PI and low-order controllers for two-channel decentralized systems

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Abstract

A systematic design method is proposed for simple low-order decentralized controllers in the cascaded form of proportional-integral and first-order blocks. The plant is linear, time-invariant and has two channels, each with a single-input and single-output; there may be any number of poles in the region of stability, but the unstable poles can only occur at the origin.

1 Introduction

We consider simple, low order decentralized controller design with integral-action for linear, time-invariant (LTI) plants, whose unstable poles can only be at the origin. These plant models are common particularly in process control problems [1, 2]. The main result is the completely systematic design procedure for decentralized controllers with integral-action explicitly (Theorem 1). The proposed design method characterizes a class of controllers with one parameter completely free. In each of its two channels, the "nominal controller" has no unstable poles other than at $s = 0$ to satisfy the integral-action requirement. The stable poles are completely arbitrary. The nominal controller in each of the two channels is in the form of one proportional-integral (PI) block cascaded with first-order blocks (lead or lag controllers). The number of these cascaded blocks depends on the number of integrators in the plant. The nominal controller is low-order, with order independent of the number of stable plant poles. Decentralized controllers *without* integral-action can be obtained as a specialization of the result leading to stable controllers.

The results apply also to discrete-time systems with appropriate modifications. *Notation:* Let $\mathcal{U}$ be the extended closed right-half-plane. Real numbers, proper rational functions with real coefficients, proper rational functions with no unstable poles are denoted by $\mathbb{R}$, $\mathrm{R_p}$, $\mathbf{S}$; $\mathcal{M}(\mathbf{S})$ denotes matrices with all entries in $\mathbf{S}$; M is stable iff $M \in \mathcal{M}(\mathbf{S})$; $M \in \mathcal{M}(\mathbf{S})$ is unimodular iff $M^{-1} \in \mathcal{M}(\mathbf{S})$. A diagonal matrix whose entries are N_1 and N_2 is denoted by $\mathrm{diag}[N_1, N_2]$. For $M \in \mathcal{M}(\mathbf{S})$, the norm $\|\cdot\|$ is defined as $\|M\| = \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ denotes the maximum singular value and $\partial\mathcal{U}$ denotes the boundary of $\mathcal{U}$. The variable s is dropped from rational functions such as $P(s)$.

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2 Main Results

Consider the LTI, MIMO, 2-channel decentralized feedback system $\Sigma(P, C_D)$: $P, C_D \in \mathrm{R_p}^{2\times 2}$ are the transfer-functions of the plant and the decentralized controller, $C_D = \mathrm{diag}[C_1, C_2]$. It is assumed that $\Sigma(P, C_D)$ is well-posed and P and C_D have no hidden modes corresponding to eigenvalues in $\mathcal{U}$; P may have poles at $s = 0$; it does not have any other $\mathcal{U}$-poles. Let $\alpha > 0$ be an arbitrary but fixed real number and define $Z = \frac{s}{s+\alpha} \in \mathbf{S}$. Since the only $\mathcal{U}$-poles are at $s = 0$, P has a left-coprime-factorization (LCF) $P = D^{-1}N$ as:

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} Z^{m-1} & 0 \\ D_{21} & Z^{w-1} \end{bmatrix}^{-1} \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}, \quad (1)$$

where $m \geq 1$, $w \geq 1$ are integers, $N, D \in \mathcal{M}(\mathbf{S})$, D is in lower-triangular Hermite-form [4].

A decentralized $C_D = \mathrm{diag}[C_1, C_2]$ is an *integral-action controller* iff C_D stabilizes P and $\hat{D}_c(0) = 0$ for any RCF $C_D = N_c \hat{D}_c^{-1}$ [4, 3, 2]. Therefore, C_D is an integral-action controller if and only if $\hat{D}_c = ZD_c$ for some $D_c := \mathrm{diag}[D_1, D_2] \in \mathcal{M}(\mathbf{S})$. The decentralized integral-action controller $C_D = \mathrm{diag}[C_1, C_2]$, $C_j = N_j(ZD_j)^{-1}$ stabilizes P if and only if $T := ZD\mathrm{diag}[D_1, D_2] + N\mathrm{diag}[N_1, N_2]$ is unimodular.

Lemma 1: An integral-action controller exists for $P = D^{-1}N$ if and only if $N(0)$ is nonsingular.

Lemma 2: Let $G \in \mathbf{S}^{r\times\rho}$. For any integer $q \geq 1$, there exists $X \in \mathbf{S}^{\rho\times r}$ such that $Z^q I + GX$ is unimodular if and only if $\mathrm{rank}G(0) = r$. $\triangle$

The necessary condition $\mathrm{rank}N(0) = 2$, i.e., P has no transmission-zeros at $s = 0$, implies $(N_{11}N_{22} - N_{12}N_{21})(0) \neq 0$. If $N_{11} = 0$, then $N_{11} = Z^n G_1$ for some $G_1 \in \mathbf{S}$, $G_1(0) \neq 0$, where $n \geq 0$ is an integer; if $N_{11}(0) \neq 0$, then $G_1 = N_{11}$. The proposed controller design is stated as two cases depending on the number of zeros of N_{11} at $s = 0$. If $N_{11} = 0$, then define $\beta := m$ and $G_1 = 0$. If $N_{11} \neq 0$, then let $N_{11} =: Z^n G_1$ for some $G_1 \in \mathbf{S}$, $G_1(0) \neq 0$. Define $\beta := \min\{n, m\}$. Let $q_1 := m - \beta$ and $q_2 := w + \beta$. Define $\tilde{N}_1, \tilde{D}_1 \in \mathbf{S}$ as follows: **i)** If $\beta = m$, i.e., if $N_{11} = 0$ or if $m \leq n$, let $\tilde{N}_1 := \tilde{Q}_1$, $\tilde{D}_1 = (1 - Z^{(n-m)} G_1 \tilde{Q}_1)$, for some $\tilde{Q}_1 \in \mathbf{S}$ such that $\tilde{Q}_1(0) \neq 0$, and $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$. **ii)** If $\beta = n < m$, let $X_1 \in \mathbf{S}$ be such that $M_1 := Z^{q_1} + G_1 X_1$ is a unit and let $\tilde{N}_1 := X_1 M_1^{-1}$, $\tilde{D}_1 = M_1^{-1}$. With $\tilde{N}_1, \tilde{D}_1$ defined as above depending on $\beta = m$ or $\beta = n$, define $G_2 \in \mathbf{S}$ as $G_2 := Z^\beta N_{22} - N_{12}(ZD_{21}\tilde{D}_1 + N_{21}\tilde{N}_1)$.

Let $X_2 \in \mathbf{S}$ be such that $M_2 := Z^{q_2} + G_2 X_2$ is a unit; let $Y := N_{12}(ZD_{21}G_1 - Z^{(m-n)}N_{21}) \in \mathbf{S}$. The design procedure in Theorem 1 uses the following: *Step (1)*: **i)** If $\beta = m$, choose any $\tilde{Q}_1 \in \mathbf{S}$ such that $\tilde{Q}_1(0) \neq 0$, and $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$. Define $\tilde{N}_1 = \tilde{Q}_1$, $\tilde{D}_1 = (1 - Z^{(n-m)}G_1\tilde{Q}_1)$. **ii)** If $\beta = n$, construct $X_1 = \frac{1}{s+\alpha}h_{11}H_1\prod_{i=2}^{q_1}\frac{(s+h_{1i})}{(s+\alpha)}$ as in X_j below. Let $\tilde{N}_1 := X_1 M_1^{-1}$, $\tilde{D}_1 := M_1^{-1}$. *Step (2)*: Choose any $f_j \in \mathbb{R}$; define $H_j := f_j s + G_j(0)^{-1}$. Choose $h_{j1} \in \mathbb{R}$ satisfying $0 < h_{j1} < \|s^{-1}(G_j H_j - 1)\|^{-1}$. If $q_j > 1$, for $v = 2, \ldots, q_j$, choose $h_{jv} \in \mathbb{R}$ satisfying $0 < h_{jv} < \|s^{-1}(1 + G_j H_j \frac{h_{j1}}{s^{v-1}} \prod_{i=2}^{v-1}(s + h_{ji}))^{-1}\|^{-1}$; let $X_j := \frac{1}{s+\alpha}h_{j1}H_j\prod_{i=2}^{q_j}\frac{(s+h_{ji})}{(s+\alpha)}$, $M_j := Z^{q_j} + G_j X_j$.

Theorem 1: Let $P \in \mathrm{R_p}^{2\times 2}$, $P = D^{-1}N$ be an LCF as (1), and $\mathrm{rank}N(0) = 2$. A class of decentralized integral-action controllers $\{C_D = \mathrm{diag}[C_1, C_2]\}$ is obtained as follows: If $\beta = m$, design C_1 as

$$C_1 = \frac{s+\alpha}{s}\tilde{N}_1\tilde{D}_1^{-1} = \frac{s+\alpha}{s}\tilde{Q}_1(1 - Z^{n-m}G_1\tilde{Q}_1)^{-1}, \quad (2)$$

where $\tilde{Q}_1 \in \mathbf{S}$ is such that $\tilde{Q}_1(0) \neq 0$, and $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$. If $\beta = n$, design C_1 as in (3) below for $j = 1$. In both cases, design C_2 as in (3) below for $j = 2$:

$$C_j = \frac{(s+\alpha)}{s}(X_j + Z^{q_j}Q_j)(1 - G_jQ_j)^{-1}$$
$$= \frac{H_j h_{j1}}{s}\prod_{i=2}^{q_j}\frac{s + h_{ji}}{s+\alpha} + \frac{s+\alpha}{s}M_jQ_j(1 - G_jQ_j)^{-1}, \quad (3)$$

where $Q_1, Q_2 \in \mathbf{S}$; Q_1 also satisfies $\tilde{W} := 1 + Y(X_2 + Z^{q_2}Q_2)M_2^{-1}M_1^{-1}Q_1$ is a unit. The controller C_j is proper if and only if $Q_j(\infty) \neq G_j(\infty)^{-1}$ for $j = 1, 2$.

Comments: **1)** Let C_j in (3) with $Q_j = 0$ be called the "nominal controller" $C_{jo} := \frac{s+\alpha}{s}X_j = \frac{H_j h_{j1}}{s}\prod_{j=2}^{q_j}\frac{(s+h_{ji})}{(s+\alpha)}$, which has important properties justifying the significance and strength of the proposed design. For $j = 1, 2$, C_{jo} is designed to have a pole at $s = 0$ for the integral-action requirement; C_{jo} has no other unstable poles; it has $(q_j - 1)$ poles at $s = -\alpha$ (α is free). If $n < m$, when $q_j = 1$, C_{jo} is a PI controller. In general, C_{jo} is in the form of one PI block $H_j h_{j1}/s = f_j h_{j1} + G_j(0)^{-1}h_{j1}/s$, cascaded with $(q_j - 1)$ first-order blocks $(s + h_{ji})/(s + \alpha)$, $i = 2, \ldots, q_j$, designed when $q_j > 1$. The initial PI block is a pure integral controller $G_j(0)^{-1}h_{j1}/s$ for $f_j = 0$. Each subsequent first-order block is minimum-phase, with a pole at $s = -\alpha$ and a zero at $-h_{ji}$; these may be interpreted as lead or lag controllers depending on α and h_{ji} (since h_{ji} are typically small and α can be chosen arbitrarily large, they are all lead controllers). The order of C_{1o} is $q_1 = m - n$, which does not exceed the number of plant poles at $s = 0$ in channel-one; the order of C_2 is $q_2 = m + \beta$, which does not exceed the total number of plant poles at $s = 0$ in channel-one and channel-two. **2)** The controllers in (3) are biproper for any $Q_j \in \mathbf{S}$. If $\beta = m < n$, C_1 in (2) is strictly-proper if and only if $Q_1 \in \mathbf{S}$ is strictly-proper. Due to the integral-action requirement, C_j has poles at $s = 0$ for any $Q \in \mathbf{S}$; C_j has no other unstable poles if and only if $Q_j \in \mathbf{S}$ is such that $(1 - G_jQ_j)$ is a unit; it is sufficient to take $\|Q_j\| < \|G_j\|^{-1}$. In the case that $\beta = m < n$, C_1 in (2) has no unstable poles other than at $s = 0$ if and only if $Q_1 \in \mathbf{S}$ is such that $(1 - Z^{(n-m)}G_1Q_1)$ is a unit; it is sufficient to take $\|Q_1\| < \|G_1\|^{-1}$. **3)** The choice of the design parameter $Q_2 \in \mathbf{S}$ for C_2 in (3) is completely arbitrary (where C_2 is proper if and only if $Q_2(\infty) \neq G_2(\infty)^{-1}$). This freedom may be used to satisfy other design objectives. The choice of the design parameter $Q_1 \in \mathbf{S}$ for C_1 in (3) is restricted so that $\tilde{W}$ is a unit (where C_1 is proper if and only if $Q_1(\infty) \neq G_1(\infty)^{-1}$). While $Q_1 = 0$ obviously makes $\tilde{W}$ a unit, another sufficient condition is to choose $Q_1 \in \mathbf{S}$ such that $\|Q_1\| < \|Y(X_2 + Z^{q_2}Q_2)M_2^{-1}M_1^{-1}\|^{-1}$. **4)** Decentralized controllers *without* integral-action can be obtained from Theorem 1 simply by removing the Z^{-1} term from the controllers. In Theorem 1, substitute m by $(m-1)$, w by $(w-1)$, and re-define $G_2 := Z^{\beta}N_{22} - N_{12}(D_{21}\tilde{D}_1 + N_{21}\tilde{N}_1)$, $Y := N_{12}(D_{21}G_1 - Z^{(m-1-n)}N_{21})$. If $\beta = m - 1$, design $C_1 = \tilde{N}_1\tilde{D}_1^{-1} = \tilde{Q}_1(1 - Z^{n-(m-1)}G_1\tilde{Q}_1)^{-1}$, with $\tilde{Q}_1 \in \mathbf{S}$, $\tilde{Q}_1(0) \neq 0$, $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$. If $\beta = n$, design C_1 as C_j below. In both cases, design C_2 as in $C_j = (X_j + Z^{q_j}Q_j)(1 - G_jQ_j)^{-1} = \frac{H_j h_{j1}}{(s+\alpha)}\prod_{j=2}^{q_j}\frac{(s+h_{ji})}{(s+\alpha)} + M_jQ_j(1 - G_jQ_j)^{-1}$, where, for $j = 1, 2$, $Q_j \in \mathbf{S}$, $Q_j(\infty) \neq G_j(\infty)^{-1}$, $Q_1 \in \mathbf{S}$ also satisfies $\tilde{W} := 1 + Y(X_2 + Z^{q_2}Q_2)M_2^{-1}M_1^{-1}Q_1$ is a unit. The nominal $C_{jo} = X_j$ is stable, with q_j poles at $-\alpha$.

3 Conclusions

The proposed design method achieves closed-loop stability and robust asymptotic tracking of step-input references. The nominal controller for each of the two channels has a pole at $s = 0$ but no other unstable poles. It is designed as a low-order controller in the form of one PI block cascaded with stable minimum-phase first-order blocks. Unlike most standard full-order observer-based controller designs, the controller order is independent of the number of stable plant poles. This low-order property and the simple explicit definition of the controllers without any computation makes this a very desirable straightforward design procedure. Other tractable extensions of this systematic method include the case of decentralized systems with more than two channels and multiple inputs and outputs in each channel.

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