentice Hall.

Braatz, R. D., Morari, M., & Skogestad, S. (1994). Robust reliable decentralized control. In *Proc. American control conference* (pp. 3384–3388).

Campo, P. J., & Morari, M. (1994). Achievable closed-loop properties of systems under decentralized control: Conditions involving the steady-state gain. *IEEE Transactions on Automatic Control*, *39*(5), 932–943.

El-Farra, N. H., Mhaskar, P., & Christofides, P. D. (2004). Hybrid predicative control of nonlinear systems: Method and applications to chemical processes. *International Journal on Robust Nonlinear Control*, *14*, 199–225.

Goodwin, G. C., Graebe, S. F., & Salgado, M. E. (2001). *Control system design*. NJ: Prentice Hall.

Gündeş, A. N., & Desoer, C. A. (1990). *LNCIS*: *Vol. 142. Algebraic theory of linear feedback systems with full and decentralized compensators*. Berlin: Springer.

Gündeş, A. N., & Kabuli, M. G. (2001). Reliable decentralized integral-action controller design. *IEEE Transactions on Automatic Control*, *46*, 296–301.

Gündeş, A. N., & Özgüler, A. B. (2007). PID stabilization of MIMO plants. *IEEE Transactions on Automatic Control*, *52*(8), 1502–1508.

Johansson, K. H. (2000). The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions of Control Systems Technology*, *8*(3), 456–465.

Seron, M. M., Zhuo, X. W., De Dona, J. A., & Martinez, J. J. (2008). Multisensor switching control strategy with fault tolerance guarantees. *Automatica*, *44*, 88–97.

Siljak, D. D. (1980). Reliable control using multiple control systems. *International Journal of Control*, *56*(4), 303–329.

Tan, X. L., Siljak, D. D., & Ikeda, M. (1992). Reliable stabilization via factorization methods. *IEEE Transactions on Automatic Control*, *37*(2), 1786–1791.

Vidyasagar, M. (1985). *Control system synthesis: A factorization approach*. Cambridge, MA: MIT Press.

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