LINEAR TIME-INVARIANT CONTROLLER DESIGN FOR TWO-CHANNEL DECENTRALIZED CONTROL SYSTEMS

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ABSTRACT

This paper analyzes a linear time-invariant 2-channel decentralized control system with a 2x2 strictly proper plant. It presents an algorithm for the algebraic design of a class of decentralized compensators which stabilize the given plant.

INTRODUCTION

It is well known (see for example [Vid.1], [Des.1]) that the set of all stabilizing full-output-feedback compensators that stabilize a given plant P can be parametrized using coprime factorizations of P. If the system has several local control stations and dynamic output-feedback is allowed only from each channel output to the input of that channel, such a parametrization is not available. It was shown in [Wan.1] that a plant P can be stabilized using such decentralized dynamic output-feedback if and only if P has no *unstable fixed modes.*

In this paper we consider a 2-channel 2x2 strictly proper plant which has no unstable fixed modes. We present an algorithm which constructs a class of proper decentralized compensators which stabilize the plant. We use a factorization approach.

Notation [Lang1]: Let $\mathcal{U} \subset \mathbb{C}_+$ be a closed subset of $\mathbb{C}$, symmetric about the real axis, and let $\mathbb{C}\setminus\mathcal{U}$ be nonempty. Let $\bar{\mathcal{U}} := \mathcal{U} \cup \{\infty\}$. Let $R_{\mathcal{U}}(s)$ be the ring of proper scalar rational functions in s (with coefficients in $\mathbb{R}$) which are analytic in $\mathcal{U}$. Let $\mathcal{I}$ be the (multiplicative) set of elements $f \in R_{\mathcal{U}}$ such that $f(\infty)$ = a nonzero constant; equivalently, $\mathcal{I} \subset R_{\mathcal{U}}$ is the set of proper but not strictly proper rational functions which are analytic in $\mathcal{U}$. Let $R_{\mathcal{U}}/\mathcal{I} := \{ n/d : n \in R_{\mathcal{U}}, d \in \mathcal{I}$ be the ring of fractions associated with $R_{\mathcal{U}}$; clearly, this ring is the ring of proper rational functions $\mathbb{R}_p(s)$.
Let $\mathbb{R}_{sp}(s)$ be the set of strictly proper rational functions; equivalently $f \in \mathbb{R}_{sp}(s)$ goes to 0 as $s \to \infty$. Let $\mathcal{J}$ be the group of units of $R_{\mathcal{U}}$; equivalently, $f \in \mathcal{J}$ has neither poles nor zeros in $\bar{\mathcal{U}}$.

ANALYSIS

Assumptions: Consider the decentralized control system $S(P, C_d)$ shown in figure 1.

(A) Let P and C_d have no hidden $\mathcal{U}$-unstable modes so that they can be specified by their I/O representations.

(B) Let $P \in \mathbb{R}_p(s)^{2x2}$ be a 2-channel plant. Let (N, D) be a right-coprime fraction representation (r.c.f.r.) of P, where $N = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$, $D = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$, $N_1, N_2, D_1, D_2 \in R_{\mathcal{U}}^{1x2}$, $\det D \in \mathcal{I}$, $P = ND^{-1}$.
Let $rank \begin{bmatrix} D_1(s) \\ N_1(s) \end{bmatrix} \geq 1$ and let $rank \begin{bmatrix} D_2(s) \\ N_2(s) \end{bmatrix} \geq 1$, for all $s \in \bar{\mathcal{U}}$.

(C) Let $C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \in \mathbb{R}_p(s)^{2x2}$ be a decentralized compensator. Let $(\tilde{D}', \tilde{N}')$ be a left-coprime fraction representation of C_d, where $\tilde{D}' = \begin{bmatrix} \tilde{d}_1' & 0 \\ 0 & \tilde{d}_2' \end{bmatrix}$, $\tilde{N}' = \begin{bmatrix} \tilde{n}_1' & 0 \\ 0 & \tilde{n}_2' \end{bmatrix}$, $\tilde{n}_1', \tilde{n}_2' \in R_{\mathcal{U}}, \tilde{d}_1', \tilde{d}_2' \in \mathcal{I}$, $C_d = \tilde{D}'^{-1}\tilde{N}'$.

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Comment: Assumption (B) implies that the plant P has no unstable fixed modes [And.1, Wan.1].

Using the representations of P and C_d as in Assumptions (B) and (C), we redraw the decentralized control system as in figure 2. The system $S(P, C_d)$ is described by equations (1)-(2) below:

$$\begin{bmatrix} 1 & 0 & \vdots & -D_1 \\ 0 & 1 & \vdots & -D_2 \\ \tilde{d}_1' & 0 & \vdots & \tilde{n}_1' N_1 \\ 0 & \tilde{d}_2' & \vdots & \tilde{n}_2' N_2 \end{bmatrix} \begin{bmatrix} y_1' \\ y_2' \\ \cdots \\ \xi_p \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \tilde{n}_1' & 0 \\ 0 & 0 & 0 & \tilde{n}_2' \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_1' \\ u_2' \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 0 & 0 & \vdots & N_1 \\ 0 & 0 & \vdots & N_2 \\ 1 & 0 & \vdots & 0 \\ 0 & 1 & \vdots & 0 \end{bmatrix} \begin{bmatrix} y_1' \\ y_2' \\ \cdots \\ \xi_p \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_1' \\ y_2' \end{bmatrix} \tag{2}$$

Using obvious notations we write equations (1)-(2) in the form

$$D_H \xi = N_L u \qquad N_R \xi = y$$

where, by inspection, (N_R, D_H) is right-coprime (r.c.) and (D_H, N_L) is left-coprime (l.c). Let $H_{yu} : u \mapsto y$. If $\det D_H \in \mathcal{I}$ (equivalently, the system $S(P, C_d)$ is well-posed), then

$$H_{yu} = N_R D_H^{-1} N_L \in \mathcal{M}(\mathbb{R}_p(s))$$

Definition: $S(P, C_d)$ is called *$\mathcal{U}$-stable* if and only if $H_{yu} \in \mathcal{M}(R_{\mathcal{U}})$.

Theorem: Let Assumptions (A), (B), (C) hold. Then $S(P, C_d)$ is $\mathcal{U}$-stable if and only if $\det D_H \in \mathcal{J}$.

Comment : This theorem implies that $S(P, C_d)$ is $\mathcal{U}$-stable if and only if $\det D_H = \det \begin{bmatrix} \tilde{d}_1' D_1 + \tilde{n}_1' N_1 \\ \tilde{d}_2' D_2 + \tilde{n}_2' N_2 \end{bmatrix} \in \mathcal{J}$

$$\Leftrightarrow \begin{bmatrix} \tilde{d}_1' & 0 & \vdots & \tilde{n}_1' & 0 \\ 0 & \tilde{d}_2' & \vdots & 0 & \tilde{n}_2' \end{bmatrix} \begin{bmatrix} D \\ \cdots \\ N \end{bmatrix} \text{ is unimodular}$$

$$\Leftrightarrow \begin{bmatrix} \tilde{d}_1' & \tilde{n}_1' & \vdots & 0 & 0 \\ 0 & 0 & \vdots & \tilde{d}_2' & \tilde{n}_2' \end{bmatrix} \begin{bmatrix} D \\ \cdots \\ N \end{bmatrix} \text{ is unimodular}$$

SYNTHESIS

Let P satisfy Assumption (B).

Definition : C_d is called a *$\mathcal{U}$-stabilizing decentralized compensator* for P (equivalently, C_d *$\mathcal{U}$-stabilizes P*) if and only if C_d satisfies Assumption (C), *and* $S(P, C_d)$ is $\mathcal{U}$-stable.

We now state an algorithm for finding a decentralized compensator C_d which $\mathcal{U}$-stabilizes a given strictly proper P.

Algorithm :
Given : $P \in \mathbb{R}_{sp}(s)^{2x2}$, and a r.c.f.r. (N, D) of P such that Assumption (B) holds.

Find : $C_d \in \mathbb{R}_p(s)^{2x2}$ satisfying Assumption (C) such that the system $S(P, C_d)$ is $\mathcal{U}$–stable.

Step 1 : Put $\begin{bmatrix} D_1 \\ N_1 \end{bmatrix}$ into the *Smith Form*; equivalently, find *unimodular* L_1 , $R_1 \in R_{\mathcal{U}}^{2x2}$ such that $L_1\begin{bmatrix} D_1 \\ N_1 \end{bmatrix}R_1 = \begin{bmatrix} 1 & 0 \\ 0 & \lambda_1 \end{bmatrix}$, $\lambda_1 \in R_{\mathcal{U}}$, where λ_1 may be 0. Let $\begin{bmatrix} D_2 \\ N_2 \end{bmatrix}R_1 =: \begin{bmatrix} d_{21} & d_{22} \\ n_{21} & n_{22} \end{bmatrix}$.

Step 2 : Find a *unimodular* $M_1 \in R_{\mathcal{U}}^{2x2}$ such that $M_1\begin{bmatrix} d_{21} & d_{22} \\ n_{21} & n_{22} \end{bmatrix} =: \begin{bmatrix} \hat{d}_{21} & \lambda_2 \\ \bar{n}_{21} & 0 \end{bmatrix} \in R_{\mathcal{U}}^{2x2}$ where $\lambda_2 \neq 0$. Find $r \in R_{\mathcal{U}}$ such that the pair $(\hat{d}_{21}+rn_{21}, \lambda_2) =: (\bar{d}_{21}, \lambda_2)$ is coprime. Let $M_2 =: \begin{bmatrix} 1 & r \\ 0 & 1 \end{bmatrix}M_1$. Then

$$\begin{bmatrix} L_1 & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & M_2 \end{bmatrix}\begin{bmatrix} D_1 \\ N_1 \\ D_2 \\ N_2 \end{bmatrix}R_1 = \begin{bmatrix} 1 & 0 \\ 0 & \lambda_1 \\ \bar{d}_{21} & \lambda_2 \\ \bar{n}_{21} & 0 \end{bmatrix}, \ (\bar{d}_{21}, \lambda_2) \text{ coprime.}$$

Step 3: Find a *unimodular* $T \in R_{\mathcal{U}}^{2x2}$ such that $T\begin{bmatrix} \lambda_2 \\ -\lambda_1\bar{d}_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Step 4 : For *all* $q_1 \in R_{\mathcal{U}}$, the decentralized compensator numerators and denominators are given by

$$\begin{bmatrix} \tilde{d}_1' & \tilde{n}_1' & \vdots & 0 & 0 \\ 0 & 0 & \vdots & \tilde{d}_2' & \tilde{n}_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 1 & 0 \end{bmatrix}\begin{bmatrix} \begin{bmatrix} 1 & q_1 \\ 0 & 1 \end{bmatrix}TL_1 & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & M_2 \end{bmatrix}$$

$$= \begin{bmatrix} [1 \ \ q_1]TL_1 & \vdots & 0 \\ \cdots & \vdots & \cdots \\ 0 & \vdots & [1 \ \ 0]M_2 \end{bmatrix}$$

■

Comments: 1) In Step 1, $\lambda_1 \neq 1$ because $P \in \mathbb{R}_{sp}(s)^{2x2}$ implies that $N_1 \in \mathbb{R}_{sp}(s)^{1x2}$. Therefore, in Step 2, $\lambda_2 \neq 0$. 2) In Step 2, the pair $(\hat{d}_{21}+rn_{21}, \lambda_2) = (\bar{d}_{21}, \lambda_2)$ may be coprime for more than one $r \in R_{\mathcal{U}}$. For *each* choice of such r, we can find a $T \in R_{\mathcal{U}}^{2x2}$ in Step 3. Then in Step 4, for this T, we find a whole class of compensator parameters $\tilde{d}_1'$, $\tilde{n}_1'$, $\tilde{d}_2'$, $\tilde{n}_2'$, where $C_1 = \tilde{d}_1'^{-1}\tilde{n}_1'$ and $C_2 = \tilde{d}_2'^{-1}\tilde{n}_2'$.

EXAMPLE

We now follow the algorithm above to find a $\mathcal{U}$–stabilizing decentralized compensator for a given strictly proper plant.

Given $P = \begin{bmatrix} \dfrac{2(s^3-7s^2+6s+2)}{(s+1)^2(s-2)(s-3)} & \dfrac{-(s-1)}{(s-2)(s-3)} \\ \dfrac{1}{s-3} & \dfrac{1}{s-3} \end{bmatrix} \in \mathbb{R}_{sp}(s)^{2x2}$

and a r.c.f.r. (N, D) of P, where

$$N = \begin{bmatrix} \dfrac{1}{s+1} & 0 \\ \dfrac{2s-1}{(s+1)^2} & \dfrac{3s^2-4s-1}{(s+1)^3} \end{bmatrix}, \ D = \begin{bmatrix} 1 & \dfrac{s-1}{s+1} \\ \dfrac{s^2-9s+2}{(s+1)^2} & \dfrac{2(s^3-7s^2+6s+2)}{(s+1)^3} \end{bmatrix}.$$

Step 1: $L_1 = \begin{bmatrix} 1 & 0 \\ \dfrac{-1}{s+1} & 1 \end{bmatrix}$, $R_1 = \begin{bmatrix} 1 & \dfrac{-(s-1)}{s+1} \\ 0 & 1 \end{bmatrix}$ and hence, $\lambda_1 = \dfrac{-(s-1)}{(s+1)^2}$.

Then $\begin{bmatrix} D_2 \\ N_2 \end{bmatrix}R_1 = \begin{bmatrix} \dfrac{s^2-9s+2}{(s+1)^2} & \dfrac{(s-2)(s-3)}{(s+1)^2} \\ \dfrac{2s-1}{(s+1)^2} & \dfrac{s-2}{(s+1)^2} \end{bmatrix}$.

Step 2: $M_1 = \begin{bmatrix} 1 & 4 \\ \dfrac{-1}{s+1} & \dfrac{s-3}{s+1} \end{bmatrix}$. Then $\hat{d}_{21} = \dfrac{-(s-2)}{s+1}$, $\bar{n}_{21} = \dfrac{1}{s+1}$, $\lambda_2 = \dfrac{s-2}{s+1}$. Now we can choose any $r \in R_{\mathcal{U}}$ such that $\hat{d}_{21}+r\bar{n}_{21}$ has no zero at 2 (where λ_2 has a zero). Choose for example $r = -1$. Then $\bar{d}_{21} = \dfrac{-(s-1)}{s+1}$ is coprime with λ_2.

Step 4 : For *all* $q_1 \in R_{\mathcal{U}}$, $[\tilde{d}_1' \ \ \tilde{n}_1'] = [1 \ \ q_1]\begin{bmatrix} \dfrac{s^2+5s+40}{(s+1)^2} & -27 \\ \dfrac{-(s-3)}{(s+1)^3} & \dfrac{s-2}{s+1} \end{bmatrix}$ and $[\tilde{d}_2' \ \ \tilde{n}_2'] = \begin{bmatrix} \dfrac{s+2}{s+1} & \dfrac{3s+7}{s+1} \end{bmatrix}$.

■

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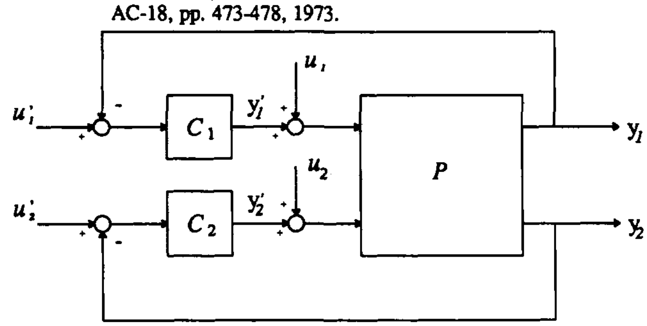

Figure 1

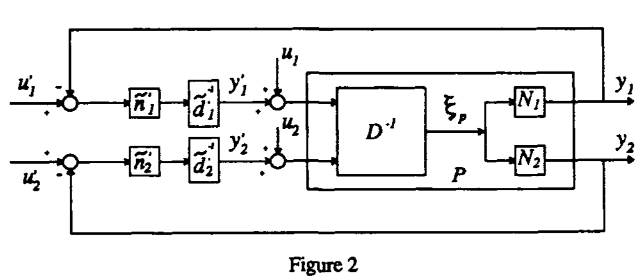

Figure 2