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Decoupled PID Controller Synthesis for MIMO Plants with I/O Delays

A. N. Mete, A. N. Gündeş and H. Özbay

***Abstract*— Decentralized Proportional+Integral+Derivative (PID) controller synthesis methods are presented for closed-loop stabilization of linear time-invariant plants with two multi-input, multi-output channels subject to I/O delays (time delays in the input and/or output channels). The plant classes considered here have a finite dimensional part with at most two poles in the unstable region. The designs are reliable, where closed-loop stability is maintained with only one of the two controllers when the other controller is turned-off and taken out of service.**

I. INTRODUCTION

Proportional+Integral+Derivative (PID) controllers are widely used in many control applications and preferred due to their simple structure. We consider two-channel decentralized stabilization of linear time-invariant (LTI), multi-input multi-output (MIMO) systems with time delays in the input and/or output channels. We propose systematic synthesis procedures for two decoupled PID-controllers. We also investigate the problem of maintaining closed-loop stability when one of the controllers fails completely and is taken out of service. Due to the integral-action, asymptotic tracking of step-input references is achieved at each output channel when both controllers are operational.

Stability and feedback stabilization of delay systems have been extensively investigated and many delay-independent and delay-dependent stability results are available [6], [10]. Most of the tuning and internal model control techniques used in process control systems apply to delay systems [1], which inherit the results on robust control of infinite dimensional systems [4]. The more specialized problem of existence of stabilizing PID-controllers for delay systems is not easy to solve [12]. Even for the delay-free case, some unstable plants cannot be stabilized using PID-controllers and strong stabilizability is a necessary condition [8]. The decentralized controller structure brings additional constraints. Decentralized PID controller designs were considered for two-by-two plants in [2]. The reliable control problem when one controller fails has been considered in e.g., [7], [9].

We consider MIMO plant classes with I/O delays that can be stabilized using PID-controllers and present systematic synthesis procedures. When the plant is stable, we propose decoupled PID designs that are reliable against the failure of either one of the controllers. It is assumed that a controller that fails is set equal to zero; i.e., the failure is recognized and the failed controller is taken out of service. Although asymptotic tracking with zero steady-state error is no longer achieved for the failed channel, closed-loop stability is still maintained. We also consider two classes of unstable MIMO plants where the plant may have one or two poles in the unstable region. A partially reliable decoupled PID design is achieved in one of these two cases, where the main channel controller must remain operational for closed-loop stability although the other controller may fail. The goal is to establish existence of stabilizing PID controllers; we do not consider performance issues but propose freedom in the design parameters that can be used towards satisfaction of performance criteria.

Notation: Let $\mathbb{C}, \mathbb{R}, \mathbb{R}_+$ denote complex, real, and positive real numbers. The extended closed right-half complex plane is $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{R}e(s) \geq 0\} \cup \{\infty\}$; $\mathbf{R_p}$ denotes real proper rational functions (of s); $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$; I_r is the $r \times r$ identity matrix. The space $\mathcal{H}_\infty$ is the set of all bounded analytic functions in $\mathbb{C}_+$. For $h \in \mathcal{H}_\infty$, the norm is defined as $\|h\|_\infty = \text{ess}\ \sup_{s \in \mathbb{C}_+} |h(s)|$, where ess sup denotes the essential supremum. A matrix-valued function H is in $\mathcal{M}(\mathcal{H}_\infty)$ if all its entries are in $\mathcal{H}_\infty$; in this case $\|H\|_\infty = \text{ess}\ \sup_{s \in \mathbb{C}_+} \overline{\sigma}(H(s))$, where $\bar{\sigma}$ denotes the maximum singular value. From the induced L^2 gain point of view, a system whose transfer matrix is H is stable iff $H \in \mathcal{M}(\mathcal{H}_\infty)$. A square transfer matrix $H \in \mathcal{M}(\mathcal{H}_\infty)$ is unimodular iff $H^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. We drop (s) in transfer matrices such as $G(s)$. Since all norms of interest here are $\mathcal{H}_\infty$ norms, we drop the norm subscript, i.e. $\|\cdot\|_\infty \equiv \|\cdot\|$. We use coprime factorizations over $\mathbf{S}$; i.e., for $G \in {\mathbf{R_p}}^{r \times r}$, $G = Y^{-1}X$ denotes a left-coprime-factorization (LCF), where $X, Y \in \mathbf{S}^{r \times r}$, $\det Y(\infty) \neq 0$.

II. PROBLEM DESCRIPTION

Consider the two-channel decentralized feedback system $Sys(\widehat{G}, C_D)$ with two MIMO channels in Fig. 1, where $C_D = \text{diag}\,[\,C_1\,,\ C_2\,] \in {\mathbf{R_p}}^{r \times r}$ is the decentralized controller and $\widehat{G}$ is the delayed plant transfer-function partitioned as

$$\widehat{G} = \begin{bmatrix} \widehat{G}_{11} & \widehat{G}_{12} \\ \widehat{G}_{21} & \widehat{G}_{22} \end{bmatrix} = \begin{bmatrix} \Lambda_{11}\, G_{11}\, \widetilde{\Lambda}_{11} & \Lambda_{12}\, G_{12}\, \widetilde{\Lambda}_{12} \\ \Lambda_{21}\, G_{21}\, \widetilde{\Lambda}_{21} & \Lambda_{22}\, G_{22}\, \widetilde{\Lambda}_{22} \end{bmatrix}\ . \quad (1)$$

It is assumed that the feedback system is well-posed and that the delay-free part of the plant and the controller have no unstable hidden-modes. The finite-dimensional part of the plant is $G \in {\mathbf{R_p}}^{r \times r}$, where $G_{jj} \in {\mathbf{R_p}}^{r_j \times r_j}$, and $\text{rank} G = r$. We assume that the delays are known. For $i, j \in \{1, 2\}$ the (ij)-th output delay term Λ_{ij} is an $r_i \times r_i$ diagonal matrix with unit-delays $e^{-sh_\ell^{(ij)}}$ as entries ($\ell = 1, \dots r_i$); the (ij)-th input delay term $\widetilde{\Lambda}_{ij}$ is an $r_j \times r_j$ diagonal matrix with unit-delays $e^{-s\tilde{h}_\ell^{(ij)}}$ as entries ($\ell = 1, \dots r_j$). For the system $Sys(\widehat{G}, C_D)$, let $w := [w_1, w_2]^T$, $v := [v_1, v_2]^T$,

$y := [y_1, y_2]^T$, $u := [u_1, u_2]^T$. The closed-loop transfer matrix H_{cl} from (w, v) to (u, y) is

$$H_{cl} = \begin{bmatrix} C_D(I+\widehat{G}C_D)^{-1} & -C_D(I+\widehat{G}C_D)^{-1}\widehat{G} \\ \widehat{G}C_D(I+\widehat{G}C_D)^{-1} & (I+\widehat{G}C_D)^{-1}\widehat{G} \end{bmatrix}. \quad (2)$$

Definition 2.1: **a)** The feedback system $Sys(\widehat{G}, C_D)$ is said to be stable iff the closed-loop map H_{cl} is in $\mathcal{M}(\mathcal{H}_\infty)$. **b)** The controller C_D is said to stabilize $\widehat{G}$ iff C_D is proper and $Sys(\widehat{G}, C_D)$ is stable. **c)** The controller C_D that stabilizes $\widehat{G}$ is said to be partially reliable iff the system $Sys(\widehat{G}, 0, C_2)$ is also stable, i.e., the transfer-function from (w_2, v) to (y, u_2) is in $\mathcal{M}(\mathcal{H}_\infty)$. **d)** The controller C_D that stabilizes $\widehat{G}$ is said to be fully reliable iff the system $Sys(\widehat{G}, 0, C_2)$ is also stable (i.e., the transfer-function from (w_2, v) to (y, u_2) is in $\mathcal{M}(\mathcal{H}_\infty)$), and the system $Sys(\widehat{G}, C_1, 0)$ is stable (i.e., the transfer-function from (w_1, v) to (y, u_1) is in $\mathcal{M}(\mathcal{H}_\infty)$). ▪

For existence of partially reliable controllers, the finite-dimensional part G of the plant $\widehat{G}$ must satisfy additional requirements [7]. In addition to the decentralized structure of the controller C_D, we restrict our attention to *proper* PID-controllers of the following form [5]: For $j = 1, 2$,

$$C_j = K_{Pj} + \frac{K_{Ij}}{s} + \frac{K_{Dj}\ s}{\tau_j\, s + 1}\ , \quad (3)$$

where $K_{Pj}, K_{Ij}, K_{Dj} \in \mathbb{R}^{r_j \times r_j}$ are the proportional, the integral, and the derivative constants, respectively, and $\tau_j \in \mathbb{R}_+$, where C_j has integral-action when $K_{Ij} \neq 0$. We include subsets of PID-controllers obtained by setting one or two of thee constants to zero; e.g., (3) is a PI-controller when $K_{Dj} = 0$ and an I-controller when $K_{Pj} = K_{Dj} = 0$.

In Section III, we propose decentralized PID-controller design for stable MIMO plants as well as two classes of unstable MIMO plants with input/output delays.

III. MAIN RESULTS

Partially or fully reliable decentralized PID-controllers can be designed for stable MIMO plants with I/O delays; we explore design for stable MIMO plants in Section III-A. In Section III-B, we consider decentralized PID-controller synthesis for MIMO plants with one or two poles in the region of instability $\mathcal{U}$, including the origin. Some restrictions on the number of $\mathcal{U}$-poles are necessary since existence of PID-controllers is not guaranteed even for delay-free plants with an arbitrary number of $\mathcal{U}$-poles. Many plants that have more than two poles in the unstable region cannot be stabilized using PID-controllers. (e.g., $\frac{1}{(s-p)^3}$ or $\frac{1}{(s-p)(s^2+p^2)}$ for $p \geq 0$).

A. Stable plants with input/output delays

If the finite dimensional part G of the delayed plant $\widehat{G}$ is stable, then decentralized PID-controllers can be partially or fully reliable. In Proposition 3.1, we first design the controller C_2 to stabilize $\widehat{G}_{22}$ and then we design C_1 to stabilize the system $\widehat{W}$ defined by

$$\widehat{W} := \widehat{G}_{11} - \widehat{G}_{12}C_2(I + \widehat{G}_{22}C_2)^{-1}\widehat{G}_{21}\ , \quad (4)$$

which contains C_2. When G is stable, $\widehat{W}$ is also stable. This method provides a partially reliable decentralized design. If C_1 is designed to stabilize $\widehat{W}$ and $\widehat{G}_{11}$ simultaneously, then the decentralized controller becomes fully reliable.

Proposition 3.1: Let $G \in \mathbf{S}^{r\times r}$, let rank$G(s) = r$. For C_j to be a PD-controller, let $M_j = 0$. For C_j to be a PID-controller (with nonzero integral constant), let $M_j = I$ and let rank$G(0) = r$, rank$G_{22}(0) = r_2$.

a) Partially reliable design: Choose any $\widehat{K}_{P2}$, $\widehat{K}_{D2} \in \mathbb{R}^{r_2\times r_2}$, $\tau_2 > 0$. Then the PID-controller C_2 in (5) stabilizes $\widehat{G}_{22}$ for any $\beta_2 \in \mathbb{R}_+$ satisfying (6):

$$C_2 =: \beta_2\widehat{C}_2 = \beta_2\widehat{K}_{P2} + \frac{\beta_2\,\widehat{K}_{D2}\,s}{\tau_2\,s+1} + \frac{\beta_2\,G_{22}(0)^{-1}}{s}\,M_2\ , \quad (5)$$

$$0 < \beta_2 < \ \|\ s^{-1}\,[\ s\,\widehat{G}_{22}(s)\ \widehat{C}_2 - M_2\ ]\ \|^{-1}. \quad (6)$$

Let $\widehat{W}$ be defined by (4); $\widehat{W}(0) = G_{11}(0) - G_{12}(0)G_{22}(0)^{-1}G_{21}(0)$. Choose any $\widehat{K}_{P1}$, $\widehat{K}_{D1} \in \mathbb{R}^{r_1\times r_1}$, $\tau_1 > 0$. Let C_1 be as in (7) for $\beta_1 \in \mathbb{R}_+$ satisfying (8):

$$C_1 =: \beta_1\widehat{C}_1 = \beta_1\widehat{K}_{P1} + \frac{\beta_1\,\widehat{K}_{D1}\,s}{\tau_1\,s+1} + \frac{\beta_1\,\widehat{W}(0)^{-1}}{s}\,M_1\ , \quad (7)$$

$$0 < \beta_1 < \ \|\ s^{-1}\,[\ s\,\widehat{W}(s)\ \widehat{C}_1 - M_1\ ]\ \|^{-1}\ . \quad (8)$$

With C_2 and C_1 as in (5) and (7), respectively, $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is a partially reliable decentralized PID-controller for the delayed plant $\widehat{G}$. For $\widehat{K}_{Dj} = 0$, the controllers (5) and (7) become P-controllers (if $M_j = 0$) or PI-controllers (if $M_j = I$); for $\widehat{K}_{Pj} = 0$, (5) and (7) become D-controllers (if $M_j = 0$) or ID-controllers (if $M_j = I$).

b) Fully reliable design: Let rank$G_{11}(0) = r_1$. Let $\widehat{W}(0)G_{11}(0)^{-1} = I - G_{12}(0)G_{22}(0)^{-1}G_{21}(0)G_{11}(0)^{-1}$ have all positive real eigenvalues. Let C_2 be as in (5) with β_2 satisfying (6). Let C_1 be as in (7) with β_1 satisfying

$$0 < \beta_1 < \ \min\ \{\ \|s^{-1}\,[\ s\,\widehat{W}(s)\widehat{C}_1 - M_1\ ]\ \|^{-1}\ ,$$
$$\|\ s^{-1}\,[\ s\,\widehat{G}_{11}(s)\widehat{C}_1 - G_{11}(0)\widehat{W}(0)^{-1}\,M_1\ ]\ \|^{-1}\}. \quad (9)$$

Then $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is a fully reliable decentralized PID-controller for the delayed plant $\widehat{G}$. ▪

An interesting problem arising within the context of the above control procedure is the "optimal" design of some of the free parameters such as $\widehat{K}_{P2}$ and $\widehat{K}_{P1}$. Let us consider optimal PI controller $\widehat{C}_2(s) = \widehat{K}_{P2} + \frac{1}{s}G_{22}(0)^{-1}$. The proportional gain $\widehat{K}_{P2}$ will be optimized so that the allowable interval for β_2 is the largest. This problem is equivalent to finding $\widehat{K}_{P2}$ such that (10) is minimized:

$$\|\,s^{-1}\ [\ s\,\widehat{G}_{22}(s)(\widehat{K}_{P2} + \frac{1}{s}G_{22}(0)^{-1}) - I\ ]\,\|\ . \quad (10)$$

Re-arranging terms in (10), defining $\widehat{K}_{P2} = G_{22}(0)^{-1}\widetilde{K}_{P2}$, and $F_{22}(s) := \widehat{G}_{22}(s)G_{22}(0)^{-1}$, we are interested in finding the optimal $\widetilde{K}_{P2}$ such that (11) is minimized:

$$\|\ \frac{F_{22}(s) - I}{s}\ +\ F_{22}(s)\,\widetilde{K}_{P2}\ \|\ . \quad (11)$$

This problem has been studied in [11], where a formula for the optimal solution is obtained for a class of $F_{22}(s)$.

Similarly, one can derive a PI controller C_1 by optimizing $\widehat{K}_{P1}$ to maximize the allowable range for β_1. The problem is again in the form (11) with F_{22} is replaced by $\widehat{W}(s)\widehat{W}(0)^{-1}$.

In Example 3.1, we apply the synthesis procedure in Proposition 3.1 to design a partially and fully reliable decentralized control system that manipulates the flow rate of two drugs (dopamine and sodium nitroprusside) to regulate two outputs (main arterial pressure and cardiac output) for critical care patients. A simplified model is used representing the input-output behavior for a particular patient [3].

Example 3.1: Let $\widehat{G} = \begin{bmatrix} \frac{-6}{0.67s+1}e^{-0.75s} & \frac{3}{2s+1}e^{-s} \\ \frac{12}{0.67s+1}e^{-0.75s} & \frac{5}{5s+1}e^{-s} \end{bmatrix} \in \mathcal{H}_\infty{}^{2\times 2}$, where $\widetilde{\Lambda}_{11} = \widetilde{\Lambda}_{21} = e^{-0.75s}$, $\widetilde{\Lambda}_{12} = \widetilde{\Lambda}_{22} = e^{-s}$; all output delay terms are $\Lambda_{ij} = 1$. Following Proposition 3.1, partially and fully reliable decentralized PID-controllers can be designed with nonzero K_{Ij}: $\mathrm{rank}G(0) = 2$, $G_{jj}(0) \neq 0$, $\widehat{W}(0)G_{11}(0)^{-1} = 2.2 > 0$. First design C_2: Choose $\widehat{K}_{P2} = 1$, $\widehat{K}_{D2} = 0.2$, $\tau_2 = 0.1$. With $\beta_2 = 0.6$ satisfying (6), the PID-controller in (5) is $C_2 = 0.6+0.12/s+0.12s/(0.1s+1)$. Now design C_1: Choose $\widehat{K}_{P1} = -0.15$, $\widehat{K}_{D1} = -0.1$, $\tau_1 = 0.1$. With $\beta_1 = 0.1$ satisfying (8), the PID-controller in (7) is $C_1 = -0.015-0.0076/s-0.01s/(0.1s+1)$. Then $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is a partially reliable decentralized controller; it is also fully reliable since $\beta_1 = 0.1$ also satisfies (9) with this $\widehat{K}_{P1}$. Fig. 2 shows the closed-loop step responses for the outputs y_1 (dashed), y_2 (solid), with unit-step references applied at both r_1, r_2. The controller C_D is active with both channels operational, and both achieve asymptotic tracking with zero steady-state error.

For this example, we can also design a PI controller $C_2 = \beta_2^o\widehat{C}_2$, where $\beta_2^o = \beta_2^{\max}/2$, and $\beta_2^{\max}$ is the largest allowable β_2 over all $\widehat{K}_{P2}$. By using the result of [11] we find that $\beta_2^{\max} = 1.015$ and the corresponding optimal $\widehat{K}_{P2}$ is 1.03. In the second stage we find the optimal PI controller C_1 using the same technique, and in this case $\beta_1^{\max} = 0.0108$ with optimal $\widehat{K}_{P1} = -0.1735$. ▪

B. Delayed plants with one or two $\mathcal{U}$-poles

Let the finite dimensional part $G(s) \in \mathbf{R_p}{}^{r\times r}$ of the plant have full (normal) rank. Let G have no transmission-zeros at $s = 0$. Without loss of generality, it can be assumed that G has a left-coprime-factorization (LCF) as in (12) [13]:

$$G = Y^{-1}X = \begin{bmatrix} Y_{11} & Y_{12} \\ 0 & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} . \qquad (12)$$

We consider two cases of unstable G with restrictions on the number of $\mathcal{U}$-poles. In all cases, G may have any number of poles in the stable region. We first design C_2 to stabilize G_{22} and then C_1 to stabilize the system $\widehat{W}$ defined in (4), which contains C_2. In Case (1), $\widehat{W}$ is unstable; in case (2), $\widehat{W}$ is stable. Clearly the channels can be re-ordered to exchange the roles of G_{11} and G_{22}. In Case (1), G has one $\mathcal{U}$-pole $p_1 \in \mathbb{R}_+$ that appears in G_{22}, and has another $\mathcal{U}$-pole $p_2 \in \mathbb{R}_+$ that appears in G_{11} (and possibly various other entries) but not in G_{22} (unless $p_1 = p_2$). In this case, a partially reliable decentralized design that relies on closed-loop stability with only C_2 active and $C_1 = 0$ is not possible because of the instability that is not reflected in G_{22}. In Case (2), G has one or two $\mathcal{U}$-poles that appear in G_{22} and these poles may appear in various other entries of G. Since all instabilities of G are reflected in G_{22}, a partially reliable decentralized design with $C_1 = 0$ is possible.

Case 1) For the finite dimensional part G of $\widehat{G}$ in (12), let

$$Y_{11} = \frac{(s-p_1)}{a_1 s+1} I_{r_1} \ , \ Y_{22} = \frac{(s-p_2)}{a_2 s+1} I_{r_2} \ , \ Y_{12} = 0 \ , \qquad (13)$$

where $p_1, p_2 \geq 0$ are the non-negative real poles of G, $a_j \in \mathbb{R}_+$, $j = 1,2$. Let $\mathrm{rank}X_{jj}(p_j) = \mathrm{rank}(s-p_j)G(s)|_{s=p_j} = r_j$ for $j = 1,2$. Therefore, all entries of G_{jk}, $j,k = 1,2$, have a pole at p_j. All entries of G have the same pole if $p_1 = p_2$. For PID-controller design with nonzero integral constant, also assume that G_{22} has no transmission zeros at $s = 0$, i.e., $\mathrm{rank}X_{22}(0) = \mathrm{rank}(s-p_2)G(s)|_{s=0} = r_2$; this assumption is not necessary for PD-controller design. Since each Y_{jj} is diagonal, the delayed plant $\widehat{G}$ can be written as

$$\widehat{G} = \begin{bmatrix} Y_{11}^{-1} & 0 \\ 0 & Y_{22}^{-1} \end{bmatrix} \begin{bmatrix} \Lambda_{11}X_{11}\widetilde{\Lambda}_{11} & \Lambda_{12}X_{12}\widetilde{\Lambda}_{12} \\ \Lambda_{21}X_{21}\widetilde{\Lambda}_{21} & \Lambda_{22}X_{22}\widetilde{\Lambda}_{22} \end{bmatrix}$$
$$=: \begin{bmatrix} Y_{11}^{-1} & 0 \\ 0 & Y_{22}^{-1} \end{bmatrix} \begin{bmatrix} \widehat{X}_{11} & \widehat{X}_{12} \\ \widehat{X}_{21} & \widehat{X}_{22} \end{bmatrix} . \qquad (14)$$

Under certain assumptions on the poles $p_1, p_2 \in \mathbb{R} \cap \mathcal{U}$, there exits decentralized PID-controllers for the delayed $\widehat{G}$. A systematic synthesis is developed in Proposition 3.2:

Proposition 3.2: Let $\widehat{G}$ be as in (14). For $j = 1,2$, let $G_{jj} = Y_{jj}^{-1}X_{jj} \in \mathbf{R_p}{}^{r_j\times r_j}$, $\mathrm{rank}X_{jj}(p_j) = \mathrm{rank}(s-p_j)G_{jj}(s)|_{s=p_j} = r_j$, where $p_j \geq 0$. Let $G_{jk} = Y_{jj}^{-1}X_{jk}$, $k = 1,2$. For C_j to be a PD-controller, let $M_j = 0$. For C_j to be a PID-controller (with nonzero integral constant), let $M_j = I$ and let G have no transmission-zeros at $s = 0$, i.e., $\mathrm{rank}X(0) = \mathrm{rank}(YG(s))|_{s=0} = r$, and let $\mathrm{rank}X_{22}(0) = \mathrm{rank}(s-p_2)G_{22}(s)|_{s=0} = r_2$. *Step 1: Design* C_2: Choose any $\widehat{K}_{Dj} \in \mathbb{R}^{r_j\times r_j}$, $\tau_j > 0$. Define $\widehat{C}_2 := X_{22}(0)^{-1} + \frac{\widehat{K}_{D2}\, s}{\tau_2\, s+1}$ and $\Phi_{\Lambda 2} := s^{-1}[\,(s-p_2)\,\widehat{G}_{22}(s)\,\widehat{C}_2 - I\,]$. If $0 \leq p_2 < \|\ \Phi_{\Lambda 2}\ \|^{-1}$, then for any $\alpha_2 \in \mathbb{R}_+$ satisfying

$$0 < \ \alpha_2 \ < \|\ \Phi_{\Lambda 2}\ \|^{-1} - p_2 \ , \qquad (15)$$

a PD-controller that stabilizes $\widehat{G}_{22}$ is $C_{pd2} = (\,\alpha_2 + p_2\,)\widehat{C}_2$, given by (16); if $\widehat{K}_{D2} = 0$, (16) is a P-controller:

$$C_{pd2} = (\,\alpha_2 + p_2\,)\,X_{22}(0)^{-1} + \frac{(\,\alpha_2 + p_2\,)\,\widehat{K}_{D2}\ s}{\tau_2 s+1} \ . \qquad (16)$$

With C_{pd2} as in (16), let $H_{pd2} := \widehat{G}_{22}(I + C_{pd2}\,\widehat{G}_{22})^{-1}$, where $H_{pd2}(0)^{-1} = \alpha_2 X_{22}(0)^{-1}$. Then the PID-controller C_2 in (17) stabilizes $\widehat{G}_{22}$ for any $\gamma_2 \in \mathbb{R}_+$ satisfying (18):

$$C_2 = C_{pd2} + \frac{\gamma_2\,\alpha_2\,X_{22}(0)^{-1}}{s}\,M_2 \ , \qquad (17)$$

$$0 < \gamma_2 < \ \|\ s^{-1}\ [\ H_{pd2}(s)H_{pd2}(0)^{-1} - I\ ]\ \|^{-1} \ . \qquad (18)$$

Step 2: Design C_1: Let $\widehat{W} =: Y_{11}^{-1}\widehat{W}_{11}$ be defined by (4), where $\widehat{W}_{11}(0) = X_{11}(0) - X_{12}(0)X_{22}(0)^{-1}X_{21}(0)$.

Let $\widehat{C}_1 := \widehat{W}_{11}(0)^{-1} + \frac{\widehat{K}_{D1}\, s}{\tau_1\, s+1}$ and $\Phi_{\Lambda 1} := s^{-1}[\,(s - p_1)\,\widehat{W}(s)\,\widehat{C}_1 - I\,]$. If $0 \le p_1 < \|\, \Phi_{\Lambda 1}\,\|^{-1}$, then let $C_{pd1} = (\,\alpha_1 + p_1\,)\widehat{C}_1$ be given by (19) for any $\alpha_1 \in \mathbb{R}_+$ satisfying (20); if $\widehat{K}_{D1} = 0$, (19) is a P-controller:

$$C_{pd1} = (\,\alpha_1 + p_1\,)\,\widehat{W}_{11}(0)^{-1} + \frac{(\,\alpha_1 + p_1\,)\,\widehat{K}_{D1}\; s}{\tau_1 s + 1}\ , \quad (19)$$

$$0 < \ \alpha_1 \ < \|\, \Phi_{\Lambda 1}\,\|^{-1} - p_1\ . \quad (20)$$

With C_{pd1} as in (19), let $H_{pd1} := \widehat{W}(I + C_{pd1}\,\widehat{W})^{-1}$, where $H_{pd1}(0)^{-1} = \alpha_1 \widehat{W}_{11}(0)^{-1}$. Let C_1 be as in (21) for any $\gamma_1 \in \mathbb{R}_+$ satisfying (22):

$$C_1 = C_{pd1} + \frac{\gamma_1\,\alpha_1\,\widehat{W}_{11}(0)^{-1}}{s}\, M_1\ , \quad (21)$$

$$0 < \gamma_1 < \ \|\, s^{-1}\ [\ H_{pd1}(s) H_{pd1}(0)^{-1} - I\ ]\ \|^{-1}\ . \quad (22)$$

With C_2, C_1 as in (17), (21), $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is a decentralized PID-controller for the delayed plant $\widehat{G}$. For $\widehat{K}_{Dj} = 0$, (17), (21) are P-controllers (if $M_j = 0$) or PI-controllers (if $M_j = I$); for $\widehat{K}_{Pj} = 0$, (17), (21) are D-controllers (if $M_j = 0$) or ID-controllers (if $M_j = I$). ▪

In Example 3.2, we apply Proposition 3.2 to design decoupled PID-controllers for an MIMO distillation column with delays in the input channels. A full-feedback proportional control design was considered for this system in [8].

Example 3.2: Let $\widehat{G} = \frac{1}{s}\begin{bmatrix} 3.04 e^{-h_1 s} & \frac{-278.2 e^{-h_2 s}}{(s+6)(s+30)} \\ 0.052 e^{-h_1 s} & \frac{206.6 e^{-h_2 s}}{(s+6)(s+30)} \end{bmatrix}$, which can be written in the form of (14):

$$\widehat{G} = \begin{bmatrix} \frac{s}{a_1 s+1} & 0 \\ 0 & \frac{s}{a_1 s+1} \end{bmatrix}^{-1} \begin{bmatrix} \frac{3.04 e^{-h_1 s}}{a_1 s+1} & \frac{-278.2 e^{-h_2 s}}{(s+6)(s+30)(a_1 s+1)} \\ \frac{0.052 e^{-h_1 s}}{a_1 s+1} & \frac{206.6 e^{-h_2 s}}{(s+6)(s+30)(a_1 s+1)} \end{bmatrix},$$

$a_1 \in \mathbb{R}_+$, $Y_{11} = Y_{22}$ and $p_1 = p_2 = 0$. Let $h_1 = 0.5$, $h_2 = 0.6$. Choose $\widehat{K}_{D2} = 0$. With $\widehat{C}_2 = X_{22}(0)^{-1} = 180/206.6$, take $\alpha_2 = 0.5$ satisfying (15). Then take $\gamma_2 = 0.1$ satisfying (18). The PI-controller $C_2 = \alpha_2 X_{22}(0)^{-1}(1 + M_2\gamma_2/s) = 0.4356 + M_2\, 0.04356/s$ stabilizes $\widehat{G}_{22}$ for $M_2 = 1$ and $M_2 = 0$, in which case C_2 is just proportional. Now choose $\widehat{K}_{D1} = 0$. With $\widehat{C}_1 = \widehat{W}_{11}(0)^{-1} = 1/3.11$, take $\alpha_1 = 1.3$ satisfying (20). Then take $\gamma_1 = 0.15$ satisfying (22). The PI-controller $C_1 = \alpha_1 \widehat{W}_{11}(0)^{-1}(1 + M_1\gamma_2/s) = 0.418 + M_1 0.0627/s$ stabilizes $\widehat{W}$ for $M_1 = 1$ and $M_1 = 0$, in which case C_1 is proportional. With the decentralized PI-controller $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ stabilizing $\widehat{G}$, Fig. 3 shows the closed-loop step responses for the outputs y_1 (dashed), y_2 (solid), with unit-step references applied at both r_1, r_2. ▪

Case 2) For the finite dimensional part G of $\widehat{G}$ in (12), let Y_{12} be either diagonal or zero, let $Y_{11} = I_{r_1}$. Let

$$d := \prod_{i=1}^{\ell}(a_i s + 1)\ ,\ n := \prod_{i=1}^{\ell}(s - p_i)\ , \quad (23)$$

$Y_{22} = \frac{n}{d} I_{r_2}$, $X_{22} = Y_{22} G_{22}$, and $\ell \in \{1, 2\}$, $a_i \in \mathbb{R}_+$, $i \in \{1, \ell\}$. Let $\mathrm{rank} X_{22}(p_i) = \mathrm{rank}\, n\, G(s)|_{s=p_i} = r_j$ for $i \in \{1, \ell\}$. Therefore, G has one or two $\mathcal{U}$-poles at $p_i \in \mathcal{U}$, and all $\mathcal{U}$-poles of G appear in G_{22}; they may also appear in any of the other entries of G. If $\ell = 1$, then $p_1 \ge 0$; if $\ell = 2$, then the two poles are either real ($p_1, p_2 \ge 0$) or they are a complex-conjugate pair ($p_1 = \bar{p}_2 \in \mathcal{U}$). For PID-controller design with nonzero integral constant, also assume that G_{22} has no transmission zeros at $s = 0$, i.e., $\mathrm{rank} X_{22}(0) = \mathrm{rank}\, n\, G_{22}(s)|_{s=0} = r_2$; this assumption is not necessary for PD-controller design. Since Y_{22} is diagonal, and Y_{12} is diagonal when it is not zero, the delayed plant $\widehat{G}$ can be written as

$$\widehat{G} = \begin{bmatrix} I & -Y_{12} Y_{22}^{-1} \\ 0 & Y_{22}^{-1} \end{bmatrix} \begin{bmatrix} \Lambda_{11} X_{11} \widetilde{\Lambda}_{11} & \Lambda_{12} X_{12} \widetilde{\Lambda}_{12} \\ \Lambda_{21} X_{21} \widetilde{\Lambda}_{21} & \Lambda_{22} X_{22} \widetilde{\Lambda}_{22} \end{bmatrix}$$

$$=: \begin{bmatrix} I & Y_{12} \\ 0 & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} \widehat{X}_{11} & \widehat{X}_{12} \\ \widehat{X}_{21} & \widehat{X}_{22} \end{bmatrix}\ . \quad (24)$$

Under certain assumptions on the $\mathcal{U}$-poles, there exits decentralized PID-controllers for the delayed plant $\widehat{G}$. Furthermore, closed-loop stability can be maintained with $C_1 = 0$. A systematic reliable decentralized PID-controller synthesis is developed in Proposition 3.3, where, for the controller C_2 that stabilizes $\widehat{G}_{22}$, we consider real and complex-conjugate pairs of poles as two separate cases:

Case a) The two $\mathcal{U}$-poles are real, i.e., $p_i \in \mathbb{R}$, $p_i \ge 0$, $i = 1, 2$.

Case b) The two $\mathcal{U}$-poles are a complex-conjugate pair, i.e., $p_1 = \bar{p}_2$, $n = s^2 - (p_1 + p_2)s + p_1 p_2 = s^2 - 2fs + g^2$, $f \ge 0$, $g > 0$, $f < g$. In this case, $X_{22}(0) = g^2 G_{22}(0)$.

Proposition 3.3: Let $\widehat{G}$ be as in (24). With n, d as (23), $G_{22} = Y_{22}^{-1} X_{22} \in \mathbf{R_p}^{r_2 \times r_2}$, $\mathrm{rank} X_{22}(p_i) = \mathrm{rank}\, n\, G(s)|_{s=p_i} = r_2$, $i \in \{1, \ell\}$, $\ell \in \{1, 2\}$. For C_j to be a PD-controller, let $M_j = 0$. For C_j to be a PID-controller (with $K_I \ne 0$), let $M_j = I$ and let $\mathrm{rank} X(0) = \mathrm{rank}(YG(s))|_{s=0} = r$, $\mathrm{rank} X_{22}(0) = \mathrm{rank}\, n\, G_{22}(s)|_{s=0} = r_2$. *Step 1: Design C_2*: If $\ell = 1$, design the PID-controller C_2 that stabilizes $\widehat{G}_{22}$ as in (17) of Proposition 3.2. If $\ell = 2$, choose any $\tau_2 > 0$. Define $\Psi_{\Lambda 1} := s^{-1}\,[\,\frac{n}{(\tau_2\, s+1)}\,\widehat{G}_{22}(s)\, X_{22}(0)^{-1} - I\,]$. Consider two cases: **a)** Let $p_i \in \mathbb{R}$, $p_i \ge 0$, $i \in \{1, 2\}$. Let $\widehat{F}_2 := (s - p_2)\widehat{G}_{22}(s) X_{22}(0)^{-1}$. If $0 \le p_1 < \|\Psi_{\Lambda 1}\|^{-1}$, then define $\Psi_{\Lambda 2} := s^{-1}[\,\alpha_1(\,I + \frac{(\alpha_1 + p_1)}{\tau_2\, s+1}\widehat{F}_2\,)^{-1}\widehat{F}_2 - I\,]$, for any $\alpha_1 \in \mathbb{R}_+$ satisfying (25):

$$0 < \alpha_1 < \ \|\, \Psi_{\Lambda 1}\,\|^{-1} - p_1\ . \quad (25)$$

If $0 \le p_2 < \|\, \Psi_{\Lambda 2}\,\|^{-1}$, then let $K_{P2} = (\alpha_1\,\alpha_2 - p_1\, p_2) X_{22}(0)^{-1}$, $K_{D2} = (\alpha_1 + p_1\,)\,(1 + \tau_2\, p_2) X_{22}(0)^{-1}$ for any $\alpha_2 \in \mathbb{R}_+$ satisfying (26):

$$0 < \alpha_2 < \ \|\, \Psi_{\Lambda 2}\,\|^{-1} - p_2\ . \quad (26)$$

Then a PD-controller that stabilizes $\widehat{G}_{22}$ is given by (27): $C_{pd2} =$

$$[(\alpha_1\alpha_2 - p_1 p_2) + \frac{(\alpha_1 + p_1)(1 + p_2\tau_2)s}{\tau_2 s + 1}\,] X_{22}(0)^{-1}\ . \quad (27)$$

With C_{pd2} as in (27), let $H_{pd2} := \widehat{G}_{22}(I + C_{pd2}\,\widehat{G}_{22})^{-1}$, $H_{pd2}(0)^{-1} = \alpha_1\,\alpha_2 X_{22}(0)^{-1}$. Then the PID-controller C_2

in (28) stabilizes $\widehat{G}_{22}$ for any $\gamma_2 \in \mathbb{R}_+$ satisfying (18):

$$C_2 = C_{pd2} + \frac{\gamma_2\,\alpha_1\,\alpha_2\,X_{22}(0)^{-1}}{s}\,M_2\,. \qquad (28)$$

b) Let $p_1 = \bar{p}_2 \in \mathbb{C}$, $n = s^2 - (p_1+p_2)s + p_1p_2 = s^2 - 2fs + g^2$, $f \geq 0$, $g > 0$, $f < g$. If $f + 2g < \|\ \Psi_{\Lambda 1}\ \|^{-1}$, then let $K_{P2} = [\delta_1\delta_2 + \delta_1(g-f) + \delta_2 g - fg\,]X_{22}(0)^{-1}$, $K_{D2} = (\delta_1 + \delta_2 + f + 2g)X_{22}(0)^{-1} - \tau_2\,K_{P2}$ for any $\delta_1, \delta_2 \in \{\mathbb{R}_+ \cup 0\}$ satisfying (29):

$$0 \leq \delta_1 + \delta_2 < \|\ \Psi_{\Lambda 1}\ \|^{-1} - (f+2g)\ . \qquad (29)$$

Then a PD-controller that stabilizes $\widehat{G}_{22}$ is given by (30): $C_{pd2} =$

$$\frac{[(\delta_1+\delta_2+f+2g)s + \delta_1\delta_2 + \delta_1(g-f) + \delta_2 g - fg]G_{22}(0)^{-1}}{g^2(\tau_2 s+1)}\,. \qquad (30)$$

With C_{pd2} as in (30), let $H_{pd2} := \widehat{G}_{22}(I + C_{pd2}\,\widehat{G}_{22})^{-1}$, where $H_{pd2}(0)^{-1} = (\delta_1+g)(\delta_2+g-f)\,X_{22}(0)^{-1}$ and $X_{22}(0) = g^2 G_{22}(0)$. Then the PID-controller C_2 in (31) stabilizes $\widehat{G}_{22}$ for any $\gamma_2 \in \mathbb{R}_+$ satisfying (18):

$$C_2 = C_{pd2} + \frac{\gamma_2\,(\delta_1+g)\,(\delta_2+g-f)\,X_{22}(0)^{-1}}{s}\,M_2\,. \quad (31)$$

If $\delta_1 = \delta_2 = 0$, (31) is a D-controller for $M_2 = 0$, and an ID-controller for $M_2 = I$. *Step 2: Design* C_1 : If $\ell = 1$, let C_2 be as in (17). If $\ell = 2$, let C_2 be as in (28) or (31) when $p_i \in \mathbb{R}_+$ or $p_i \in \mathcal{U} \setminus \mathbb{R}_+$, respectively. Let $\widehat{W}$ be defined by (4), where $\widehat{W}(0) = X_{11}(0) - X_{12}(0)X_{22}(0)^{-1}X_{21}(0)$. Choose any $\widehat{K}_{P1}, \widehat{K}_{D1} \in \mathbb{R}^{r_1\times r_1}$, $\tau_1 > 0$. Let C_1 be as in (32) for $\beta_1 \in \mathbb{R}_+$ satisfying (33):

$$C_1 = \beta_1\widehat{C}_1 = \beta_1\widehat{K}_{P1} + \frac{\beta_1\,\widehat{K}_{D1}\,s}{\tau_1\,s+1} + \frac{\beta_1\,\widehat{W}(0)^{-1}}{s}\,M_1\,, \quad (32)$$

$$0 < \beta_1 < \ \|\ s^{-1}\ [\ s\,\widehat{W}(s)\ \widehat{C}_1 - M_1\ ]\ \|^{-1}\ . \qquad (33)$$

For $\widehat{K}_{D1} = 0$, (32) is a P-controller (if $M_j = 0$) or a PI-controller (if $M_j = I$); for $\widehat{K}_{P1} = 0$, (32) is a D-controller (if $M_j = 0$) or an ID-controller (if $M_j = I$). With C_1 as in (32), $C_D = \mathrm{diag}\,[\,C_1, C_2\,]$ is a partially reliable decentralized PID-controller for the delayed plant $\widehat{G}$. ∎

IV. CONCLUSIONS

We showed existence of stabilizing decoupled PID-controllers for LTI plants with two MIMO channels subject to input and/or output delays. For stable plants, the decentralized controllers are designed to be partially or fully reliable to provide closed-loop stability even when one of the controllers is set to zero. For plants with only one or two unstable poles (with no restriction on the number of stable poles) we presented systematic methods to define the PID-controller parameters explicitly. Reliable stabilization is achieved also for unstable plants if the channel that always remains operational contains all poles in the unstable region. Since these results are obtained from small gain based arguments, they are conservative.

Plants with more than two poles in the unstable region do not necessarily admit PID-controllers even if they are strongly stabilizable. Further assumptions are needed on such plants, which would impose restrictions on the plant's transmission-zeros. It may be possible to extend the reliable decentralized PID synthesis methods presented here to delayed plants with more than two MIMO channels.

APPENDIX: PROOFS

Proof of Proposition 3.1: *a)* The decentralized PID-controller $C_D = ND^{-1}$, where $D = \mathrm{diag}\,[D_1, D_2]$, with $D_j = I - \frac{\beta_j}{s+\beta_j}M_j$, $N_j = C_jD_j$, $\beta_j \in \mathbb{R}_+$, $j = 1, 2$, stabilizes $\widehat{G} \in \mathcal{M}(\mathcal{H}_\infty)$ if and only if $U_D := D + \widehat{G}N$ is unimodular. Similarly, C_2 stabilizes $\widehat{G}_{22}$ if and only if $U_2 := D_2 + \widehat{G}_{22}N_2 = D_2 + \widehat{G}_{22}C_2D_2 = I + \beta_2[\widehat{G}_{22}(\widehat{K}_{P2} + \frac{s\,\widehat{K}_{D2}}{\tau_2 s+1})D_2 + \frac{s}{s+\beta_2}\frac{(\widehat{G}_{22}G_{22}(0)^{-1}-I)}{s}M_2\,]$ is unimodular if (6) holds. Hence, C_2 in (5) stabilizes $\widehat{G}_{22}$ and $C_2\,(I + \widehat{G}_{22}C_2)^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$ implies $\widehat{W} \in \mathcal{M}(\mathcal{H}_\infty)$; $C_2\,(I + \widehat{G}_{22}C_2)^{-1}(0) = G_{22}(0)^{-1}$ implies $\widehat{W}(0) = G_{11}(0) - G_{12}(0)G_{22}(0)^{-1}G_{21}(0)$. By (8), $U_w := D_1 + \widehat{W}C_1D_1$ is unimodular; hence, C_1 stabilizes $\widehat{W}$. Therefore, $Sys(\widehat{G}, C_D)$ is stable and C_D is partially reliable since diag $[0, C_2]$ also stabilizes $\widehat{G}$. *b)* By assumption, $\Theta =: G_{11}(0)\widehat{W}(0)^{-1}$ has positive real eigenvalues implies $\|sI(sI + \beta_1\Theta)^{-1}\| = 1$ for $\beta_1 > 0$. Define $\tilde{D}_1 = I - \beta_1\Theta\,(sI + \beta_1\Theta)^{-1}M_1$, $\tilde{N}_1 = C_1\tilde{D}_1$. Then $U_1 := \tilde{D}_1 + \widehat{G}_{11}\tilde{N}_1 = I + \beta_1\,[\,\widehat{G}_{11}\,(\widehat{K}_{P1} + \frac{s\,\widehat{K}_{D1}}{\tau_1 s+1})\,\tilde{D}_1 + \frac{(\widehat{G}_{11}\widehat{W}(0)^{-1}-\Theta)}{s}\,sI(sI+\beta_1\Theta)^{-1}\,M_1\,]$ is unimodular. Hence, C_1 stabilizes $\widehat{G}_{11}$ and C_D is fully reliable since diag $[C_1, 0]$ also stabilizes $\widehat{G}$. ∎

Proof of Proposition 3.2: *i)* By (14), the decentralized PID-controller $C_D = ND^{-1}$ (as in the proof of Proposition 3.1) stabilizes $\widehat{G}$ if and only if $U_D := YD + \widehat{X}N$ is unimodular. The PD-controller C_{pd2} stabilizes $\widehat{G}_{22}$ if and only if $U_{pd} := Y_{22} + \widehat{X}_{22}C_{pd2} = \frac{(s-p_2)}{a_2 s+1}[I + \widehat{G}_{22}C_{pd2}\,]$ is unimodular. Writing $U_{pd} = \frac{(s-p_2)}{a_2 s+1}[I + (\alpha_2 + p_2)\,\widehat{G}_{22}C_{pd2}\,] = [\,I + \frac{(\alpha_2+p_2)\,s}{s+\alpha_2}\,\Phi_{\Lambda 2}\,]\,\frac{(s+\alpha_2)}{(a_2 s+1)}$, a sufficient condition for U_{pd} to be unimodular is that (15) holds. Hence, C_{pd2} in (16) stabilizes $\widehat{G}_{22}$ and $H_{pd2} := U_{pd}^{-1}\widehat{X}_{22} = \widehat{G}_{22}(I + C_{pd2}\widehat{G}_{22})^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, where $H_{pd2}(0)^{-1} = G_{22}^{-1}(0) + K_{P2} = X_{22}(0)^{-1}Y_{22}(0) + (\alpha_2+p_2)X_{22}(0)^{-1} = \alpha_2\,X_{22}(0)^{-1}$. Using similar steps as in the proof of Proposition 3.1, the I-controller $K_{I2}/s = \gamma_2 H_{pd2}(0)^{-1}/s$ stabilizes H_{pd2} for any $\gamma_2 \in \mathbb{R}$ satisfying (18). Therefore, $C_2 = C_{pd2} + K_{I2}/s$ in (17) stabilizes $\widehat{G}_{22}$ and $C_2\,(I + \widehat{G}_{22}C_2)^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. Now since $U_2 := \frac{s}{s+\gamma_2}Y_{22} + \widehat{X}_{22}\frac{s}{s+\gamma_2}C_2$ is unimodular, U_D is unimodular if and only if $Y_{11}D_1 + \widehat{W}_{11}N_1$ is unimodular, equivalently, C_1 stabilizes $\widehat{W}$. Using a C_1 design for $\widehat{W}$ similar to C_2 for $\widehat{G}_{22}$, it follows that C_D stabilizes $\widehat{G}$. ∎

Proof of Proposition 3.3: **a)** Let $p_i \in \mathbb{R}_+$. If $\ell = 2$, let $V_1 := \frac{(s-p_1)}{a_1 s+1}I + (\alpha_1 + p_1)\,\frac{(a_2 s+1)}{\tau_2 s+1}\widehat{X}_{22}X_{22}(0)^{-1} = \frac{(s-p_1)}{a_1 s+1}[I + \frac{(\alpha_1+p_1)}{\tau_2\,s+1}\widehat{F}_2\,] = [\,I + \frac{(\alpha_1+p_1)\,s}{s+\alpha_1}\,\Psi_{\Lambda 1}\,]\frac{(s+\alpha_1)}{a_1 s+1}$. If (25) holds then V_1 is unimodular. By (27), $C_{pd2} = (\alpha_1+p_1)\frac{(s-p_2)}{\tau_2\,s+1}X_{22}(0)^{-1} + \alpha_1\,(\alpha_2+p_2)X_{22}(0)^{-1}$. Let $V_{pd} := Y_{22} + \widehat{X}_{22}C_{pd} = \frac{(s-p_2)}{a_2 s+1}[\frac{(s-p_1)}{a_1 s+1}I +$

$(\alpha_1 + p_1)\frac{(a_2s+1)}{\tau_2 s+1}\widehat{X}_{22}(s)X_{22}(0)^{-1}] + \alpha_1(\alpha_2 + p_2)\widehat{X}_{22}(s)X_{22}(0)^{-1} = V_1[\frac{(s-p_2)}{a_2s+1}I + \alpha_1(\alpha_2 + p_2)V_1^{-1}\widehat{X}_{22}(s)X_{22}(0)^{-1}] =: V_1V_2$. Since V_1 is unimodular, V_{pd} is unimodular if and only if V_2 is unimodular, where $V_2 = \frac{(s-p_2)}{a_2s+1}[I + \frac{(a_2s+1)}{s-p_2}\alpha_1(\alpha_2 + p_2)V_1^{-1}\widehat{X}_{22}(s)X_{22}(0)^{-1}] = \frac{(s-p_2)}{a_2s+1}[I + \alpha_1(\alpha_2 + p_2)(I + \frac{(a_1s+1)}{\tau_2 s+1}\widehat{F}_2)^{-1}\widehat{G}_{22}(s)\,X_{22}(0)^{-1}] = [I + \frac{(\alpha_2+p_2)}{s+\alpha_2}(\alpha_1V_1^{-1}\widehat{X}_{22}(s)X_{22}(0)^{-1}(a_2s+1) - I)]\frac{(s+\alpha_2)}{a_2s+1} = [I + \frac{(\alpha_2+p_2)s}{s+\alpha_2}\Psi_{\Lambda 2}]\frac{(s+\alpha_2)}{a_2s+1}$. If (26) holds then V_2 is unimodular. Hence, C_{pd2} in (27) stabilizes $\widehat{G}_{22}$ and $H_{pd2} := V_{pd}^{-1}\widehat{X}_{22} = \widehat{G}_{22}(I + C_{pd2}\widehat{G}_{22})^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, where $H_{pd2}(0)^{-1} = G_{22}^{-1}(0) + K_{P2} = X_{22}(0)^{-1}Y_{22}(0) + (\alpha_1\alpha_2 - p_1p_2)X_{22}(0)^{-1} = \alpha_1\alpha_2X_{22}(0)^{-1}$. Using similar steps as in the proof of Proposition 3.1, the I-controller $K_{I2}/s = \gamma_2H_{pd2}(0)^{-1}/s$ stabilizes H_{pd2} for any $\gamma_2 \in \mathbb{R}$ satisfying (18). Therefore, $C_2 = C_{pd2} + K_{I2}/s$ in (28) stabilizes $\widehat{G}_{22}$. **b)** Let $p_i \in \mathcal{U} \setminus \mathbb{R}_+$. Define $y := (s + \delta_1 + g)(s + \delta_2 + g - f)$, where $g - f > 0$ by assumption. Let $x := y - n = (\delta_1 + \delta_2 + f + 2g)s + \delta_1\delta_2 + \delta_1(g - f) + \delta_2 g - fg$. Then $\|\frac{s\,x}{y}\| \leq (\delta_1 + \delta_2 + f + 2g)$, where $\dfrac{p_1 + p_2}{2} + 2\sqrt{p_1p_2} = f + 2g$. If (29) holds, then $\|\frac{s\,x}{y}\,\Psi_{\Lambda 1}\| \leq (\delta_1 + \delta_2 + f + 2g)\,\|\Psi_{\Lambda 1}\| < 1$ implies $V_{pd} := Y_{22} + \widehat{X}_{22}C_{pd2} = \frac{n}{d}\,[I + \widehat{G}_{22}C_{pd2}] = \frac{y}{d}\,[I + \frac{x}{y}\,\Psi_{\Lambda 1}]$ is unimodular. Hence, C_{pd2} in (30) stabilizes $\widehat{G}_{22}$ and $H_{pd2} := V_{pd}^{-1}\widehat{X}_{22} = \widehat{G}_{22}(I + C_{pd2}\widehat{G}_{22})^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, $H_{pd2}(0)^{-1} = G_{22}^{-1}(0) + K_{P2} = (g^2I + K_{P2})X_{22}(0)^{-1}$. For any $\gamma_2 \in \mathbb{R}$ satisfying (18), the I-controller $K_{I2}/s = H_{pd2}(0)^{-1}\gamma_2/s$ stabilizes H_{pd2}. Therefore, $C_2 = C_{pd2} + K_{I2}/s$ in (31) stabilizes $\widehat{G}_{22}$. Now we prove C_1 guarantees stability of the overall system, with C_2 as in (28) when $p_i \in \mathbb{R}_+$ and as in (31) when $p_i \notin \mathbb{R}_+$; hence $C_2\,(I + \widehat{G}_{22}C_2)^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. Write $C_2 = (N_2U_2^{-1} + D_2U_2^{-1})^{-1}$, with $U_2 := \frac{s}{s+\gamma_2}Y_{22} + \widehat{X}_{22}\frac{s}{s+\gamma_2}C_2$ unimodular. Then $\widehat{W} = \widehat{X}_{11} - (Y_{12}D_2 + \widehat{X}_{12}N_2)U_2^{-1}\widehat{X}_{21} \in \mathcal{M}(\mathcal{H}_\infty)$. Therefore, $U_D := \frac{s}{s+\beta}Y + \widehat{X}\frac{s}{s+\beta}C_D$ is unimodular for any $\beta \in \mathbb{R}_+$ if and only if $\frac{s}{s+b}I_{r1} + \frac{s}{s+b}\widehat{W}C_1$ is unimodular for $b > 0$, equivalently, C_1 stabilizes $\widehat{W}$. Designing C_1 for $\widehat{W}$ as in Proposition 3.1, C_D stabilizes $\widehat{G}$. Furthermore, C_D is partially reliable because $U_D = \begin{bmatrix} I_{r1} & Y_{12}D_2 + \widehat{X}_{12}N_2 \\ 0 & U_2 \end{bmatrix}$ is unimodular when $C_1 = 0$. ∎

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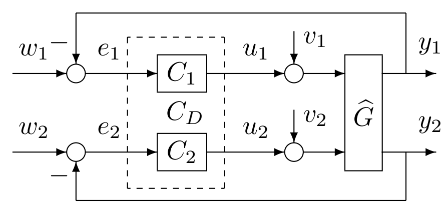

Fig. 1. Two-channel decentralized system $Sys(\widehat{G}, C)$ with I/O delays.

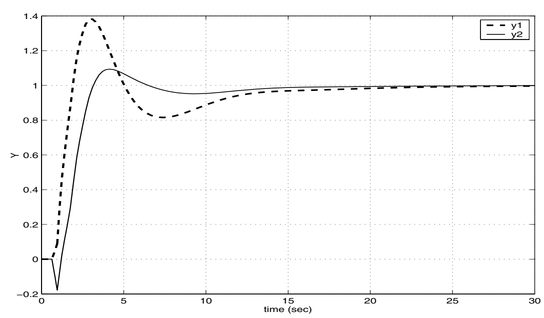

Fig. 2. Example 3.1 step-responses

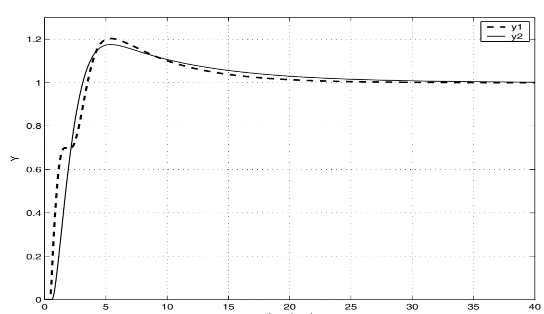

Fig. 3. Example 3.2 step-responses.