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Simultaneous stabilization of a class of multiplicatively perturbed plants[1]

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Abstract

Necessary and sufficient conditions are obtained for stabilizability of a given linear, time-invariant, multi-input multi-output plant under unknown sensor failures of a prespecified channel. This problem is equivalent to simultaneous stabilization of a nominal plant and a multiplicatively perturbed plant, where the multiplicative perturbation is unknown but stable. A class of simultaneously stabilizing controllers are proposed for the nominal plant under such unknown perturbations.

1. Introduction

In the standard linear, time-invariant (LTI), multi-input multi-output (MIMO) unity-feedback system, the well-known parametrization of all stabilizing controllers has lead to necessary and sufficient conditions for simultaneous stabilization of two given plants [4]. For the case of three or more plants, necessary and sufficient conditions to check simultaneous stabilizability are not available [1]. Although simultaneous stabilizability of three or more plants cannot be determined in the most general case, it may be possible to obtain conditions and develop design methods for special classes of plants [2].

We consider simultaneous stabilization of a given nominal plant represented by its transfer-function P and a class of perturbed plants $(I-\Delta)P$, where the multiplicative perturbation Δ is diagonal and stable; no small-gain restrictions are imposed on Δ. The perturbed plant $(I-\Delta)P$ represents the nominal plant under various perturbations, such as sensor failures. It is assumed that the diagonal Δ has at most one non-zero stable entry δ_j, representing possible unknown sensor failures at a prespecified output channel. Although the location of the failure is known to be the j-th output, δ_j is unknown. Since δ_j is arbitrary, simultaneous stabilizability of P and $(I-\Delta)P$ cannot be determined by small-gain results. Since the plant is not necessarily stable, P and $(I-\Delta)P$ are not necessarily simultaneously stabilizable. We derive necessary and sufficient conditions for simultaneous stabilizability and determine the class of plants for which simultaneously stabilization is possible, and we propose a class of simultaneously stabilizing controllers.

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Due to the algebraic framework described in the following notation, the results apply to continuous-time systems as well as discrete-time systems:

Notation: Let $\mathcal{U}$ contain the extended closed right-half-plane (for continuous-time systems) or the complement of the open unit-disk (for discrete-time systems). The set of proper rational functions which have no poles in the region of instability $\mathcal{U}$, the sets of proper and strictly-proper rational functions with real coefficients are denoted by $\mathcal{R}$, $\mathrm{R_p}$, $\mathrm{R_{sp}}$, respectively. The set of matrices whose entries are in $\mathcal{R}$ is denoted by $\mathcal{M}(\mathcal{R})$; M is called $\mathcal{R}$-stable iff $M \in \mathcal{M}(\mathcal{R})$; $M \in \mathcal{M}(\mathcal{R})$ is called $\mathcal{R}$-unimodular iff M^{-1} is also $\mathcal{R}$-stable. A right-coprime-factorization (RCF) and a left-coprime-factorization (LCF) of $P \in \mathrm{R_p}^{n_o \times n_i}$ are denoted by $P = ND^{-1} = \widetilde{D}^{-1}\widetilde{N}$; $N, D, \widetilde{N}, \widetilde{D} \in \mathcal{M}(\mathcal{R})$; D, $\widetilde{D}$ are biproper. The identity map is denoted by I; the j-th column of I is denoted by e_j.

2. Main Results

Consider the LTI, MIMO system $\mathcal{S}(I-\Delta, P, C)$ shown in Fig. 1, where $P \in \mathrm{R_p}^{n_o \times n_i}$, $C \in \mathrm{R_p}^{n_i \times n_o}$, $\Delta \in \mathcal{R}^{n_o \times n_o}$ represent the plant, the controller, and the multiplicative perturbation. The nominal plant and the perturbed plant are denoted by P and $(I-\Delta)P$, respectively. It is assumed that P and C do not have any hidden modes corresponding to eigenvalues in $\mathcal{U}$ and that the system $\mathcal{S}(I-\Delta, P, C)$ is well-posed. The multiplicative perturbation is represented by the diagonal matrix $\Delta = e_j \delta_j e_j^T$, where $j \in \{1, \ldots, n_o\}$ is the location of the failure and $\delta_j \in \mathcal{R}$ is arbitrary. Under normal operation of the j-th channel, $\delta_j = 0$; all other values of the $\mathcal{R}$-stable δ_j imply a failure; $\delta_j = 1$ corresponds to a disconnection failure. If $\delta_j = 0$, then $\mathcal{S}(I-\Delta, P, C)$ becomes the unity-feedback system $\mathcal{S}(P, C)$, called the nominal system.

The system $\mathcal{S}(I-\Delta, P, C)$ is said to be $\mathcal{R}$-stable iff the closed-loop transfer-function H from $u := [\,u_P^T\ u_C^T\,]^T$ to $y := [\,y_P^T\ y_C^T\,]^T$ is $\mathcal{R}$-stable; the nominal system $\mathcal{S}(P, C)$ is $\mathcal{R}$-stable iff $H : u \mapsto y$ is $\mathcal{R}$-stable when $\Delta = 0$. The controller C is said to $\mathcal{R}$-stabilize $P \in \mathrm{R_p}^{n_o \times n_i}$ iff $C \in \mathrm{R_p}^{n_i \times n_o}$ and $\mathcal{S}(P, C)$ is $\mathcal{R}$-stable; C is said to simultaneously $\mathcal{R}$-stabilize P and $(I-\Delta)P$ iff $C \in \mathrm{R_p}^{n_i \times n_o}$ and the systems $\mathcal{S}(P, C)$ and $\mathcal{S}(I-\Delta, P, C)$ are both $\mathcal{R}$-stable. The plants P and $(I-\Delta)P$ are said to be simultaneously $\mathcal{R}$-stabilizable iff there exists a simultaneously $\mathcal{R}$-stabilizing controller C.

The controller C $\mathcal{R}$-stabilizes P if and only if C is given by (see for example [4], [3]):

$$C = (V - Q\widetilde{N})^{-1}(U + Q\widetilde{D}) = (\widetilde{U} + DQ)(\widetilde{V} - NQ)^{-1}, \quad (1)$$

where $Q \in \mathcal{M}(\mathcal{R})$ is such that $(V - Q\widetilde{N})$ is biproper (holds for all $Q \in \mathcal{M}(\mathcal{R})$ when $P \in \mathcal{M}(\mathrm{R_{sp}})$); $U, V, \widetilde{U}, \widetilde{V} \in \mathcal{M}(\mathcal{R})$ satisfy $VD + UN = I$, $\widetilde{D}\widetilde{V} + \widetilde{N}\widetilde{U} = I$, $V\widetilde{U} = U\widetilde{V}$. The controller C simultaneously $\mathcal{R}$-stabilizes P and $(I - \Delta)P$ if and only if C is given by (1), where $Q \in \mathcal{M}(\mathcal{R})$ is such that $(V - Q\widetilde{N})$ is biproper and

$$I - (\,U + Q\widetilde{D})\,\Delta N \quad \text{is } \mathcal{R}\text{-unimodular.} \quad (2)$$

The plants P and $(I - \Delta)P$ are simultaneously $\mathcal{R}$-stabilizable if and only if there exists $Q \in \mathcal{M}(\mathcal{R})$ such that (2) holds. It follows from standard strong $\mathcal{R}$-stabilizability results that condition (2) holds if and only if the pair $(I - U\Delta N, \widetilde{D}\Delta N)$ satisfies the parity-interlacing-property [4], [1]. For the class of diagonal perturbations $\Delta = e_j \delta_j e_j^T$, we have the necessary and sufficient conditions of Lemma 2.1 for simultaneous $\mathcal{R}$-stabilizability of P and $(I - e_j\delta_j e_j^T)P$. For plants satisfying these conditions, we propose a class of simultaneously $\mathcal{R}$-stabilizing controllers in Proposition 2.2. Since $\delta_j \in \mathcal{R}$ is unknown, these conditions are in fact for simultaneous $\mathcal{R}$-stabilizability of a special class of infinitely many plants, and the controllers proposed simultaneously $\mathcal{R}$-stabilize all plants in this class.

2.1. Lemma *(Stabilizability conditions)*:

Let $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta = e_j \delta_j e_j^T$, for some $j \in \{1, \ldots, n_o\}$, where $\delta_j \in \mathcal{R}$. Let $d_j \in \mathcal{R}^{n_o \times 1}$ denote the j-th column of $\widetilde{D}$ in any LCF $P = \widetilde{D}^{-1}\widetilde{N}$. The plants P and $(I - e_j\delta_j e_j^T)P$ are simultaneously $\mathcal{R}$-stabilizable for all $\delta_j \in \mathcal{R}$ if and only if $\mathrm{rank} d_j(s) = 1$ for all $s \in \mathcal{U}$. □

2.2. Proposition *(Stabilizing controllers)*:

Let $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta = e_j \delta_j e_j^T$, for some $j \in \{1, \ldots, n_o\}$, where $\delta_j \in \mathcal{R}$. Let $\widetilde{D}^{-1}\widetilde{N}$ be any LCF of P, where $d_j \in \mathcal{R}^{n_o \times 1}$ denotes the j-th column of $\widetilde{D}$. Let $\mathrm{rank} d_j(s) = 1$ for all $s \in \mathcal{U}$, equivalently, let d_j have a left-inverse denoted by $y_j^T \in \mathcal{R}^{1 \times n_o}$. Let $U, V, \widetilde{U}, \widetilde{V} \in \mathcal{M}(\mathcal{R})$ satisfy $VD + UN = I$, $\widetilde{D}\widetilde{V} + \widetilde{N}\widetilde{U} = I$, $V\widetilde{U} = U\widetilde{V}$. Define $\hat{C} := (V - \hat{Q}\widetilde{N})^{-1}(U + Q\widetilde{D})$, The controller

$$C = (V - \hat{Q}\widetilde{N} + (U + \hat{Q}\widetilde{D})\, e_j y_j^T \widetilde{N})^{-1}(U + \hat{Q}\widetilde{D})(I - e_j y_j^T \widetilde{D})$$

$$= \hat{C}(I + e_j\, y_j^T\, \widetilde{D}\, P\, \hat{C})^{-1}(I - e_j\, y_j^T\, \widetilde{D}) \quad (3)$$

simultaneously $\mathcal{R}$-stabilizes P and $(I - e_j\delta_j e_j^T)P$ for all $\delta_j \in \mathcal{R}$, where $\hat{Q} \in \mathcal{M}(\mathcal{R})$ satisfies $\det(V - \hat{Q}\widetilde{N})(\infty) \neq 0$ and $y_j^T(\widetilde{V} - N\hat{Q})^{-1}e_j(\infty) = 1 + y_j^T \widetilde{N}(V - \hat{Q}\widetilde{N})^{-1}(U + \hat{Q}\widetilde{D})e_j(\infty) = 1 + y_j^T \widetilde{D}P\hat{C}e_j(\infty) \neq 0$ (holds for all $\hat{Q} \in \mathcal{M}(\mathcal{R})$ when $P \in \mathcal{M}(\mathrm{R_{sp}})$). The controller $C \in \mathcal{M}(\mathrm{R_{sp}})$ when $\hat{Q} \in \mathcal{M}(\mathcal{R})$ satisfies $\hat{Q}(\infty) = -U(\infty)\widetilde{D}^{-1}(\infty)$. □

Example: Let $\mathcal{U}$ be the extended closed right-half plane. Let $P = \begin{bmatrix} \frac{(s-4)(s-1)}{(s+2)^2(s-2)} & \frac{(s+8)(s-1)(s-2)}{(s+2)^3} \\ \frac{-(s-1)^2}{(s+2)^2(s-2)} & \frac{-3(s-1)(s-2)}{(s+2)^3} \end{bmatrix} = ND^{-1} = \begin{bmatrix} \frac{(s-4)(s-1)}{(s+2)^3} & \frac{(s+8)(s-1)(s-2)}{(s+2)^3} \\ \frac{-(s-1)^2}{(s+2)^3} & \frac{-3(s-1)(s-2)}{(s+2)^3} \end{bmatrix} \begin{bmatrix} \frac{s-2}{s+2} & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \widetilde{D}^{-1}\widetilde{N}$
$= \begin{bmatrix} \frac{-3(s-2)}{(s+2)^2} & \frac{-(s+8)(s-2)}{(s+2)^2} \\ \frac{s-1}{s+2} & \frac{s-4}{s+2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{s-1}{(s+2)^2} & 0 \\ 0 & \frac{(s-1)(s-2)}{(s+2)^3} \end{bmatrix}$.
Since each column of $\widetilde{D}$ is full-rank for all $s \in \mathcal{U}$, P and $(I - e_j\delta_j e_j^T)P$ are simultaneously $\mathcal{R}$-stabilizable for all $\delta_j \in \mathcal{R}$ for $j = 1, 2$. A solution for $U, V \in \mathcal{M}(\mathcal{R})$ satisfying $VD + UN = I$, $\widetilde{D}\widetilde{V} + \widetilde{N}\widetilde{U} = I$, $V\widetilde{U} = U\widetilde{V}$ is $U = \begin{bmatrix} \frac{-48}{s+2} & \frac{-16(s+8)}{s+2} \\ 0 & 0 \end{bmatrix}$, $V = \begin{bmatrix} \frac{s-10}{s+2} & 0 \\ 0 & 1 \end{bmatrix}$. Consider failures in the first channel, i.e., let $\Delta = e_1\delta_1 e_1^T$. A left-inverse $y_1^T \in \mathcal{M}(\mathcal{R})$ of the first column d_1 of $\widetilde{D}$ is $y_1^T = \begin{bmatrix} 3 & \frac{s+14}{s+2} \end{bmatrix}$. Let $\hat{C} = (V - \hat{Q}\widetilde{N})^{-1}(U + Q\widetilde{D})$, where $\hat{Q} \in \mathcal{M}(\mathcal{R})$ is arbitrary. By (3), $C = \hat{C}(\,I + \begin{bmatrix} \frac{3(s-1)}{(s+2)^2} & \frac{(s+14)(s-1)(s-2)}{(s+2)^4} \\ 0 & 0 \end{bmatrix} \hat{C}\,)^{-1} \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$ simultaneously $\mathcal{R}$-stabilizes P and $(I - e_1\delta_1 e_1^T)P$. Now consider failures in the second channel, i.e., let $\Delta = e_2\delta_2 e_2^T$. A left-inverse $y_2^T \in \mathcal{M}(\mathcal{R})$ of the second column d_2 of $\widetilde{D}$ is $y_2^T = \begin{bmatrix} \frac{-3(s+2)}{s+8} & 2 \end{bmatrix}$. Let $\hat{C} = (V - \hat{Q}\widetilde{N})^{-1}(U + Q\widetilde{D})$, where $\hat{Q} \in \mathcal{M}(\mathcal{R})$ is arbitrary. By (3), $C = \hat{C}(I + \begin{bmatrix} 0 & 0 \\ \frac{-3(s-1)}{(s+2)(s+8)} & \frac{-2(s-1)(s-2)}{(s+2)^3} \end{bmatrix} \hat{C})^{-1} \begin{bmatrix} 1 & 0 \\ \frac{2s^2+5s+2}{(s+2)(s+8)} & 0 \end{bmatrix}$ simultaneously $\mathcal{R}$-stabilizes P and $(I - e_2\delta_2 e_2^T)P$. □

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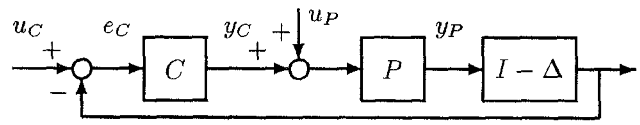

Figure 1: The system $\mathcal{S}(I - \Delta, P, C)$