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Two-channel decentralized controller design with integral action[1]

A. N. Gündeş and M. G. Kabuli
Electrical and Computer Engineering, University of California, Davis, CA 95616
gundes@ece.ucdavis.edu kabuli@ece.ucdavis.edu

Abstract

Stabilizing controller design with integral action is considered for linear time-invariant, multi-input multi-output, two-channel decentralized systems with stable plants. Design for reliable stabilization with integral action is also considered, where the goal is to maintain closed-loop stability and integral action in the active channel when both controllers act together and when each controller acts alone.

1 Introduction

Decentralized stabilizing controller design with integral action is considered for linear time-invariant (LTI), multi-input multi-output (MIMO), two-channel decentralized systems. The objective is to achieve closed-loop stability with (at least) type-1 integral action in each output channel so that step-input references applied at each input are asymptotically tracked (with zero steady-state error). Reliable stabilization with integral action is also considered, where the goal is to maintain closed-loop stability when both controllers act together and when each controller acts alone. The plant is stable. The failure model assumes that a controller that fails is replaced by zero; the failure is recognized and the corresponding controller is taken out of service (i.e., the states in the controller implementation are all set to zero, the initial conditions and the outputs of the channel that failed are set to zero for all inputs). Integral action is present in the outputs of the channel with the active controller due to its integrators. The results apply to continuous-time and discrete-time systems. Although a continuous-time setting was assumed here for simplicity, all evaluations and discussions involving poles and zeros at $s = 0$ should be interpreted at $z = 1$ in the discrete-time case.

Notation and algebraic framework: Let $\mathcal{U}$ be the extended closed right-half-plane (for continuous-time systems) or the complement of the open unit-disk (for discrete-time systems). The sets of real numbers, proper rational functions with no poles in the region of instability $\mathcal{U}$, proper and strictly-proper rational functions with real coefficients are denoted by $\mathbb{R}$, $\mathcal{R}$, $\mathrm{R_p}$, $\mathrm{R_s}$; $\mathcal{M}(\mathcal{R})$ denotes the set of matrices whose entries are in $\mathcal{R}$; M is called stable iff $M \in \mathcal{M}(\mathcal{R})$; $M \in \mathcal{M}(\mathcal{R})$ is called unimodular iff $M^{-1} \in \mathcal{M}(\mathcal{R})$. The notation $\mathrm{diag}[N_1, N_2]$ denotes a block-diagonal matrix. For $M \in \mathcal{M}(\mathcal{R})$, $\|M\| := \sup_{s \in \partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ denotes the maximum singular value, $\partial\mathcal{U}$ denotes the boundary of $\mathcal{U}$.

2 Analysis and Design

Consider the LTI, MIMO, two-channel decentralized feedback system $\mathcal{S}(P, C_D)$ in Figure 1: $\mathcal{S}(P, C_D)$ is well-posed, where $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \in \mathcal{R}^{n_y \times n_u}$, $C_D = \mathrm{diag}[C_1, C_2] \in \mathrm{R_p}^{n_u \times n_y}$ are the transfer-functions of the plant and the decentralized controller, $P_{jj} \in \mathcal{R}^{n_{yj} \times n_{uj}}$, $C_j \in \mathrm{R_p}^{n_{uj} \times n_{yj}}$, $j = 1, 2$, $n_y = n_{y1} + n_{y2}$, $n_u = n_{u1} + n_{u2}$. It is assumed that P and C_D have no hidden modes corresponding to eigenvalues in $\mathcal{U}$. When a controller fails, it is set equal to zero; the failure is recognized and the corresponding controller is taken out of service. When $C_2 = 0$, the system is called $\mathcal{S}(P, C_1)$; when $C_1 = 0$, it is called $\mathcal{S}(P, C_2)$. Let H_{er} denote the (input-error) transfer-function from r to e, where $r := [r_1^T \;\; r_2^T]^T$, $e := [e_1^T \;\; e_2^T]^T$. Let H denote the transfer-function from (r, u) to (y, y_c), where $u := [u_1^T \;\; u_2^T]^T$, $y := [y_1^T \;\; y_2^T]^T$, $y_c := [y_{c1}^T \;\; y_{c2}^T]^T$. In $\mathcal{S}(P, C_1)$, the outputs y_{c2} of the second control channel are not observed; in $\mathcal{S}(P, C_2)$, the outputs y_{c1} are not observed. For $j = 1, 2$, let H_j denote the transfer-function of $\mathcal{S}(P, C_j)$ from (r_j, u) to (y, y_{Cj}).

2.1. Definitions [2, 1]: The system $\mathcal{S}(P, C_D)$ is *stable* iff $H \in \mathcal{M}(\mathcal{R})$. For $j = 1, 2$, $\mathcal{S}(P, C_j)$ is stable iff $H_j \in \mathcal{M}(\mathcal{R})$. The stable $\mathcal{S}(P, C_D)$ has *integral action* in each output channel iff $H_{er}(0) = 0$. For $j = 1, 2$, the stable $\mathcal{S}(P, C_j)$ has integral action iff the transfer-function from r_j to e_j, $H_{ej,rj}(0) = 0$. The decentralized controller $C_D = \mathrm{diag}[C_1, C_2]$ is a *stabilizing controller* for the plant P (or C_D is said to stabilize P) iff $C_D \in \mathcal{M}(\mathrm{R_p})$ and the system $\mathcal{S}(P, C_D)$ is stable; C_D is a *stabilizing controller with integral action* iff C_D stabilizes P, and $\mathcal{S}(P, C_D)$ has integral action, i.e., $H_{er}(0) = 0$; C_D is a *reliable stabilizing controller with integral action* iff all three systems $\mathcal{S}(P, C_D)$, $\mathcal{S}(P, C_1)$ and $\mathcal{S}(P, C_2)$ are stable and have integral action. □

Let $C_D = N_c D_c^{-1}$ be a right-coprime-factorization (RCF) of $C_D \in \mathrm{R_p}^{n_u \times n_y}$ ($N_c, D_c \in \mathcal{M}(\mathcal{R})$, $\det D_c(\infty) \neq 0$). The controller C_D stabilizes $P \in \mathcal{M}(\mathcal{R})$ if and only if $(D_c + P N_c)$ is unimodular [2]; $H_{er} = 0$ if and only if $D_c(0) = 0$. Let $N_j \hat{D}_j^{-1}$ be any RCF of C_j, $j = 1, 2$; $N_c := \mathrm{diag}[N_1, N_2]$,

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$D_c := \mathrm{diag}[\hat{D}_1, \hat{D}_2]$; then $N_c D_c^{-1}$ is an RCF of C_D; $D_c(0) = 0$ if and only if $\hat{D}_j = \frac{s}{s+\alpha} D_j$ for some $D_j \in \mathcal{M}(\mathcal{R})$, where $-\alpha \in \mathbb{R} \backslash \mathcal{U}$. In the stable $\mathcal{S}(P, C_D)$, $H_{er}(0) = 0$ implies $\mathrm{rank}P(0) = n_y \le n_u$; $\hat{D}_j(0) = 0$ implies $\mathrm{rank}N_j(0) = n_{yj} \le n_{uj}$. Since these conditions are necessary for integral action, it is assumed that $P \in \mathcal{R}^{n_y \times n_u}$ is full row-rank and has no (transmission) zeros at zero ($\mathrm{rank}P(0) = n_y \le n_u$), and $n_{yj} \le n_{uj}$, $j = 1, 2$. For $j = 1, 2$, $\mathcal{S}(P, C_j)$ is stable and has integral action if and only if $D_{Hj} := [\, P_{jj} N_j + \frac{s}{s+\alpha} D_j \,]$ is unimodular; D_{Hj} unimodular implies $\mathrm{rank}D_{Hj}(0) = \mathrm{rank}(P_{jj} N_j)(0) = n_{yj}$. When $\mathcal{S}(P, C_j)$ is stable in addition to $\mathcal{S}(P, C_D)$, an additional necessary condition is $\mathrm{rank}P_{jj}(0) = n_{yj} \le n_{uj}$.

The controller design approaches are divided into two cases depending on the rank of $P_{jj}(0)$.

Case 1: Let at least one of P_{11} or P_{22} have no transmission-zeros at zero. Without loss of generality, $\mathrm{rank}P_{11}(0) = n_{y1}$. Proposition 2.2 presents controllers such that $\mathcal{S}(P, C_D)$ (and also $\mathcal{S}(P, C_1)$) is stable and has integral action. If $\mathrm{rank}P_{22}(0) = n_{y2}$, then stability and integral action may be achievable for $\mathcal{S}(P, C_2)$as well. Lemma 2.3 gives conditions for existence of reliable stabilizing controllers with integral action.

2.2. Proposition: Let $P \in \mathcal{R}^{n_y \times n_u}$, $\mathrm{rank}P(0) = n_y \le n_u$, $n_{yj} \le n_{uj}$, $j = 1, 2$. Let $\mathrm{rank}P_{11}(0) = n_{y1}$, $\mathrm{rank}(P_{22} - P_{21} P_{11}^I P_{12})(0) = n_{y2}$, where $P_{11}^I(0) = P_{11}^T(0)(P_{11}(0) P_{11}^T(0))^{-1}$ is a right-inverse of $P_{11}(0)$. Define $K_1 := k_1 P_{11}^I(0)$, $0 < k_1 < \| \, s^{-1}(I - P_{11} P_{11}^I(0)) \|^{-1}$. Let $C_1 = (I - Q_1 P_{11})^{-1}(\frac{K_1}{s} + Q_1)$; $Q_1 \in \mathcal{R}^{n_{u1} \times n_{y1}}$ satisfies $\det(I - Q_1 P_{11})(\infty) \ne 0$. For fixed Q_1, define $G := P_{22} - P_{21}(I + \frac{K_1}{s} P_{11})^{-1}(\frac{K_1}{s} + Q_1) P_{12} \in \mathcal{M}(\mathcal{R})$, $K_2 := k_2 G^I(0)$, $0 < k_2 < \| \, s^{-1}(I - G G^I(0)) \|^{-1}$, where $G^I(0)$ is any right-inverse of $G(0) = (P_{22}(0) - P_{21}(0) P_{11}^I(0) P_{12}(0))$. Let $C_2 = (I - Q_2 G)^{-1}(\frac{K_2}{s} + Q_2)$; $Q_2 \in \mathcal{R}^{n_{u2} \times n_{y2}}$ satisfies $\det(I - Q_2 G)(\infty) \ne 0$. With $C_D = \mathrm{diag}[C_1, C_2]$, the system $\mathcal{S}(P, C_D)$ (and also $\mathcal{S}(P, C_1)$) is stable and has integral action. □

The choice of $Q_1 = 0$, $Q_2 = 0$ in Proposition 2.2 corresponds to integral controllers $C_1 = \frac{K_1}{s}$, $C_2 = \frac{K_2}{s}$.

2.3. Lemma: Let $P \in \mathcal{R}^{n_y \times n_u}$, $\mathrm{rank}P(0) = n_y \le n_u$, $n_{yj} \le n_{uj}$, $j = 1, 2$. Let $\mathrm{rank}P_{11}(0) = n_{y1}$ and $\mathrm{rank}P_{22}(0) = n_{y2}$. There exist reliable stabilizing controllers with integral action if $\det(G(0) P_{22}^I(0)) = \det(I - P_{21}(0) P_{11}^I(0) P_{12}(0) P_{22}(0)^I) > 0$, for some right-inverse $P_{11}^I(0)$ of $P_{11}(0)$ and $P_{22}^I(0)$ of $P_{22}(0)$. When $P_{12} \in \mathcal{M}(\mathrm{R_s})$ or $P_{21} \in \mathcal{M}(\mathrm{R_s})$, or when C_1 or C_2 is designed strictly-proper, then the sufficient condition $\det(G(0) P_{22}^I(0)) > 0$ is necessary and sufficient. □

Explicit controller design such that all three systems $\mathcal{S}(P, C_D)$, $\mathcal{S}(P, C_1)$, $\mathcal{S}(P, C_2)$ are stable and have integral action is challenging even with $\det(G(0) P_{22}^I(0) > 0$. A reliable design method is proposed in Corollary 2.4 under the stronger assumption that $G(0) P_{22}^I(0)$ is symmetric, positive definite. This assumption is equivalent to $\det(G(0) P_{22}^I(0)) > 0$ for example: i) when ($n_{y2} = 1$); ii) when $P_{12}(0) = 0$ or $P_{21}(0) = 0$.

2.4. Corollary: Let $P(0)$, $P_{11}(0)$, $P_{22}(0)$ satisfy the rank assumptions in Lemma 2.3, C_1, G and C_2 be as in Proposition 2.2, $G(0) P_{22}^I(0)$ be symmetric, positive definite, where $P_{22}^I(0) = P_{22}^T(0)(P_{22}(0) P_{22}^T(0))^{-1}$, $K_2 := k_2 P_{22}^I(0)$, $0 < k_2 < \min\{\|s^{-1}(I - P_{22} P_{22}^I(0))\|^{-1}, \|s^{-1}(G(0) - G) P_{22}^I(0)\|^{-1}\}$; $Q_2 \in \mathcal{M}(\mathcal{R})$ satisfies $I + P_{21}(I + \frac{K_1}{s} P_{11})^{-1}(\frac{K_1}{s} + Q_1) P_{12}(I + \frac{K_2}{s} P_{22})^{-1} Q_2$ unimodular, $\det(I - Q_2 G)(\infty) \ne 0$. Then $C_D = \mathrm{diag}[C_1, C_2]$ is a reliable stabilizing controller with integral action. □

Case 2: Let both P_{11} and P_{22} have transmission-zeros at zero; let at least one of these sub-blocks has blocking-zeros at zero; without loss of generality, let $P_{11}(0) = 0$. Proposition 2.5 presents a class of controllers such that $\mathcal{S}(P, C_D)$ is stable and has integral action. Since $\mathrm{rank}P_{jj}(0) < n_{yj}$, the systems $\mathcal{S}(P, C_1)$ and $\mathcal{S}(P, C_2)$ cannot be stable and therefore, reliable stabilizing controller design with integral action is not attempted in this case.

2.5. Proposition: Let $P \in \mathcal{R}^{n_y \times n_u}$, $\mathrm{rank}P(0) = n_y \le n_u$, $n_{yj} \le n_{uj}$, $j = 1, 2$; $n_{y1} = n_{y2}$. Let $P_{11}(0) = 0$, $\mathrm{rank}P_{22} < n_{y2}$, $\mathrm{rank}P_{12} = n_{y1}$, and $\mathrm{rank}P_{21} = n_{y2} = n_{y1}$. Let $-\alpha \in \mathbb{R} \backslash \mathcal{U}$. Let $C_1 = Q_1(\frac{s}{s+\alpha} I - P_{11} Q_1)^{-1}$; $Q_1 \in \mathcal{R}^{n_{u1} \times n_{y1}}$ satisfies $Q_1(0) = P_{21}^I(0)$ for some right-inverse $P_{21}^I(0)$ of $P_{21}(0)$, and $\det(I - P_{11} Q_1)(\infty) \ne 0$. For fixed Q_1, define $\widetilde{G} := \frac{s}{s+\alpha} P_{22} - P_{21} Q_1 P_{12}$, $\widetilde{K}_1 := \widetilde{k}_1 \widetilde{G}^I(0)$, $0 < \widetilde{k}_1 < \|s^{-1}(I - \widetilde{G} \widetilde{G}^I(0))\|^{-1}$, where $\widetilde{G}^I(0) = -P_{12}^I(0)$ is any right-inverse of $\widetilde{G}(0) = -P_{12}(0)$. Choose $\widetilde{k}_2 \in \mathbb{R}$, $0 < \widetilde{k}_2 < \|s^{-1}(I + \widetilde{G} \frac{\widetilde{K}_1}{s})^{-1}\|^{-1}$. Let $C_2 = (I - Q_2 \widetilde{G})^{-1}(\frac{\widetilde{K}_1(s + \widetilde{k}_2)}{s(s+\alpha)}) + \frac{s}{s+\alpha} Q_2)$; $Q_2 \in \mathcal{R}^{n_{u2} \times n_{y2}}$ satisfies $\det(I - Q_2 \widetilde{G})(\infty) \ne 0$. With $C_D = \mathrm{diag}[C_1, C_2]$, the system $\mathcal{S}(P, C_D)$ is stable and has integral action. □

The design methods presented here can be extended to multi-channel decentralized control systems that satisfy additional rank requirements.

References

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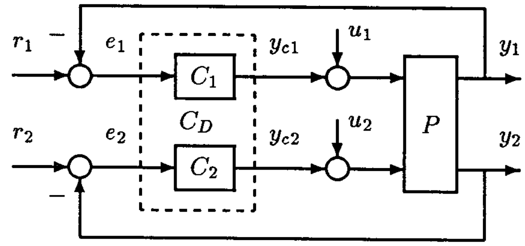

Figure 1: Two-channel system $\mathcal{S}(P, C_D)$.