ALGEBRAIC THEORY OF TWO-CHANNEL DECENTRALIZED CONTROL SYSTEMS

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Abstract

A two-channel multiinput-multioutput linear time-invariant decentralized control system is analyzed in a general algebraic framework. Necessary and sufficient conditions for decentralized stabilizability are obtained in an algebraic setting and interpreted in terms of fixed-eigenvalues in the case of rational transfer functions. The class of all decentralized stabilizing compensators is given; this class is parametrized by two parameter matrices, which are not completely free. The results apply to distributed or lumped, discrete-time or continuous-time systems.

I. Introduction

In large scale systems, we often encounter restrictions on the feedback controller structure. These systems have several local control stations; each local controller observes only the corresponding outputs. Such decentralized control results in a block-diagonal controller-matrix structure.

A multi-channel plant P with rational function entries can be stabilized by a decentralized dynamic output-feedback compensator if and only if P has no unstable *decentralized fixed-eigenvalues* (misleadingly called fixed-modes in the literature) with respect to block-diagonal real constant output-feedback [Wan.1]. Decentralized fixed-eigenvalues can be characterized in various ways and interpreted in terms of transmission-zeros [And.1, Cor.1, Dav.1, 2]. An algebraic characterization of fixed-eigenvalues using left-factorizations of the plant is given in [And.1].

Decentralized compensator synthesis methods for linear time-invariant systems are available in the literature; these procedures do not result in an explicit expression for the class of all stabilizing compensators. The original method in [Wan.1] uses state-space techniques to move all unstable controllable and observable modes to the left-half complex plane by applying feedback to each channel sequentially; an algorithm that includes improper plants is given in [Dav.2]. In [Cor.1], if the plant is *strongly-connected*, the system is made stabilizable and detectable through one channel by applying appropriate feedback to all other channels (see also [Vid.2]). An N x N plant, which has no unstable fixed-eigenvalues with respect to *diagonal* constant feedback, is considered in [Güç.1]; using polynomial algebra, an N-step algorithm is given to determine a compensator which moves the poles of this square plant to a symmetric region of stability.

In this paper, we obtain necessary and sufficient conditions on P for stabilizability by a decentralized dynamic compensator in a completely general algebraic framework; hence the results are applicable to distributed and lumped, continuous-time and discrete-time systems. Decentralized stabilizability conditions turn out to be certain Smith-form-like structures that must be satisfied by coprime factorizations of the plant P. When the compensator structure is required to be block-diagonal as in decentralized output-feedback, finding the class of all stabilizing decentralized compensators is complicated; the task is to find a *structured* Bezout identity where the coprime factorizations of P satisfy decentralized stabilizability conditions. For plants that satisfy these conditions, we parametrize the class of all stabilizing decentralized compensators; this class has two parameter matrices (the parameter matrices satisfy a unimodularity condition).

The paper is organized as follows: The algebraic setting is explained in Section II. Section III gives the system description; to simplify derivations, we consider a two-channel multiinput-multioutput system in detail (see Figure 1). All results can be extended to m–channels [Des.1]. Conditions on coprime factorizations of P for decentralized stabilizability and the set of all stabilizing decentralized compensators C_d are given in Section IV. In Section V, the main results of Section IV are interpreted when the plant can be represented by a transfer matrix with rational function entries; it is shown that the decentralized stabilizability conditions of Section IV in fact generalize the requirement that the system has no fixed-eigenvalues [And.1, Wan.1]. An algorithm is given for designing stabilizing decentralized compensators for a given strictly proper P based on any of its right-coprime factorizations.

II. Algebraic framework

2.1. Notation [Lan.1, Vid.1]: H is a principal ring (i.e., an entire commutative ring in which every ideal is principal). $\mathcal{m}(H)$ is the set of matrices with elements in H. $J \subset H$ is the group of units of H. $I \subset H$ is a multiplicative subset, $0 \notin I$, $1 \in I$. $G = H / I := \{ n/d : n \in H, d \in I \}$ is the ring of fractions of H associated with I. G_s is the Jacobson radical of G;
$G_s := \{ x \in G : (1+xy)^{-1} \in G, \text{ for all } y \in G \}$.

2.2. Example (Rational functions in s) : Let $\mathcal{U} \supset \mathbb{C}_+$ be a closed subset of $\mathbb{C}$, symmetric about the real axis, and let $\mathbb{C} \setminus \mathcal{U}$ be nonempty; let $\mathcal{U}_e := \mathcal{U} \cup \{ \infty \}$. The ring of proper scalar rational functions (with real coefficients) which are analytic in $\mathcal{U}$, denoted by $R_{\mathcal{U}}(s)$, is a principal ring. Let H be $R_{\mathcal{U}}(s)$; by definition of J, $f \in J$ implies that f is a proper rational function, which has neither poles nor zeros in $\mathcal{U}_e$. We choose I to be the multiplicative subset of $R_{\mathcal{U}}(s)$ such that $f \in I$ implies that $f(\infty)$ is a nonzero constant in $\mathbb{R}$; equivalently, $I \subset R_{\mathcal{U}}(s)$ is the set of proper, but not *strictly proper*, real rational functions which are analytic in $\mathcal{U}$. Then $R_{\mathcal{U}}(s)/I$ is the ring of *proper* rational functions $\mathbb{R}_p(s)$. The Jacobson radical of $\mathbb{R}_p(s)$ is the set of strictly proper rational functions $\mathbb{R}_{sp}(s)$.

2.3. Definitions (Coprime factorizations in H):
(i) The pair (N_p, D_p), where $N_p, D_p \in \mathcal{m}(H)$, is called *right-coprime* (**r.c.**) iff there exist $U_p, V_p \in \mathcal{m}(H)$ such that $V_p D_p + U_p N_p = I$; **(ii)** the pair (N_p, D_p) is called a

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right-fraction representation (r.f.r.) of $P \in \mathcal{M}(G)$ iff D_p is square, $\det D_p \in \mathcal{I}$ and $P = N_p D_p^{-1}$; (iii) the pair (N_p, D_p) is called a *right-coprime-fraction representation* (r.c.f.r.) of $P \in \mathcal{M}(G)$ iff (N_p, D_p) is an r.f.r. of P and (N_p, D_p) is r.c. (iv) The pair $(\tilde{D}_p, \tilde{N}_p)$, where $\tilde{D}_p, \tilde{N}_p \in \mathcal{M}(H)$, is called *left-coprime* (l.c.) iff there exist $\tilde{U}_p, \tilde{V}_p \in \mathcal{M}(H)$ such that $\tilde{N}_p \tilde{U}_p + \tilde{D}_p \tilde{V}_p = I$; (v) the pair $(\tilde{D}_p, \tilde{N}_p)$ is called a *left-fraction representation* (l.f.r.) of $P \in \mathcal{M}(G)$ iff $\tilde{D}_p$ is square, $\det \tilde{D}_p \in \mathcal{I}$ and $P = \tilde{D}_p^{-1}\tilde{N}_p$; (vi) the pair $(\tilde{D}_p, \tilde{N}_p)$ is called a *left-coprime-fraction representation* (l.c.f.r.) of $P \in \mathcal{M}(G)$ iff $(\tilde{D}_p, \tilde{N}_p)$ is an l.f.r. of P and $(\tilde{D}_p, \tilde{N}_p)$ is l.c. (vii) The triple (N_{pr}, D, N_{pl}), where $N_{pr}, D, N_{pl} \in \mathcal{M}(H)$, is called a *bicoprime-fraction representation* (b.c.f.r.) of $P \in \mathcal{M}(G)$ iff the pair (N_{pr}, D) is *right-coprime*, the pair (D, N_{pl}) is *left-coprime*, $\det D \in \mathcal{I}$ and $P = N_{pr} D^{-1} N_{pl}$. Note that $P \in \mathcal{M}(G)$ is sometimes given as $P = N_{pr} D^{-1} N_{pl} + S_p$, where $S_p \in \mathcal{M}(H)$ and (N_{pr}, D, N_{pl}) is a bicoprime (b.c.) triple. In this case, the b.c.f.r. is given by (N_{pr}, D, N_{pl}, S_p) [Vid.1].

III. System description

Consider the decentralized control system $S(P, C_d)$ shown in Figure 1.

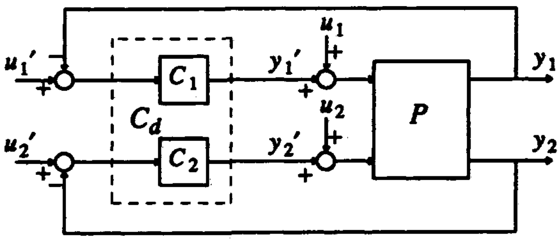

Figure 1: The two-channel decentralized control system $S(P, C_d)$.

3.1. Assumptions:

(A) Let $P \in G^{n_o \times n_i}$ be a two-channel plant, where $n_o =: n_{o1} + n_{o2}$, $n_i =: n_{i1} + n_{i2}$. Let (N_p, D_p) be an r.c.f.r. of P, where $N_p =: \begin{bmatrix} N_{p1} \\ N_{p2} \end{bmatrix}$, $D_p =: \begin{bmatrix} D_{p1} \\ D_{p2} \end{bmatrix}$, $N_{p1} \in H^{n_{o1} \times n_i}$, $N_{p2} \in H^{n_{o2} \times n_i}$, $D_{p1} \in H^{n_{i1} \times n_i}$, $D_{p2} \in H^{n_{i2} \times n_i}$. Let $(\tilde{D}_p, \tilde{N}_p)$ be an l.c.f.r. of P, where $\tilde{D}_p =: \begin{bmatrix} \tilde{D}_{p1} & \vdots & \tilde{D}_{p2} \end{bmatrix}$, $\tilde{N}_p =: \begin{bmatrix} \tilde{N}_{p1} & \vdots & \tilde{N}_{p2} \end{bmatrix}$, $\tilde{D}_{p1} \in H^{n_o \times n_{o1}}$, $\tilde{D}_{p2} \in H^{n_o \times n_{o2}}$, $\tilde{N}_{p1} \in H^{n_o \times n_{i1}}$, $\tilde{N}_{p2} \in H^{n_o \times n_{i2}}$. Let (N_{pr}, D, N_{pl}) be a b.c.f.r. of P where $N_{pr} =: \begin{bmatrix} N_{pr1} \\ N_{pr2} \end{bmatrix} \in H^{n_o \times n}$, $D \in H^{n \times n}$, $N_{pl} =: \begin{bmatrix} N_{pl1} & \vdots & N_{pl2} \end{bmatrix} \in H^{n \times n_i}$, $N_{pr1} \in H^{n_{o1} \times n}$, $N_{pr2} \in H^{n_{o2} \times n}$, $N_{pl1} \in H^{n \times n_{i1}}$, $N_{pl2} \in H^{n \times n_{i2}}$.

(B) Let $C_d \in G^{n_i \times n_o}$ be a *decentralized* compensator, where $C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix}$, $C_1 \in G^{n_{i1} \times n_{o1}}$, $C_2 \in G^{n_{i2} \times n_{o2}}$. Let $(\tilde{D}_{c1}, \tilde{N}_{c1})$ be an l.c.f.r. of C_1 and let $(\tilde{D}_{c2}, \tilde{N}_{c2})$ be an l.c.f.r. of C_2, where $\tilde{D}_{c1} \in H^{n_{i1} \times n_{i1}}$, $\tilde{D}_{c2} \in H^{n_{i2} \times n_{i2}}$, $\tilde{N}_{c1} \in H^{n_{i1} \times n_{o1}}$, $\tilde{N}_{c2} \in H^{n_{i2} \times n_{o2}}$. Let $\tilde{D}_c := \begin{bmatrix} \tilde{D}_{c1} & 0 \\ 0 & \tilde{D}_{c2} \end{bmatrix}$, $\tilde{N}_c := \begin{bmatrix} \tilde{N}_{c1} & 0 \\ 0 & \tilde{N}_{c2} \end{bmatrix}$; note that $(\tilde{D}_c, \tilde{N}_c)$ is an l.c.f.r. of C_d if and only if $(\tilde{D}_{c1}, \tilde{N}_{c1})$ is an l.c.f.r. of C_1 and $(\tilde{D}_{c2}, \tilde{N}_{c2})$ is an l.c.f.r. of C_2. Let (N_{c1}, D_{c1}) be an r.c.f.r. of C_1 and let (N_{c2}, D_{c2}) be an r.c.f.r. of C_2, where $N_{c1} \in H^{n_{i1} \times n_{o1}}$, $N_{c2} \in H^{n_{i2} \times n_{o2}}$, $D_{c1} \in H^{n_{o1} \times n_{o1}}$, $D_{c2} \in H^{n_{o2} \times n_{o2}}$. Let $D_c := \begin{bmatrix} D_{c1} & 0 \\ 0 & D_{c2} \end{bmatrix}$, $N_c := \begin{bmatrix} N_{c1} & 0 \\ 0 & N_{c2} \end{bmatrix}$; note that (N_c, D_c) is an r.c.f.r. of C_d if and only if (N_{c1}, D_{c1}) is an r.c.f.r. of C_1 and (N_{c2}, D_{c2}) is an r.c.f.r. of C_2. □

If P satisfies Assumption 3.1 (A) we have the generalized Bezout identity in (3.1) below: Let (N_p, D_p) be an r.c. pair and let $(\tilde{D}_p, \tilde{N}_p)$ be an l.c. pair, and let $\tilde{N}_p D_p = \tilde{D}_p N_p$; then there are matrices $V_p, U_p, \tilde{U}_p, \tilde{V}_p \in \mathcal{M}(H)$ such that

$$\begin{bmatrix} V_p & U_p \\ -\tilde{N}_p & \tilde{D}_p \end{bmatrix} \begin{bmatrix} D_p & -\tilde{U}_p \\ N_p & \tilde{V}_p \end{bmatrix} = \begin{bmatrix} I_{n_i} & 0 \\ 0 & I_{n_o} \end{bmatrix}. \quad (3.1)$$

For the b.c.f.r. (N_{pr}, D, N_{pl}) of P we have two generalized Bezout identities: For the r.c. pair (N_{pr}, D), there are matrices $V_{pr}, U_{pr}, \tilde{X}, \tilde{Y}, \tilde{U}, \tilde{V} \in \mathcal{M}(H)$ such that

$$\begin{bmatrix} V_{pr} & U_{pr} \\ -\tilde{X} & \tilde{Y} \end{bmatrix} \begin{bmatrix} D & -\tilde{U} \\ N_{pr} & \tilde{V} \end{bmatrix} =:$$

$$\begin{bmatrix} V_{pr} & U_{pr1} & U_{pr2} \\ -\tilde{X} & \tilde{Y}_1 & \tilde{Y}_2 \end{bmatrix} \begin{bmatrix} D & -\tilde{U} \\ N_{pr1} & \tilde{V}_1 \\ N_{pr2} & \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ 0 & I_{n_o} \end{bmatrix}; \quad (3.2)$$

for the l.c. pair (D, N_{pl}), there are matrices $V_{pl}, U_{pl}, X, Y, U, V \in \mathcal{M}(H)$ such that

$$\begin{bmatrix} D & -N_{pl} \\ U & V \end{bmatrix} \begin{bmatrix} V_{pl} & X \\ -U_{pl} & Y \end{bmatrix} =:$$

$$\begin{bmatrix} D & -N_{pl1} & -N_{pl2} \\ U & V_1 & V_2 \end{bmatrix} \begin{bmatrix} V_{pl} & X \\ -U_{pl1} & Y_1 \\ -U_{pl2} & Y_2 \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ 0 & I_{n_i} \end{bmatrix}. \quad (3.3)$$

Let $\bar{y} := \begin{bmatrix} y_1 \\ y_2 \\ y_1' \\ y_2' \end{bmatrix}$, $\bar{u} := \begin{bmatrix} u_1 \\ u_2 \\ u_1' \\ u_2' \end{bmatrix}$; the map $H_{\bar{y}\bar{u}} : \bar{u} \mapsto \bar{y}$ is called the I/O map. In terms of P and C_d, $H_{\bar{y}\bar{u}}$ is given by

$$H_{\bar{y}\bar{u}} = \begin{bmatrix} P(I_{n_i} + C_d P)^{-1} & P(I_{n_i} + C_d P)^{-1} C_d \\ -C_d P(I_{n_i} + C_d P)^{-1} & (I_{n_i} + C_d P)^{-1} C_d \end{bmatrix}. \quad (3.4)$$

3.2. Definition (*H*-**stability**): The system $S(P, C_d)$ is said to be *H-stable* iff $H_{\bar{y}\bar{u}} \in \mathcal{M}(H)$.

3.3. Analysis: Let $P = N_p D_p^{-1}$, let $C = \tilde{D}_c^{-1}\tilde{N}_c$, where (N_p, D_p) is an r.c. pair as in Assumption 3.1 (A), and $(\tilde{D}_c, \tilde{N}_c)$ is an l.c. pair as in Assumption 3.1 (B) (see Figure 2); ξ_p denotes the pseudo-state of P. $S(P, C_d)$ is then described by (3.5)-(3.6) below:

$$\begin{bmatrix} \tilde{D}_{c1} D_{p1} + \tilde{N}_{c1} N_{p1} \\ \tilde{D}_{c2} D_{p2} + \tilde{N}_{c2} N_{p2} \end{bmatrix} \xi_p = \begin{bmatrix} \tilde{D}_{c1} & 0 & \tilde{N}_{c1} & 0 \\ 0 & \tilde{D}_{c2} & 0 & \tilde{N}_{c2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_1' \\ u_2' \end{bmatrix}, \quad (3.5)$$

$$\begin{bmatrix} N_{p1}\\ N_{p2}\\ D_{p1}\\ D_{p2}\end{bmatrix}\xi_p = \begin{bmatrix} y_1\\ y_2\\ y_1'\\ y_2'\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&0&0&0\\ I_{n_{i1}}&0&0&0\\0&I_{n_{i2}}&0&0\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_1'\\u_2'\end{bmatrix}. \tag{3.6}$$

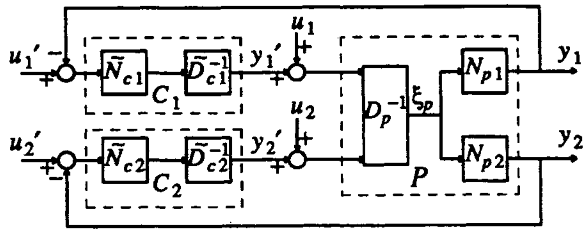

Figure 2: $S(P, C_d)$ with $P = N_p D_p^{-1}$, $C_d = \tilde D_c^{-1}\tilde N_c$.

Equations (3.5)-(3.6) are in the form $D_{H1}\xi_p = N_{L1}u$, $N_{R1}\xi_p = y - S_{H1}u$, where (N_{R1}, D_{H1}) is an r.c. pair and (D_{H1}, N_{L1}) is an l.c. pair. If $\det D_{H1} \in I$, then

$$H_{\bar y\bar u} = N_{R1}D_{H1}^{-1}N_{L1} + S_{H1} \in \mathfrak{m}(G).$$

$S(P, C_d)$ is H-stable if and only if $D_{H1}^{-1} \in \mathfrak{m}(H)$ (equivalently, $\det D_{H1} \in J$ and hence, D_{H1} is H-unimodular). D_{H1} can be expressed several ways:

$$D_{H1} = \tilde D_c D_p + \tilde N_c N_p = \begin{bmatrix}\tilde D_{c1}D_{p1}+\tilde N_{c1}N_{p1}\\ \tilde D_{c2}D_{p2}+\tilde N_{c2}N_{p2}\end{bmatrix} =$$

$$\begin{bmatrix}\tilde D_{c1}&0&\tilde N_{c1}&0\\0&\tilde D_{c2}&0&\tilde N_{c2}\end{bmatrix}\begin{bmatrix}D_p\\N_p\end{bmatrix} = \begin{bmatrix}\tilde D_{c1}&\tilde N_{c1}&0&0\\0&0&\tilde D_{c2}&\tilde N_{c2}\end{bmatrix}\begin{bmatrix}D_{p1}\\N_{p1}\\D_{p2}\\N_{p2}\end{bmatrix};$$

and $\det D_{H1}$ can also be written as $\det D_{H1} = \det\tilde D_c \det(I + C_d P)\det D_p$. By normalization and due to the block-diagonal compensator structure, $D_{H1} \in \mathfrak{m}(H)$ is H-unimodular if and only if there are *block-diagonal* matrices $V_p := \tilde D_c$, $U_p := \tilde N_c \in \mathfrak{m}(H)$ such that $V_pD_p + U_pN_p = I_{n_i}$.

IV. Main results

In this section the plant P satisfies Assumption 3.1 (A).

4.1. Definition (H-stabilizing decentralized compensator): C_d is called an *H-stabilizing decentralized compensator for P* (later abbreviated as C_d H-stabilizes P) iff $C_d \in G^{n_i \times n_o}$ satisfies Assumption 3.1 (B) *and* the system $S(P, C_d)$ is H-stable.

4.2. Definition (Class of all H-stabilizing decentralized compensators): The set $\mathbf{S}_d(P) := \{ C_d : C_d \text{ } H\text{-stabilizes } P \}$ is called the set of *all H-stabilizing decentralized compensators for P*.

4.3. Comment: The set $\mathbf{S}(P)$ of all *centralized* (full-feedback) compensators that H-stabilize P is given by

$$\mathbf{S}(P) = \{ C = (V_p - Q\tilde N_p)^{-1}(U_p + Q\tilde D_p) : Q \in \mathfrak{m}(H) \},$$

$$\mathbf{S}(P) = \{ C = (\tilde U_p + D_pQ)(\tilde V_p - N_pQ)^{-1} : Q \in \mathfrak{m}(H) \},$$

where V_p, U_p, $\tilde V_p$, $\tilde U_p$ are as in (3.1). If $P \in \mathfrak{m}(G)$ instead of $\mathfrak{m}(G_S)$, then $Q \in \mathfrak{m}(H)$ should be such that $\det(\tilde V_p - N_pQ) \in I$ (equivalently, $\det(V_p - Q\tilde N_p) \in I$).

4.4R. Theorem (Conditions on $P = N_pD_p^{-1}$ for decentralized H-stabilizability): Let $P \in \mathfrak{m}(G_S)$ satisfy Assumption 3.1 (A); then there exists an H-stabilizing decentralized compensator C_d for P if and only if P has an r.c.f.r. (N_p, D_p) such that

$$\begin{bmatrix}D_{p1}\\N_{p1}\end{bmatrix} =: \begin{bmatrix}D_{11}&D_{12}\\N_{11}&N_{12}\end{bmatrix} = E_1\begin{bmatrix}I_{n_{i1}}&0\\0&W_1\end{bmatrix}, \tag{4.1}$$

where $E_1 \in H^{(n_{i1}+n_{o1})\times(n_{i1}+n_{o1})}$ is H-unimodular and $W_1 \in H^{n_{o1}\times n_{i2}}$, and

$$\begin{bmatrix}D_{p2}\\N_{p2}\end{bmatrix} =: \begin{bmatrix}D_{21}&D_{22}\\N_{21}&N_{22}\end{bmatrix} = E_2\begin{bmatrix}0&I_{n_{i2}}\\W_2&0\end{bmatrix}, \tag{4.2}$$

where $E_2 \in H^{(n_{i2}+n_{o2})\times(n_{i2}+n_{o2})}$ is H-unimodular and $W_2 \in H^{n_{o2}\times n_{i1}}$. □

Equation (4.1) implies that the pair (N_{11}, D_{11}) is r.c. and similarly, (4.2) implies that (N_{22}, D_{22}) is r.c.

4.5L. Theorem (Conditions on $P = \tilde D_p^{-1}\tilde N_p$ for decentralized H-stabilizability): Let $P \in \mathfrak{m}(G_S)$ satisfy Assumption 3.1 (A); then there exists an H-stabilizing decentralized compensator C_d for P if and only if P has an l.c.f.r. $(\tilde D_p, \tilde N_p)$ such that

$$\begin{bmatrix}-\tilde N_{p1}&\tilde D_{p1}\end{bmatrix} = \begin{bmatrix}0&I_{n_{o1}}\\-W_2&0\end{bmatrix}E_1^{-1}, \text{ and} \tag{4.3}$$

$$\begin{bmatrix}-\tilde N_{p2}&\tilde D_{p2}\end{bmatrix} = \begin{bmatrix}-W_1&0\\0&I_{n_{o2}}\end{bmatrix}E_2^{-1}, \tag{4.4}$$

where the H-unimodular matrices E_1, E_2 and the matrices $W_1 \in \mathfrak{m}(H)$, $W_2 \in \mathfrak{m}(H)$ are defined in (4.1)-(4.2).

4.6. Comments: (i) Let (N_p, D_p) be an r.c.f.r. of P; then (X_p, Y_p) is another r.c.f.r. of P if and only if $(X_p, Y_p) = (N_pR, D_pR)$ for some H-unimodular matrix $R \in H^{n_i\times n_i}$. By Theorem 4.4R, P can be H-stabilized by a decentralized compensator if and only if any r.c.f.r. (X_p, Y_p), $X_p := \begin{bmatrix}X_{p1}\\X_{p2}\end{bmatrix}$, $Y_p := \begin{bmatrix}Y_{p1}\\Y_{p2}\end{bmatrix}$, of P is of the form

$$\begin{bmatrix}Y_{p1}\\X_{p1}\\Y_{p2}\\X_{p2}\end{bmatrix} = \begin{bmatrix}D_{p1}\\N_{p1}\\D_{p2}\\N_{p2}\end{bmatrix}R = \begin{bmatrix}E_1&\vdots&0\\ \cdots&&\cdots\\0&\vdots&E_2\end{bmatrix}\begin{bmatrix}I_{n_{i1}}&0\\0&W_1\\0&I_{n_{i2}}\\W_2&0\end{bmatrix}R, \tag{4.5}$$

for some H-unimodular matrix $R \in H^{n_i\times n_i}$, where E_1, $E_2 \in \mathfrak{m}(H)$ are H-unimodular and $W_1, W_2 \in \mathfrak{m}(H)$.

Similarly, let $(\tilde D_p, \tilde N_p)$ be an l.c.f.r. of P; then $(\tilde Y_p, \tilde X_p)$ is another l.c.f.r. of P if and only if $(\tilde Y_p, \tilde X_p) = (L\tilde D_p, L\tilde N_p)$ for some H-unimodular matrix $L \in H^{n_o\times n_o}$. By Theorem 4.5L, P can be H-stabilized by C_d if and only if any l.c.f.r. $(\tilde Y_p, \tilde X_p)$, $\tilde Y_p := \begin{bmatrix}\tilde Y_{p1}&\tilde Y_{p2}\end{bmatrix}$, $\tilde X_p := \begin{bmatrix}\tilde X_{p1}&\tilde X_{p2}\end{bmatrix}$ of P is of the form

$$\begin{bmatrix}-\tilde X_{p1}&\tilde Y_{p1}&\vdots&-\tilde X_{p2}&\tilde Y_{p2}\end{bmatrix}$$

$$= L\begin{bmatrix}0&I_{n_{o1}}&\vdots&-W_1&0\\-W_2&0&\vdots&0&I_{n_{o2}}\end{bmatrix}\begin{bmatrix}E_1^{-1}&\vdots&0\\ \cdots&&\cdots\\0&\vdots&E_2^{-1}\end{bmatrix}, \tag{4.6}$$

for some H-unimodular matrix $L \in H^{n_o\times n_o}$. (ii) Suppose that P is given by a b.c.f.r. (N_{pr}, D, N_{pl}) and C_d is given by an l.c.f.r. $(\tilde D_c, \tilde N_c)$; apply Theorem 4.4R to the r.c.f.r. $(N_p, D_p) := (N_{pr}X, Y)$ of P; $P = N_{pr}D^{-1}N_{pl} \in \mathfrak{m}(G_S)$ can be H-stabilized by a decentralized compensator C_d if and only if there exists an H-unimodular matrix $R \in H^{n_i\times n_i}$ such that

$$\begin{bmatrix} Y_1 \\ N_{pr1}X \end{bmatrix} = E_1 \begin{bmatrix} I_{n_{i1}} & 0 \\ 0 & W_1 \end{bmatrix} R \text{ , and} \tag{4.7}$$

$$\begin{bmatrix} Y_2 \\ N_{pr2}X \end{bmatrix} = E_2 \begin{bmatrix} 0 & I_{n_{i2}} \\ W_2 & 0 \end{bmatrix} R \text{ ,} \tag{4.8}$$

where $E_1 \in \mathcal{M}(H)$ and $E_2 \in \mathcal{M}(H)$ are H–unimodular, and $W_1 \in H^{n_{o1} \times n_{i2}}$, $W_2 \in H^{n_{o2} \times n_{i1}}$. Similarly if C_d is given by an r.c.f.r. (N_c, D_c) then we apply Theorem 4.5L to the l.c.f.r. $(\tilde{D}_p, \tilde{N}_p) := (\tilde{Y}, \tilde{X} N_{pl})$ of P and conclude that P can be H–stabilized by a decentralized compensator C_d if and only if there exists an H–unimodular matrix $L \in H^{n_o \times n_o}$ such that

$$\begin{bmatrix} -\tilde{X} N_{pl1} & \vdots & \tilde{Y}_1 \end{bmatrix} = L \begin{bmatrix} 0 & I_{n_{o1}} \\ -W_2 & 0 \end{bmatrix} E_1^{-1} \text{ , and} \tag{4.9}$$

$$\begin{bmatrix} -\tilde{X} N_{pl2} & \vdots & \tilde{Y}_2 \end{bmatrix} = L \begin{bmatrix} -W_1 & 0 \\ 0 & I_{n_{o2}} \end{bmatrix} E_2^{-1} \text{ ,} \tag{4.10}$$

where $E_1^{-1} \in \mathcal{M}(H)$ and $E_2^{-1} \in \mathcal{M}(H)$ are H–unimodular and $W_1 \in \mathcal{M}(H)$, $W_2 \in \mathcal{M}(H)$.

4.7. Theorem (Set of all H–stabilizing decentralized compensators): Let $P \in \mathcal{M}(G_s)$ satisfy Assumption 3.1 (A); let in addition an r.c.f.r. (N_p, D_p) of P satisfy conditions (4.1) and (4.2) of Theorem 4.4R; equivalently, let an l.c.f.r. $(\tilde{D}_p, \tilde{N}_p)$ of P satisfy conditions (4.3) and (4.4) of Theorem 4.5L. Under these conditions the set $\mathbf{S}_d(P)$ of *all* H–stabilizing decentralized compensators for P is given by

$$\mathbf{S}_d(P) := \Big\{ \; C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix} = \begin{bmatrix} \tilde{D}_{c1}^{-1}\tilde{N}_{c1} & 0 \\ 0 & \tilde{D}_{c2}^{-1}\tilde{N}_{c1} \end{bmatrix} :$$

$\begin{bmatrix} \tilde{D}_{c1} & \vdots & \tilde{N}_{c1} \end{bmatrix} = \begin{bmatrix} I_{n_{i1}} & \vdots & Q_1 \end{bmatrix} E_1^{-1}$, $\begin{bmatrix} \tilde{D}_{c2} & \vdots & \tilde{N}_{c2} \end{bmatrix} = \begin{bmatrix} I_{n_{i2}} & \vdots & Q_2 \end{bmatrix} E_2^{-1}$, for some $Q_1 \in H^{n_{i1} \times n_{o1}}$, $Q_2 \in H^{n_{i2} \times n_{o2}}$ such that $\det(I_{n_{i2}} - Q_2 W_2 Q_1 W_1) \in J \Big\}$; (4.12)

equivalently,

$$\mathbf{S}_d(P) := \Big\{ \; C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix} = \begin{bmatrix} N_{c1}D_{c1}^{-1} & 0 \\ 0 & N_{c2}D_{c2}^{-1} \end{bmatrix} :$$

$\begin{bmatrix} -N_{c1} \\ D_{c1} \end{bmatrix} = E_1 \begin{bmatrix} -Q_1 \\ I_{n_{o1}} \end{bmatrix}$, $\begin{bmatrix} -N_{c2} \\ D_{c2} \end{bmatrix} = E_2 \begin{bmatrix} -Q_2 \\ I_{n_{o2}} \end{bmatrix}$, for some $Q_1 \in H^{n_{i1} \times n_{o1}}$, $Q_2 \in H^{n_{i2} \times n_{o2}}$ such that

$$\det(I_{n_{i2}} - Q_2 W_2 Q_1 W_1) \in J \Big\} \text{ .} \tag{4.13}$$

The map $(Q_1, Q_2) \mapsto C_d$, Q_1, $Q_2 \in \mathcal{M}(H)$, such that $\det(I_{n_{i2}} - Q_2 W_2 Q_1 W_1) \in J$, $C_d \in \mathbf{S}_d(P)$, is a bijection; for the same pair (Q_1, Q_2), (4.12) and (4.13) give the same H–stabilizing C_d.

4.8. Comments: (i) In (4.1)-(4.2) (equivalently, (4.3)-(4.4)) if either W_1 or W_2 is zero (i.e., if both of $D_{12} = 0$ and $N_{12} = 0$ in (4.1) or both of $D_{21} = 0$ and $N_{21} = 0$ in (4.2)), then for *all* Q_1, $Q_2 \in \mathcal{M}(H)$, $\det(I_{n_i} + QW) := \det\begin{bmatrix} I_{n_{i1}} & Q_1 W_1 \\ Q_2 W_2 & I_{n_{i2}} \end{bmatrix} = \det(I_{n_{i2}} - Q_2 W_2 Q_1 W_1) = 1$ and hence, the set $\mathbf{S}_d(P)$ in (4.12) (or (4.13)) is parametrized by two *free* parameters Q_1 and $Q_2 \in \mathcal{M}(H)$. (ii) In Theorem 4.7, if $P \in \mathcal{M}(G)$ instead of $\mathcal{M}(G_s)$, then $Q_1 \in \mathcal{M}(H)$ and $Q_2 \in \mathcal{M}(H)$ should be chosen so that $\det\tilde{D}_{c1} := \det(\begin{bmatrix} I_{n_{i1}} & \vdots & Q_1 \end{bmatrix} E_1^{-1} \begin{bmatrix} I_{n_{i1}} \\ 0 \end{bmatrix}) \in I$ and $\det\tilde{D}_{c2} := \det(\begin{bmatrix} I_{n_{i2}} & \vdots & Q_2 \end{bmatrix} E_2^{-1} \begin{bmatrix} I_{n_{i2}} \\ 0 \end{bmatrix}) \in I$ *in addition to* $\det(I_{n_i} + QW) \in J$.

V. Application to stable rational functions

Now we consider the case when $H = R_u(s)$ as in Example 2.2. This principal ring allows us to show the connection between our results and those of [Wan.1, And.1]. In [And.1], a rank test for fixed-eigenvalues was given in terms of a left-fraction representation of P. A similar test is useful in our approach; we give rank conditions in terms of an r.c.f.r., an l.c.f.r. and a b.c.f.r. of P. We start by considering real *constant* decentralized compensators.

Consider the system $S(P, K_d)$, which is the same as $S(P, C_d)$ shown in Figure 1, where $C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix}$ is replaced by the *real constant* matrix $K_d := diag\begin{bmatrix} K_1 & K_2 \end{bmatrix}$, $K_1 \in \mathbb{R}^{n_{i1} \times n_{o1}}$, $K_2 \in \mathbb{R}^{n_{i2} \times n_{o2}}$. The plant P still satisfies Assumption 3.1 (A), where H is replaced by $R_u(s)$. Equations (3.5)-(3.6) are now replaced by (5.1)-(5.2) describing the system $S(P, K_d)$ with constant decentralized output-feedback control:

$$\begin{bmatrix} D_{p1} + K_1 N_{p1} \\ D_{p2} + K_2 N_{p2} \end{bmatrix} \xi_p = \begin{bmatrix} I_{n_{i1}} & 0 & K_1 & 0 \\ 0 & I_{n_{i2}} & 0 & K_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_1' \\ u_2' \end{bmatrix} \text{ ,} \tag{5.1}$$

$$\begin{bmatrix} N_{p1} \\ N_{p2} \\ D_{p1} \\ D_{p2} \end{bmatrix} \xi_p = \begin{bmatrix} y_1 \\ y_2 \\ y_1' \\ y_2' \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ I_{n_{i1}} & 0 & 0 & 0 \\ 0 & I_{n_{i2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_1' \\ u_2' \end{bmatrix} \text{.} \tag{5.2}$$

The closed-loop system $S(P, K_d)$, described by (5.1)-(5.2), is H–stable if and only if $\det\begin{bmatrix} D_{p1} + K_1 N_{p1} \\ D_{p2} + K_2 N_{p2} \end{bmatrix} \in J$. Furthermore, $s_o \in \mathcal{U}_e$ is an eigenvalue of the closed-loop system if and only if

$$\det\begin{bmatrix} D_{p1}(s_o) + K_1 N_{p1}(s_o) \\ D_{p2}(s_o) + K_2 N_{p2}(s_o) \end{bmatrix} = 0 \text{ .} \tag{5.3}$$

5.1. Definition (Decentralized fixed-eigenvalue): The plant P is said to have a *decentralized fixed-eigenvalue* at $s_o \in \mathcal{U}_e$ (with respect to $K_d = diag\begin{bmatrix} K_1 & K_2 \end{bmatrix}$) iff (5.3) holds for all $K_1, K_2 \in \mathcal{M}(\mathbb{R})$.

If $s_o \in \mathcal{U}_e$ is a fixed-eigenvalue, then obviously $s_o \in \mathcal{U}_e$ is an eigenvalue of the open-loop system (i.e., with $K_1 = 0$, $K_2 = 0$, $\det\begin{bmatrix} D_{p1}(s_o) \\ D_{p2}(s_o) \end{bmatrix} = 0$ and hence, s_o is an eigenvalue of P); this eigenvalue $s_o \in \mathcal{U}_e$ remains as an eigenvalue of the closed-loop system for *all* real constant decentralized feedback compensators. We prefer to call such $s_o \in \mathcal{U}_e$ a *fixed-eigenvalue* rather than a *fixed-mode*; although the eigenvalue at $s_o \in \mathcal{U}_e$ remains fixed irrespective of the constant decentralized compensator, the *eigenvector* v_o associated with the fixed-eigenvalue $s_o \in \mathcal{U}_e$ *depends* on K_1 and K_2.

Fixed-eigenvalues are those eigenvalues of the plant which cannot be moved by any *real constant* decentralized feedback. These fixed-eigenvalues remain fixed even under *dynamic* decentralized output-feedback, in particular, under *complex* constant decentralized output-feedback.

5.2R. Theorem (Rank test on (N_p, D_p) for fixed-eigenvalues and H-stabilizability): Let $P \in \mathcal{M}(\mathbb{R}_{sp}(s))$, $P = N_p D_p^{-1}$ satisfy Assumption 3.1 (A) where H is $R_u(s)$; then statements (i)-(iv) below are equivalent:

(i) P has no decentralized fixed-eigenvalues in $\mathcal{U}_e$;

(ii) for any r.c.f.r. (N_p, D_p) of P as in Assumption 3.1 (A),

$$rank \begin{bmatrix} D_{p1}(s) \\ N_{p1}(s) \end{bmatrix} \geq n_{i1}, \quad \text{for all } s \in \mathcal{U}_e, \text{ and} \tag{5.4}$$

$$rank \begin{bmatrix} D_{p2}(s) \\ N_{p2}(s) \end{bmatrix} \geq n_{i2}, \quad \text{for all } s \in \mathcal{U}_e; \tag{5.5}$$

(iii) conditions (4.1)-(4.2) of Theorem 4.4R hold; i.e., an r.c.f.r. (N_p, D_p) of P can be chosen so that

$$\begin{bmatrix} D_{p1}(s) \\ N_{p1}(s) \end{bmatrix} = E_1(s) \begin{bmatrix} I_{n_{i1}} & 0 \\ 0 & W_1(s) \end{bmatrix}, \tag{5.6}$$

$$\begin{bmatrix} D_{p2}(s) \\ N_{p2}(s) \end{bmatrix} = E_2(s) \begin{bmatrix} 0 & I_{n_{i2}} \\ W_2(s) & 0 \end{bmatrix}, \tag{5.7}$$

where $E_1(s)$, $E_2(s) \in \mathcal{M}(R_u(s))$ are R_u-unimodular and $W_1(s)$, $W_2(s) \in \mathcal{M}(R_u(s))$;

(iv) there exists a compensator $C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix}$ (satisfying Assumption 3.1 (B)) which H-stabilizes P.

5.3L. Theorem (Rank test on $(\tilde{D}_p, \tilde{N}_p)$ for fixed-eigenvalues and H-stabilizability): Let $P \in \mathcal{M}(\mathbb{R}_{sp}(s))$, $P = \tilde{D}_p^{-1}\tilde{N}_p$ satisfy Assumption 3.1 (A) where H is $R_u(s)$; then statements (i)-(iv) below are equivalent:

(i) P has no decentralized fixed-eigenvalues in $\mathcal{U}_e$;

(ii) for any l.c.f.r. $(\tilde{D}_p, \tilde{N}_p)$ of P as in Assumption 3.1 (A),

$$rank \begin{bmatrix} -\tilde{N}_{p1}(s) & \vdots & \tilde{D}_{p1}(s) \end{bmatrix} \geq n_{o1}, \text{ for all } s \in \mathcal{U}_e \text{ and} \tag{5.8}$$

$$rank \begin{bmatrix} -\tilde{N}_{p2}(s) & \vdots & \tilde{D}_{p2}(s) \end{bmatrix} \geq n_{o2}, \text{ for all } s \in \mathcal{U}_e; \tag{5.9}$$

(iii) conditions (4.3)-(4.4) of Theorem 4.5L hold; i.e., an l.c.f.r. $(\tilde{D}_p, \tilde{N}_p)$ of P can be chosen so that

$$\begin{bmatrix} -\tilde{N}_{p1}(s) & \vdots & \tilde{D}_{p1}(s) \end{bmatrix} = \begin{bmatrix} 0 & I_{n_{o1}} \\ -W_2(s) & 0 \end{bmatrix} E_1(s)^{-1}, \tag{5.10}$$

$$\begin{bmatrix} -\tilde{N}_{p2}(s) & \vdots & \tilde{D}_{p2}(s) \end{bmatrix} = \begin{bmatrix} -W_1(s) & 0 \\ 0 & I_{n_{o2}} \end{bmatrix} E_2(s)^{-1}, \tag{5.11}$$

where $E_1(s)$, $E_2(s) \in \mathcal{M}(R_u(s))$ are R_u-unimodular and $W_1(s)$, $W_2(s) \in \mathcal{M}(R_u(s))$;

(iv) there exists a compensator $C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix}$ (satisfying Assumption 3.1 (B)) which H-stabilizes P.

5.4B. Theorem (Rank test on (N_{pr}, D, N_{pl}) for fixed-eigenvalues and H-stabilizability): Let $P \in \mathcal{M}(\mathbb{R}_{sp}(s))$, $P = N_{pr}D^{-1}N_{pl}$ satisfy Assumption 3.1 (A) where $H = R_u(s)$; then statements (i)-(iii) below are equivalent:

(i) P has no decentralized fixed-eigenvalues in $\mathcal{U}_e$; (ii) for any b.c.f.r. (N_{pr}, D, N_{pl}) of P as in Assumption 3.1 (A),

$$rank \begin{bmatrix} D(s) & -N_{pl2}(s) \\ N_{pr1}(s) & 0 \end{bmatrix} \geq n, \text{ for all } s \in \mathcal{U}_e, \text{ and} \tag{5.12}$$

$$rank \begin{bmatrix} D(s) & -N_{pl1}(s) \\ N_{pr2}(s) & 0 \end{bmatrix} \geq n, \quad \text{for all } s \in \mathcal{U}_e; \tag{5.13}$$

(iii) there exists a compensator $C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix}$ (satisfying Assumption 3.1 (B)) which H-stabilizes P.

5.5S. Remark (State-space description of P): Consider $P = C(sI_n - A)^{-1}B$, where (C, A, B) is $\mathcal{U}_e$-stabilizable and $\mathcal{U}_e$-detectable. Let $N_{pr} := \frac{C}{s+a} = \frac{1}{s+a}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$, $D := \frac{sI_n - A}{s+a}$, $N_{pl} := B = \begin{bmatrix} B_1 \vdots B_2 \end{bmatrix}$, where $-a \in \mathbb{C} \setminus \mathcal{U}_e$, $-a \in \mathbb{R}$; then (N_{pr}, D, N_{pl}) is a b.c.f.r. of P. By Theorem 5.4B, the plant has no fixed-eigenvalues in $\mathcal{U}_e$ iff conditions (5.14)-(5.15) below hold [And.1]:

$$rank \begin{bmatrix} sI_n - A & -B_2 \\ C_1 & 0 \end{bmatrix} \geq n, \text{ for all } s \in \mathcal{U}_e, \text{ and} \tag{5.14}$$

$$rank \begin{bmatrix} sI_n - A & -B_1 \\ C_2 & 0 \end{bmatrix} \geq n, \text{ for all } s \in \mathcal{U}_e; \tag{5.15}$$

we omitted the factor $\frac{1}{s+a}$ in (5.14) and (5.15) for simplicity. Note that conditions (5.14)-(5.15) need to be checked only for those $s \in \mathcal{U}_e$ such that $\det(sI_n - A) = 0$. The derivation of conditions (5.14)-(5.15) is very simple due to Theorem 5.4B.

5.6. Comments: (i) Theorem 5.2R states that $s_o \in \mathcal{U}_e$ is a fixed-eigenvalue if and only if *either* $rank \begin{bmatrix} D_{p1}(s_o) \\ N_{p1}(s_o) \end{bmatrix} < n_{i1}$ *or* $rank \begin{bmatrix} D_{p2}(s_o) \\ N_{p2}(s_o) \end{bmatrix} < n_{i2}$. Note that conditions (5.4) and (5.5) cannot *both* fail at the same time since this would contradict that (N_p, D_p) is a r.c. pair. Therefore, if $rank \begin{bmatrix} D_{p1}(s_o) \\ N_{p1}(s_o) \end{bmatrix} = \alpha < n_{i1}$, then $rank \begin{bmatrix} D_{p2}(s_o) \\ N_{p2}(s_o) \end{bmatrix} \geq n_{i2} + \alpha$ so that $s_o \in \mathcal{U}_e$ is a fixed-eigenvalue but *not* an eigenvalue associated with a hidden-mode. Similarly, (5.8) and (5.9), (5.12)-(5.13) or (5.14)-(5.15) cannot fail at the same time. (ii) Theorem 5.2R states that if the system has no fixed-eigenvalues in $\mathcal{U}_e$, then the Smith form of $\begin{bmatrix} D_{p1} \\ N_{p1} \end{bmatrix}$ is $\begin{bmatrix} I_{n_{i1}} & 0 \\ 0 & W_1 \end{bmatrix}$ (we assume that W_1 is also put in the Smith form), *and* at the same time the Smith form of $\begin{bmatrix} D_{p2} \\ N_{p2} \end{bmatrix}$ is $\begin{bmatrix} 0 & I_{n_{i2}} \\ W_2 & 0 \end{bmatrix}$ (W_2 is also put in the Smith form and appropriate column permutations are made). Hence, $s_o \in \mathcal{U}_e$ is a fixed-eigenvalue of P iff *either* the $n_{i1}th$ invariant factor of $\begin{bmatrix} D_{p1} \\ N_{p1} \end{bmatrix}$ is zero at $s_o \in \mathcal{U}_e$ *or* the $n_{i2}th$ invariant factor of $\begin{bmatrix} D_{p2} \\ N_{p2} \end{bmatrix}$ is zero at $s_o \in \mathcal{U}_e$. (iii) Let $P \in \mathcal{M}(\mathbb{R}_{sp}(s))$; then in (5.6)-(5.7), since N_{p1}, $N_{p2} \in \mathcal{M}(\mathbb{R}_{sp}(s))$, W_1 and $W_2 \in \mathcal{M}(\mathbb{R}_{sp}(s))$; hence, for $k = 1, 2$,

$rank\begin{bmatrix} D_{pk}(\infty) \\ N_{pk}(\infty) \end{bmatrix} \le n_{ik}$. Hence if (5.4)-(5.5) hold, then $\begin{bmatrix} D_{pk} \\ N_{pk} \end{bmatrix}$ has *exactly* n_{ik} invariant factors that are equal to 1.
(iv) From conditions (5.12)-(5.13) of Theorem 5.4B, we obtain the following conditions on fixed-eigenvalues: Rewrite P as $\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} N_{pr1}D^{-1}N_{pl1} & N_{pr1}D^{-1}N_{pl2} \\ N_{pr2}D^{-1}N_{pl1} & N_{pr2}D^{-1}N_{pl2} \end{bmatrix}$. (a) (*A sufficient condition for no fixed-eigenvalues in* $\mathcal{U}_e$): If (N_{pr1}, D, N_{pl1}) is a b.c.f.r. of P_{11}, then the plant P has no fixed-eigenvalues in $\mathcal{U}_e$; (the same holds if (N_{pr2}, D, N_{pl2}) is a b.c.f.r. of P_{22}). We can state this same condition in the state-space setting of Remark 5.5S where $P_{11} = C_1(sI_n - A)^{-1}B_1$: if $(C_1, (sI_n - A), B_1)$ is $\mathcal{U}_e$-stabilizable and $\mathcal{U}_e$-detectable, then P has no fixed-eigenvalues in $\mathcal{U}_e$. (b) (*Some necessary conditions on the transmission-zeros of the partial maps* P_{ij} *if* $s_o \in \mathcal{U}_e$ *is a fixed-eigenvalue*): (1) Let $s_o \in \mathcal{U}_e$ be a fixed-eigenvalue; then either (5.12) fails (and hence $s_o \in \mathcal{U}_e$ is a transmission-zero (t.z.) of P_{12}) *or* (5.13) fails (and hence $s_o \in \mathcal{U}_e$ is a t.z. of P_{21}). (2) Let $n_{o1} = n_{i1}$ and $n_{o2} = n_{i2}$; if $s_o \in \mathcal{U}_e$ is a fixed-eigenvalue, then s_o is a t.z. of P_{11}, P_{22} and of the plant P.

5.8. Algorithm (Decentralized compensator design): **Given:** $P \in \mathcal{M}(\mathbb{R}_{sp}(s))$ satisfying Assumption 3.1 (A) and conditions (5.4)-(5.5) in Theorem 5.2R. *Step 1*: Find $R_{\mathcal{U}}$-unimodular matrices L_1, R_1 such that $L_1\begin{bmatrix} D_{p1} \\ N_{p1} \end{bmatrix} R_1 = \begin{bmatrix} I_{n_{i1}} & 0 \\ 0 & \tilde{N}_{12} \end{bmatrix}$. *Step 2:* Find an $R_{\mathcal{U}}$-unimodular matrix $L_2 \in \mathcal{M}(R_{\mathcal{U}}(s))$ such that $L_2(\begin{bmatrix} D_{p2} \\ N_{p2} \end{bmatrix} R_1) = \begin{bmatrix} -\tilde{D}_{21}\tilde{D}_{22} \\ \tilde{N}_{21} & 0 \end{bmatrix}$, where $\tilde{D}_{22} \in R_{\mathcal{U}}(s)^{n_{i2} \times n_{i2}}$, and $(\tilde{D}_{22}, \tilde{D}_{21})$ is an l.c. pair. *Step 3:* Find a Bezout identity for the l.c. pair $(\tilde{D}_{22}, \tilde{D}_{21})$: $\begin{bmatrix} V_2 & U_2 \\ -\tilde{D}_{21} & \tilde{D}_{22} \end{bmatrix} \begin{bmatrix} Y_2 & -U_{2l} \\ X_2 & V_{2l} \end{bmatrix} = \begin{bmatrix} I_{n_{i1}} & 0 \\ 0 & I_{n_{i2}} \end{bmatrix}$. Find a Bezout identity for the r.c. pair $(\tilde{N}_{12}, \tilde{D}_{22})$: $\begin{bmatrix} V_{2r} & U_{2r} \\ -\tilde{X}_2 & \tilde{Y}_2 \end{bmatrix} \begin{bmatrix} \tilde{D}_{22} & -\tilde{U}_2 \\ \tilde{N}_{12} & \tilde{V}_2 \end{bmatrix} = \begin{bmatrix} I_{n_{i2}} & 0 \\ 0 & I_{n_{o1}} \end{bmatrix}$. *Step 4*: Let $E_1^{-1} := \begin{bmatrix} V_2 + U_2V_{2r}\tilde{D}_{21} & U_2U_{2r} \\ -\tilde{X}_2\tilde{D}_{21} & \tilde{Y}_2 \end{bmatrix} L_1$, $E_2^{-1} := \begin{bmatrix} I_{n_{i2}} & 0 \\ \tilde{N}_{21}U_{2l}\tilde{U}_2\tilde{X}_2 & I_{n_{o2}} \end{bmatrix} L_2$, and let $W_1 := \tilde{X}_2$, $W_2 := \tilde{N}_{21}Y_2$. *Step 5*: $C_d = diag\begin{bmatrix} C_1 & C_2 \end{bmatrix} = \begin{bmatrix} \tilde{D}_{c1}^{-1}\tilde{N}_{c1} & 0 \\ 0 & \tilde{D}_{c2}^{-1}\tilde{N}_{c1} \end{bmatrix}$ H-stabilizes the given $P \in \mathcal{M}(\mathbb{R}_{sp}(s))$, where $\begin{bmatrix} \tilde{D}_{c1} \vdots \tilde{N}_{c1} \end{bmatrix} = \begin{bmatrix} I_{n_{i1}} \vdots Q_1 \end{bmatrix} E_1^{-1}$, $\begin{bmatrix} \tilde{D}_{c2} \vdots \tilde{N}_{c2} \end{bmatrix} = \begin{bmatrix} I_{n_{i2}} \vdots Q_2 \end{bmatrix} E_2^{-1}$, for some Q_1, $Q_2 \in \mathcal{M}(R_{\mathcal{U}}(s))$ such that $\det(I_{n_{i2}} - Q_2W_2Q_1W_1) \in J$.

VI. Conclusions

In this paper we consider the linear, time-invariant multiinput-multioutput decentralized control system shown in Figure 1. We state necessary and sufficient conditions on P for decentralized stabilizability in a completely general algebraic setting (see Theorems 4.4R and 4.5L) and interpret these conditions in terms of the fixed-eigenvalues of the system in the case of stable rational functions (see Theorems 5.2R and 5.3L). In Theorem 4.7 we give a complete description of the set of all H-stabilizing compensators; this class is parametrized by two parameter matrices; these matrices cannot be chosen completely independently of each other. The class of all H-stabilizing compensators is easily extended to m-channel decentralized control systems [Des.1]. For the stable rational functions case Algorithm 5.8 shows how to find all H-stabilizing compensators starting with any right-coprime factorization of P.

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