Now it is possible to choose such $d$ that for any $y_0 \in W$ the inequality $(M/\gamma)[\mu d + C[y_0]] \leq d$ is true. This means that operator $\mathcal{P}$ transforms the space $\ell^2$ into itself. Similarly
\[
\exp(\gamma t/\mu) |\mathcal{P}(\mathcal{H})(t, \hat{x}, \mu) - \mathcal{P}(\mathcal{H})(t, \hat{x}, \mu)| \\
\leq \exp(\gamma t/\mu) \int_t^\infty |\Phi(\theta, \phi(\theta, \mu) + \mathcal{H}, y(\theta, \mu), \mu) - \Phi(\theta, \phi(\theta, \mu) + \mathcal{H}, y(\theta, \mu), \mu)| \, d\theta \\
\leq \exp(\gamma t/\mu) M |\mathcal{H} - \mathcal{H}| \leq \mu \frac{M}{\gamma} \rho(\mathcal{H}, \mathcal{F})
\]
which means that operator $\mathcal{P}$ is a contraction operator on $\ell^2$. Then, the operator $\mathcal{P}$ has the unique fixed point corresponding to the function $\Pi x = \mathcal{H}(t, \hat{x}, \mu)$. Moreover, from (37), one can conclude that the inequality $|\mathcal{H}(t, \hat{x}, \mu)| < \mu d \exp(-\gamma t/\mu)$ holds for all $(t, \hat{x}, \mu) \in \mathbb{R}^+ \times \mathbb{R}^n \times (0, \mu_0]$.

Two-Channel Decentralized Integral-Action Controller Design

A. N. Gündüz and A. B. Özgüler

Abstract—We propose a systematic controller design method that provides integral-action in linear time-invariant two-channel decentralized control systems. Each channel of the plant is single-input-single-output, with any number of poles at the origin but no other poles in the instability region. An explicit parametrization of all decentralized stabilizing controllers incorporating the integral-action requirement is provided for this special case of plants. The main result is a design methodology that constructs simple low-order controllers in the cascaded form of proportional-integral and first-order blocks.

Index Terms—Decentralized control, integral-action, stability.

I. INTRODUCTION

We consider decentralized controller design with integral-action for linear time-invariant (LTI) plants, whose unstable poles can only occur at the origin. These plant models occur in many applications and are common in process control [7]. The decentralized controller structure is preferred for simplicity of implementation and the integral-action in the controllers achieves asymptotic tracking of step-input references applied at each input. We apply and explicitly define the parametrization of all decentralized controllers and incorporate integral-action into the controllers for this important class of plants, where the $2 \times 2$ plant transfer-function matrix may have simple or multiple poles at the origin in any or all of its entries.

The theory of decentralized control has produced relatively few systematic and explicit design methods despite the wide practical demand. The main difficulty is that the decentralized structure imposed on the free parameter of the set of all stabilizing controllers renders the optimization problem nonconvex [10]. Alternatively, when viewed as a problem of making the plant stabilizable and detectable from one of its channels, the decentralized stabilizing controllers are constructed relying on genericity arguments [2], [9], [12]. The decentralized controller parametrizations obtained previously (see, for example [5] and [8]) all characterize controllers at the conceptual level and do not provide explicit descriptions. The usual computational methods that would be used to convert such conceptual designs to explicit descriptions would typically produce unnecessarily high-order controllers since the standard (robust) control designs are not tailored to special type of plants as considered here.

The integral-action problem for the case of stable plants has been considered in the decentralized setting with single-input–single-output channels in [7], and [1], and design procedures were proposed for achieving reliable stability under the possible failure of controllers in [6]. For the case of unstable plants, controller designs were presented in [3] based on choosing the free design parameter to achieve a desired sensitivity function for a suitable diagonal or triangular model of the plant. However, explicit decentralized integral-action controller designs for plants with integrators are not available.

REFERENCES

[12] I. A. Shkolnikov and Y. B. Shtessel, “Tracking a class of nonminimum sliding modes in control and optimization systems. Each channel of the plant is single-input-single-output, with any number of poles at the origin but no other poles in the instability region. An explicit parametrization of all decentralized stabilizing controllers incorporating the integral-action requirement is provided for this special case of plants. The main result is a design methodology that constructs simple low-order controllers in the cascaded form of proportional-integral and first-order blocks.

I. INTRODUCTION

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Decentralized designs such as the reliable controller design described in [6] developed for stable plants obviously cannot be applied to plants with poles at the origin. Since only some of the entries of the plant transfer-function matrix may have poles at the origin or these poles may appear with different multiplicities, the integrators in the unstable plant cannot be extracted and incorporated into the controller, i.e., the plant $P$ cannot simply be expressed as $P = (1/s)\hat{P}$ with $\hat{P}$ stable, and a controller of the form $(1/s)C$ cannot be designed for the resulting stable $\hat{P}$ following the methods in [6]. Note that $P = (1/s)\hat{P}$ would result in improper $\hat{P}$ except when $P$ is strictly-proper, but more importantly, would generally mean $\hat{P}$ has transmission-zeros at the origin and cannot be (internally) stabilized by $(1/s)C$. Furthermore, reliable design as described in [6] assumes controllers may fail arbitrarily; the integrators of the channel with the failure would not be compensated by feedback and hence, reliable stabilization is not attempted in this case of unstable plants. Therefore, an entirely different methodology is developed here for this important class of plants with poles at the origin.

The main results here are the explicit parametrization of all decentralized controllers with integral-action (Theorem 1), and the completely systematic design procedure that defines all controller transfer-functions explicitly (Theorem 2). The significance and strength of the proposed design method can be explained as follows. 1) The set of all controllers (Theorem 1) is described based on two “semi-free” parameters. 2) A subclass of controllers is characterized with one parameter completely free (Theorem 2). 3) In each of its two channels, the “nominal controller” (Theorem 2) has no unstable poles other than at $s = 0$, which it contains by design to satisfy the integral-action requirement. The location of the stable poles is completely arbitrary. 4) The nominal controller in each of the two channels is in the form of one proportional-integral (PI) block cascaded with first-order blocks (lead or lag controllers). The number of these cascaded blocks depends on the number of integrators in the plant. 5) The nominal controller is a low-order controller, with order independent of the number of stable plant poles. 6) The parametrization of all decentralized controllers without integral-action derived from Theorem 1 leads to stable controllers so the proposed design achieves strong stabilization.

The design method is illustrated by an example, where the plant is the linearized model of a sugar mill process [3], [4]. Two of the entries of the $2 \times 2$ transfer-function matrix each have a simple pole at the origin. A PI controller is designed for the first channel and a PI cascaded with one lead block is designed for the second channel.

**Notation:** Let $\mathcal{U}$ be the extended closed right-half plane (for continuous-time systems) or the complement of the open unit disk (for discrete-time systems). The sets of real numbers, proper rational functions with no unstable poles are denoted by $\mathbb{R}$, $\mathbb{P}$, $\mathcal{S}$. The set of matrices with all entries in $\mathcal{S}$ is denoted by $\mathcal{M}(\mathcal{S})$; $M$ is called stable iff $M \in \mathcal{M}(\mathcal{S})$; a square matrix $M \in \mathcal{M}(\mathcal{S})$ is unimodular iff $M^{-1} \in \mathcal{M}(\mathcal{S})$; $M \in \mathcal{S}$ is a unit in $\mathcal{S}$ if $M^{-1} \in \mathcal{S}$. A diagonal matrix whose entries are $N_1$ and $N_2$ is denoted by $\text{diag}[N_1, N_2]$. For $M \in \mathcal{M}(\mathcal{S})$, the norm $||M||$ is defined as $||M|| = \sup_{s \in \mathcal{S}} ||\sigma(M(s))||$, where $\sigma$ denotes the maximum singular value and $\mathcal{U}$ denotes the boundary of $\mathcal{U}$. For simplicity, the variable $s$ is dropped and rational functions such as $P(s)$ are denoted by $P$.

Our discussion here is constrained to continuous-time systems although the results apply also to discrete-time systems with appropriate modifications.

**II. ANALYSIS**

Consider the LTI, multiple-input–multiple-output, two-channel decentralized feedback system $\Sigma(P, C_D)$ shown in Fig. 1: $P \in \mathbb{R}_{p}^{2 \times 2}$ and $C_D \in \mathbb{R}_{p}^{2 \times 2}$ represent the transfer-functions of the plant and the decentralized controller, partitioned as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \quad C_D = \text{diag}[C_1, C_2].$$

(1)

It is assumed that $\Sigma(P, C_D)$ is a well-posed system (i.e., all closed-loop transfer-functions are proper), and that $P$ and $C_D$ have no hidden modes corresponding to eigenvalues in $\mathcal{U}$. The plant $P \in \mathbb{R}_{p}^{2 \times 2}$ may have poles at $s = 0$ but it does not have any other $\mathcal{U}$-poles. Let $\alpha > 0$ be an arbitrary but fixed real number and define $Z \in \mathcal{S}$ as

$$Z = \frac{s}{s + \alpha}.$$  

(2)

Since the only $\mathcal{U}$-poles are at $s = 0$, the plant $P$ has a left-coprime factorization (LCF) $P = D^{-1}N$ of the form

$$P = \begin{bmatrix} Z^{m_1 - 1} & 0 \\ D_{21} & Z^{n_1 - 1} \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$$

(3)

where $m \geq 1$, $w \geq 1$ are integers, $N, D \in \mathcal{M}(\mathcal{S})$, $D$ is in lower-triangular Hermite form [11].

A decentralized controller $C D = \text{diag}[C_1, C_2]$ is said to be an integral-action controller iff $C_D$ stabilizes $P$ and $D_r(0) \neq 0$ for any right-coprime factorization (RCF) $C_D = N_r D_r^{-1}$ [11], [7]. Let $C_D = N_r D_r^{-1} := \text{diag}[N_1, N_2] \text{diag}[D_r^{-1}, D_r^{-1}], D_r(\infty) \neq 0$, be any RCF over $\mathcal{S}$ of $C_D = \text{diag}[C_1, C_2]$. Therefore, $C_D = N_r D_r^{-1}$ is an integral-action controller if and only if $D_r = ZD_r$ for some $D_r := \text{diag}[D, D] \in \mathcal{M}(\mathcal{S})$. This implies $C_D = N_r(ZD_r)^{-1}$ is a decentralized integral-action controller for $P$ if and only if $N, D^{-1}$ is a decentralized stabilizing controller for $Z^{-1}P$.

**Lemma 1:** An integral-action controller exists for $P = D^{-1}N$ if and only if $N(0)$ is nonsingular.△

By Lemma 1, a necessary condition due to the integral-action requirement is that $\text{rank}(N(0)) = 2$. The decentralized integral-action controller $C_D = \text{diag}[C_1, C_2], C_j = N_j(ZD_j)^{-1}$, stabilizes the plant $P$ if and only if $T$ in (4) is unimodular

$$T := ZD \text{diag}[D_1, D_2] + N \text{diag}[N_1, N_2]$$

$$= \begin{bmatrix} Z^{n_1} D_1 + N_{11} N_{12} N_{22} \\ Z D_{21} D_1 + N_{21} N_{12} + Z^n D_2 + N_{22} N_{22} \end{bmatrix}.$$  

(4)

The controller design problem here is to determine $D_r, N_r \in \mathcal{S}$ such that $T$ in (4) is unimodular. The following lemma is used to construct simple explicit solutions for $D_r, N_r \in \mathcal{S}$ and these solutions are used in parametrizing all decentralized integral-action controllers for $P$.

**Lemma 2:** Let $G \in \mathcal{S}^{r \times r}$. For any integer $q \geq 1$, there exists $X \in \mathcal{S}^{r \times r}$ such that $Z^q I + GX$ is unimodular if and only if $\text{rank}(G(0)) = r$.

**III. DESIGN**

In this section, we propose design methods for two-channel decentralized integral-action controllers. The necessary condition $\text{rank}(N) = 2$, i.e., $P$ has no transmission-zeros at $s = 0$, implies $(N_{11} N_{22} - N_{12} N_{21})(0) \neq 0$. In (3), the diagonal entry $N_{11}$ may or may not be identically zero or zero at $s = 0$. If $N_{11} \neq 0$, then it is

![Fig. 1. The two-channel decentralized system $\Sigma(P, C_D)$.](image-url)
expressed as $N_{11} = Z^s G_1$ for some $G_1 \in \mathcal{S}$, $G_1(0) \neq 0$, where $n \geq 0$ is an integer corresponding to the number of zeros of $N_{11}$ at $s = 0$; if $N_{11}(0) \neq 0$, then $G_1 = N_{11}$.

Theorem 1 gives a complete parametrization of all two-channel decentralized integral-action controllers for $P$, stated as two cases depending on the number of zeros of $N_{11}$ at $s = 0$. If $N_{11}(0) = 0$, then define $\beta = m$ and $G_1 = 0$. If $N_{11}(0) \neq 0$, then let $N_{11} := Z^s G_1$ for some $G_1 \in \mathcal{S}$, $G_1(0) \neq 0$. Define $\beta := \min \{n, m\}$, $q_1 = m - \beta$ and $q_2 := \beta + \beta$. Define $N_1, D_1 \in \mathcal{S}$ as follows.

i) If $\beta = m$, i.e., if $N_{11}(0) = 0$ or if $m \leq n$, let

$$N_1 = \tilde{Q}_1, \quad D_1 = \left(1 - Z^{(n-m)} G_1 \tilde{Q}_1\right)(5)$$

for some $\tilde{Q}_1 \in \mathcal{S}$ such that $\tilde{Q}_1(0) \neq 0$, and $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$.

ii) If $\beta \neq m$, choose any $\tilde{Q}_1 \in \mathcal{S}$ such that $\tilde{Q}_1(0) \neq 0$, and $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$. Define $\tilde{N}_1 = \tilde{Q}_1, D_1 = (1 - Z^{(n-m)} G_1 \tilde{Q}_1)$ as in (5).

ii) If $\beta = n$, construct $X_1 \in \mathcal{S}$ satisfying (6) as in (14) for $j = 1$.

Define $N_1 = X_1 M_1^{-1}, D_1 = M_1^{-1}$ as in (6). With $N_1, D_1$ defined as (5) when $\beta = m$ or as (6) when $\beta = n$, with $X_1$ is constructed as in (14) for $j = 1$, define $G_2$ as (7). Construct $X_2 \in \mathcal{S}$ satisfying (8) as in (15) for $j = 2$.

**Step 2:** Choose any $f_j \in \mathcal{R}$; define $H_j := f_j s + G_j(0)^{-1}$. Choose $h_j, h_{j+1} \in \mathcal{R}$ satisfying (12)

$$0 < h_j < \left\| s^{-1}(G_j H_j - 1) \right\|^{-1}.$$ (12)

If $q_j > 1$, for $v = 2, \ldots, q_j$, choose $h_{vj} \in \mathcal{R}$ satisfying (13); let $X_j, M_j \in \mathcal{S}$ be as in (14) and (15)

$$0 < h_{vj} < \left\| s^{-1} \left(1 + G_j H_j h_{vj} \prod_{i=2}^{j-1} (s + h_i) \right)^{-1} \right\|^{-1}.$$ (13)

$$X_j = \frac{1}{s + \alpha} h_j H_j \sum_{i=2}^{j} \left(\frac{s + h_i}{s + \alpha}\right)$$ (14)

$$h_j := f_j s + G_j(0)^{-1}, M_j := Z^{q_j} + G_j X_j.$$ (15)

**Theorem 2 (Decentralized Integral-Action Controller Design):** Let the assumptions of Theorem 1 hold. A class of decentralized integral-action controllers $\{C_d = \text{diag}[C_1, C_2]\}$ is obtained as follows: if $\beta = m$, design $C_1$ as

$$C_1 = \frac{s + \alpha}{s} \hat{N}_1 \hat{D}_1^{-1} = \frac{s + \alpha}{s} \tilde{Q}_1 \left(1 - Z^{(n-m)} G_1 \tilde{Q}_1\right)^{-1}$$ (16)

where $\tilde{Q}_1 \in \mathcal{S}$ is such that $\tilde{Q}_1(0) \neq 0$, and $\tilde{Q}_1(\infty) \neq G_1(\infty)^{-1}$.

If $\beta = n$, design $C_1$ as (17) for $j = 1$.

In both cases, design $C_2$ as in (17) for $j = 2$.

$$C_j = \frac{s + \alpha}{s} \left(X_j + Z^{q_j} Q_j\right) (1 - G_j Q_j)^{-1}$$ (17)

where $Q_j, Z_j \in \mathcal{S}$; $Q_j \in \mathcal{S}$ is also such that

$$W := 1 + Y (X_2 + Z^{q_2} Q_2) M_1^{-1} M_2^{-1} Q_1$$ (18)

is a unit. The controller $C_j$ is proper if and only if $Q_j(\infty) \neq G_j(\infty)^{-1}$ for $j = 1, 2$.

**Comments 1:**

1) PI and first-order cascade structure of the controllers: Let $C_j$ in (17) obtained by setting $Q_j = 0$ be called the “nominal controller” $C_{j*}$ shown in (19)

$$C_{j*} = \frac{s + \alpha}{s} X_j = \frac{h_j h_{j+1}}{s} \sum_{i=2}^{j} \left(\frac{s + h_i}{s + \alpha}\right)$$ (19)

This controller has important properties justifying the significance and strength of the proposed design. For $j = 1, 2$, $C_{j*}$ is designed to have a pole at $s = -\alpha$ to satisfy the integral-action requirement; $C_{j*}$ has no other unstable poles and it has $(q_j-1)$ poles at $s = -\alpha$, where $\alpha$ is completely free. If $n < m$, when $q_j = 1$, $C_{j*}$ is simply a PI controller. In general, $C_{j*}$ is in the form of one PI block $H_j h_{j*} s / s = f_j h_{j*} + G_j(0)^{-1} h_{j*} s$, cascaded with $(q_j-1)$ first-order blocks $(s + h_{j*}) / (s + \alpha)$, $i = 2, \ldots, q_j$, designed as needed when $q_j > 1$. The initial PI block can be designed as a pure integral controller $G_j(0)^{-1} h_{j*} s$ by choosing $f_j = 0$. Each subsequent first-order block is minimum-phase, with a pole at $s = -\alpha$ and a zero at $-h_{j*}$; these may be interpreted as lead or lag controllers depending on $\alpha$ and $h_{j*}$ (they would likely all be lead controllers since $h_{j*}$ satisfying (13) are typically small and $\alpha$ can be chosen arbitrarily large at the beginning of the design procedure).
The order of $C_{1v}$ is $q_1 = m - n$, which does not exceed the number of unstable poles of the plant in channel-one; the order of $C_2$ is $q_2 = m + \beta$, which does not exceed the total number of unstable poles of the plant in channel-one and channel-two (these unstable poles are all at $s = 0$ here). This low-order controller design where the controller order is independent of the number of stable poles of the plant has obvious advantages over full-order observer based controller designs.

2) Properties of the proposed controller class: The controllers $C_{ij}$ in (17), expressed as $C_{ij} = C_{ij,0} + (s + \alpha)/s(Z^{ij} + G_iX_i)Q_i(1 - G_iQ_i)^{-1}$, are biproper for any choice of the stable parameter $Q_i$, because $X_i$ is biproper by design. If $\beta = m < n$, $C_1$ in (16) is strictly-proper if and only if $Q_1 \in S$ is strictly-proper. Due to the integral-action requirement, $C_{ij}$ have poles at $s = 0$ for any $Q \in S$; $C_j$ can be restricted to have no other unstable poles if and only if $Q_1 \in S$ is such that $(1 - G_jQ_1)$ is a unit; it is sufficient to take $\|Q_1\| < \|C_{ij}\|^{-1}$. In the case that $\beta = m < n$, $C_1$ in (16) has no unstable poles other than at $s = 0$ if and only if $Q_1 \in S$ is such that $(1 - Z^{i(m-n)}G_iQ_1)$ is a unit; it is sufficient to take $\|Q_1\| < \|G_i\|^{-1}$.

3) Freedom in the design parameter: The choice of the design parameter $Q_2 \in S$ for $C_2$ in (17) is completely arbitrary [where $C_2$ is proper if and only if $Q_2(\infty) \neq G_2(\infty)^{-1}$]. This freedom may be used to satisfy other design objectives. The choice of the design parameter $Q_1 \in S$ for $C_1$ in (17) is restricted so that $W$ is a unit [where $C_1$ is proper if and only if $Q_1(\infty) \neq G_1(\infty)^{-1}$]. While $Q_1 = 0$ obviously makes $W$ a unit, another sufficient condition is to choose $Q_1 \in S$ such that $\|Q_1\| < \|X_i - ZQ_2M_JM_i\|^{-1}$.

4) Design without integral action in the controllers: The integral-action in the controllers is due to the Z term in the denominators of $C_{ij}$. It is obvious that the parametrization of all decentralized controllers without-integral-action can be obtained from Theorem 1 simply by removing the $Z^{-1}$ term from the controllers. We outline the parametrization and design for this case. The decentralized controller $C_{ij} = \text{diag}[C_{ij,1}, C_{ij,2}]$ is a unit, stabilizes the plant $P$ and if only if $T = D \text{diag}[D_1, D_2] + N \text{diag}[N_1, N_2]$ is unimodular. Since dropping the integral-action requirement from the controllers reduces the number of integrators by one, in Theorems 1 and 2, substitute $m$ by $(m - 1)$, $w$ by $(w - 1)$, and re-define $G_{ij} = Z^{i(m-n)} + (w + 1)N_{ij}$, $Y := N_{ij}(D_2G_1D_1 + N_2N_1)$, and then all decentralized controllers are obtained from (11). In Theorem 2, if $\beta = m - 1$, design $C_1 = \text{diag}[D_1, D_2] = \text{diag}[1 - Z^{-1}]$, with $\hat{Q}_1 \in S$, $\hat{Q}_1(0) \neq 0$, $\hat{Q}_1(\infty) \neq G_1(\infty)^{-1}$. If $\beta = n$, design $C_1$ in (20) for $j = 1$. In both cases, design $C_2$ in (20)

$$C_j = (X_j + Z^{i2}Q_j)(1 - G_jQ_j)^{-1}$$

$$= \frac{H_{ij}(s + \alpha)}{(s + \alpha)} \prod_{j=2}^{j} \frac{M_j}{(s + \alpha)} + M_jQ_j(1 - G_jQ_j)^{-1}$$

where, for $j = 1, 2$, $Q_j \in S$, $Q_j(\infty) \neq G_2(\infty)^{-1}$, $Q_1 \in S$ also satisfies $W$ in (18) is a unit. Since the term $(s + \alpha)/s$ is now removed from the controllers, the nominal decentralized controller $C_{ij}$ is stable, with $q_2$ poles at $s = -\alpha$. This design is in the form of $q_2$ cascaded stable first-order blocks. The initial block $H_{ij}(s + \alpha)$ has a zero at $s = -G_jQ_j(0)/f_j$ (negative if we choose $f_j$ with the same sign as $G_j(0)$). It is followed by $(q_2 - 1)$ minimum-phase blocks $(s + \alpha)/f_j$, $i = 2, \ldots, q_2$, each with a pole at $s = -\alpha$ and a zero at $-h_{ij}$. These blocks may be interpreted as lead or lag controllers. The nominal controllers in this design are strongly stabilizing; they can even be made units by choosing $f_j$ appropriately.

Example 1 (Control of a Sugar Mill): We apply the design in Theorem 2 to the linearized model of a sugar mill process [3], [4]. The two-input–two-output plant and an LCF $P = D^{-1}N$ as (3) are

$$P = \begin{bmatrix}
-5 & s^2 - 0.005s - 0.003 \\
25s + 1 & s(s + 1) \\
1 & 25s + 1 \\
-0.0023 & s
\end{bmatrix}$$

This freedom may be used to satisfy other design objectives. The choice of the design parameter $Q_1 \in S$ for $C_1$ in (17) is restricted so that $W$ is a unit [where $C_1$ is proper if and only if $Q_1(\infty) \neq G_1(\infty)^{-1}$]. While $Q_1 = 0$ obviously makes $W$ a unit, another sufficient condition is to choose $Q_1 \in S$ such that $\|Q_1\| < \|X_i - ZQ_2M_JM_i\|^{-1}$.

IV. CONCLUSION

We presented a systematic method to explicitly design decentralized controllers with integral-action for two-channel plants that have integrators of any multiplicity in one or more entries of the $2 \times 2$ transfer-function matrix. The design achieves closed-loop stability and robust asymptotic tracking of step-input references. The nominal controller of the proposed class for each of the two channels has a pole at $s = 0$ but no other unstable poles. It is designed as a low-order controller in the form of one PI block cascaded with stable minimum-phase first-order blocks. Unlike most standard full-order observer-based controller designs, the controller order is independent of the number of stable plant poles. This low-order property and the simple explicit definition of the controllers without any computation makes this a very desirable straightforward design procedure.

In some cases the plant may have stable poles that could be considered undesirable. In Example 1, the plant pole at $s = -0.04$ appears as a pole in the closed-loop system as well since the instability region $\mathcal{H}$

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is the extended closed right-half-plane. If the stability region is de-
mixed to exclude such poles in order to achieve better performance, it
may be possible to modify the design method and extend it to include
plants with unstable poles in addition to those at the origin.

Other tractable extensions of the results presented here include the
case of decentralized systems with more than two channels and mul-
tiple inputs and outputs in each channel.

APPENDIX

Proof of Lemma 1: If \( N(0) \) is nonsingular, then \( (DZ)^{-1}N \) is an
LCF of \( Z^{-1}P \). By standard results on decentralized fixed-modes
[12], [9], it follows that \( s = 0 \) is not a decentralized fixed-mode of
\( (DZ)^{-1}N \). Hence, decentralized stabilizing controllers exist for
\( (DZ)^{-1}N \). The necessity follows from (4); if the decentralized inte-
gral-action controller \( CD_0 \) stabilizes the plant \( P \), then \( T \) unimodular
implies \( \text{rank} T(0) = \text{rank} (NN_1)(0) = \text{rank} N(0) \).

Proof of Lemma 2: If \( Z^{-1}I + GX = M \) is unimodular, then \( \text{rank} M(0) = \text{rank} (NN_1)(0) = r \leq \min \{ \text{rank} (GN_1), \text{rank} X(0) \} \). Conversely, if \( \text{rank} G(0) = r \), then \( X \) can be constructed as follows. Let \( G(0)^t \in \mathbb{R}^{r \times r} \) denote (any)
right-inverse of \( G(0) \in \mathbb{R}^{r \times r} \). Choose any \( F \in \mathbb{R}^{r \times r} \); define
\( H := Fs + G(0)^t \). Choose \( h \in \mathbb{R} \) and define \( M_1 \) as
\[
0 < h_1 < \| s^{-1}(G/H - I) \|^{-1}_2 \Rightarrow \frac{s}{s + h_1} I + \frac{G}{s + h_1} \Rightarrow ZI + G \tilde{X} \tag{22}
\]
for any \( h_1 \) satisfying (22). \( \tilde{M}_1 = \frac{s + h_1}{s} \frac{I}{s + h_1} \frac{G}{s + h_1} \Rightarrow ZI + G \tilde{X} \)
then, for any \( h_1 \) satisfying (22), \( \tilde{M}_1 = I + (s + h_1)^{-1} sh_1 [s^{-1}(GH - I)](s + h_1)^{-1} \frac{I}{s + h_1} \Rightarrow ZI + G \tilde{X} \tag{22}
\]
the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23). Therefore, the product \( \tilde{M}_2 \tilde{M}_1 \) is unimodular for any \( h_2 \) satisfying (23).

Proof of Theorem 2: The proposed controllers are obtained from
Theorem 1 by choosing \( R_{ij}, R_j, j = 1, 2 \) as follows. If \( \beta = \gamma \),
choose \( R_{ij} = 1, R_1 = 0, R_{22} = 1, R_2 = Q_2 M_2^{-1} \); then \( W = 1 \) is a unit. If \( \beta < \gamma \), choose \( R_{ij} = 1 + Y_{ij} X_2 X_1 Q_1^{-1} \), \( R_1 = H \), \( R_3 = Q_3^{-1} M_2^{-1} \); then \( W = 1 + Y_{ij} X_2 + Z_{ij} Q_2^{-1} M_2^{-1} Q_1^{-1} \) is a unit due to the choice of \( Q_1 \) in \( S \). With this \( R_{ij}, R_1 \), we have \( N_j = (X_j - Z_{ij} Q_2^{-1} M_2^{-1} D_j = (1 - G_{ij} Q_2^{-1} M_2^{-1}) \). It was shown in the
proof of Theorem 1 that \( G_{ij}(0) \neq 0, j = 1, 2 \). It follows that
\( M_2 \) is a unit for \( X_j \) in (14) constructed according to Step 2 by
applying the proof of Lemma 2 to \( G_{ij}, j = 1, 2 \) (where \( G_{ij} \) is scalar,
i.e., \( \rho = 1 \)).

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