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Simultaneous stabilization under stable multiplicative perturbations

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Abstract

Simultaneous stabilization of a given linear, time-invariant, multi-input multi-output nominal plant and a given multiplicatively perturbed plant is considered. The multiplicative perturbation is stable and no restrictions are imposed on its gain. Necessary and sufficient conditions are derived for existence of simultaneously stabilizing controllers.

1. Introduction

In the standard linear, time-invariant (LTI), multi-input multi-output (MIMO) unity-feedback system, we consider simultaneous stabilization of a given nominal plant P and a given perturbed plant $(I+\Delta)P$ using a common controller C. The multiplicative perturbation Δ is stable; we do not impose small-gain restrictions on Δ. The nominal plant is not necessarily stable. We develop necessary and sufficient conditions for existence of controllers that stabilize P and $(I+\Delta)P$ simultaneously. These conditions are based on the real-axis poles of P in the region of instability and are derived using the parity-interlacing-property.

Notation: Let $\mathcal{U}$ contain the extended closed right-half-plane (for continuous-time systems) or the complement of the open unit-disk (for discrete-time systems). Let $\mathbb{R}$, $\mathrm{R_p}$, $\mathcal{R}$, $\mathcal{M}(\mathcal{R})$ be the set of real numbers, proper rational functions with real coefficients, proper rational functions with no poles in the region of instability $\mathcal{U}$, the set of matrices whose entries are in $\mathcal{R}$. A matrix M is called $\mathcal{R}$-stable iff $M \in \mathcal{M}(\mathcal{R})$; $M \in \mathcal{M}(\mathcal{R})$ is called $\mathcal{R}$-unimodular iff M^{-1} is also $\mathcal{R}$-stable. For $M \in \mathcal{M}(\mathcal{R})$, the norm $\|\cdot\|$ is defined as $\| M \| = \sup_{s\in\partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$, $\partial\mathcal{U}$ denote the maximum singular value and the boundary of $\mathcal{U}$. A right-coprime-factorization (RCF) and a left-coprime-factorization (LCF) of $P \in \mathrm{R_p}^{n_o\times n_i}$ are denoted by (N,D), $(\tilde{D},\tilde{N})$; $N, D, \tilde{N}, \tilde{D} \in \mathcal{M}(\mathcal{R})$; D, $\tilde{D}$ are biproper, $P = ND^{-1} = \tilde{D}^{-1}\tilde{N}$. Let $\mathrm{rank}P = r$; $s_o \in \mathcal{U}$ is called a (transmission) $\mathcal{U}$-zero of P iff $\mathrm{rank}P(s_o) < r$, i.e., $\mathrm{rank}N(s_o) = \mathrm{rank}\tilde{N}(s_o) < r$; $s_o \in \mathcal{U}$ is called a blocking $\mathcal{U}$-zero of P iff $P(s_o) = 0$, i.e., $N(s_o) = 0 = \tilde{N}(s_o)$; $s_o \in \mathcal{U}$ is called a $\mathcal{U}$-pole of P iff it is a pole of some entry of P, i.e., $\det D(s_o) = 0 = \det \tilde{D}(s_o)$. The identity map is denoted by I. $a := b$ means a is defined as b.

2. Main Results

Consider the LTI, MIMO system $\mathcal{S}(\Delta, P, C)$ (Fig. 1), where $P \in \mathrm{R_p}^{n_o\times n_i}$, $C \in \mathrm{R_p}^{n_i\times n_o}$, $\Delta \in \mathcal{R}^{n_o\times n_o}$ represent the given plant, the controller and the known multiplicative perturbation. The nominal plant and the perturbed plant are denoted by P and $(I+\Delta)P$, respectively. The nominal plant P is not necessarily $\mathcal{R}$-stable; the perturbed plant $(I+\Delta)P$ is $\mathcal{R}$-stable if and only if P is $\mathcal{R}$-stable. The perturbed plant has the same $\mathcal{U}$-poles as P. If $\Delta = 0$, then $\mathcal{S}(\Delta, P, C)$ becomes the standard unity-feedback system $\mathcal{S}(P,C)$, called the nominal system. It is assumed that P and C do not have any hidden modes associated with eigenvalues in $\mathcal{U}$ and that the system $\mathcal{S}(\Delta, P, C)$ is well-posed.

Conditions for stability: The nominal system $\mathcal{S}(P,C)$ is said to be $\mathcal{R}$-stable iff the closed-loop transfer-function H from $u := [u_P^T, u_C^T]^T$ to $y := [y_P^T, y_C^T]^T$ is $\mathcal{R}$-stable. Similarly, when $\Delta \in \mathcal{M}(\mathcal{R})$, the system $\mathcal{S}(\Delta, P, C)$ is said to be $\mathcal{R}$-stable iff the closed-loop transfer-function $H : u \mapsto y$ is $\mathcal{R}$-stable [2]. The controller C is said to be an $\mathcal{R}$-stabilizing controller for $P \in \mathrm{R_p}^{n_o\times n_i}$ iff $C \in \mathrm{R_p}^{n_i\times n_o}$ and $\mathcal{S}(P,C)$ is $\mathcal{R}$-stable. The controller C is said to be a simultaneously $\mathcal{R}$-stabilizing controller for P and $(I+\Delta)P$ iff $C \in \mathrm{R_p}^{n_i\times n_o}$ and $\mathcal{S}(P,C)$ and $\mathcal{S}(\Delta, P, C)$ are both $\mathcal{R}$-stable; i.e., P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable iff there exists a simultaneously $\mathcal{R}$-stabilizing controller C.

The controller $C \in \mathcal{M}(\mathrm{R_p})$ is an $\mathcal{R}$-stabilizing controller for P if and only if C is given by ([4], [3])

$$C = (V - Q\tilde{N})^{-1}(U + Q\tilde{D}) = (\tilde{U} + DQ)(\tilde{V} - NQ)^{-1} \qquad (1)$$

for some $\mathcal{R}$-stable Q such that $(V - Q\tilde{N})$ is biproper (which holds for all $Q \in \mathcal{M}(\mathcal{R})$ when P is strictly-proper), where $U, V, \tilde{U}, \tilde{V} \in \mathcal{M}(\mathcal{R})$ satisfy

$$\begin{bmatrix} V & U \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & -\tilde{U} \\ N & \tilde{V} \end{bmatrix} = I . \qquad (2)$$

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2.1. Lemma (*Closed-loop stability of* $\mathcal{S}(\Delta, P, C)$): Let (N, D), $(\widetilde{D}, \widetilde{N})$ be an RCF and an LCF of $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta \in \mathcal{M}(\mathcal{R})$.
i) Let (N_C, D_C) be an RCF of $C \in \mathrm{R_p}^{n_i \times n_o}$. The system $\mathcal{S}(\Delta, P, C)$is $\mathcal{R}$-stable if and only if $\begin{bmatrix} \widetilde{D} & -\widetilde{N}\, N_C \\ (I+\Delta) & D_C \end{bmatrix}$ is $\mathcal{R}$-unimodular, equivalently, $[\,\widetilde{D}_C D + \widetilde{N}_C (I+\Delta) N\,]$ is $\mathcal{R}$-unimodular.
ii) Let U, V be as in (2). The controller C is a simultaneously $\mathcal{R}$-stabilizing controller for P and $(I+\Delta)P$ if and only if C is given by (1), where $Q \in \mathcal{M}(\mathcal{R})$ is such that $(V - Q\widetilde{N})$ is biproper and

$$M_\Delta := I_{n_i} + (U + Q\widetilde{D})\,\Delta N \quad \text{is } \mathcal{R}\text{-unimodular.} \quad (3)$$

2.2. Corollary (*Necessary condition*): Let $(\widetilde{D}, \widetilde{N})$ be any LCF of $P \in \mathrm{R_p}^{n_o \times n_i}$. If the system $\mathcal{S}(\Delta, P, C)$ is $\mathcal{R}$-stable, then $((I+\Delta), \widetilde{D})$ is right-coprime. □

2.3. Therem (*Necessary and sufficient conditions*): Let (N, D), $(\widetilde{D}, \widetilde{N})$ be an RCF and an LCF of $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta \in \mathcal{M}(\mathcal{R})$. Let $((I+\Delta), \widetilde{D})$ be right-coprime. Then P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable if and only if $\det(I_{n_i} + U\Delta N)$ has the same sign at all real blocking $\mathcal{U}$-zeros of $(\widetilde{D}\Delta N)$. □

2.4. Corollary (*Necessary conditions for simultaneous stabilizability*): Let (N, D), $(\widetilde{D}, \widetilde{N})$ be an RCF and an LCF of $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta \in \mathcal{M}(\mathcal{R})$. Let $((I+\Delta), \widetilde{D})$ be right-coprime. If P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable, then $\det(I_{n_o} + \Delta(s))$ has the same sign for all $s \in \mathbb{R} \cap \mathcal{U}$ such that $\widetilde{D}(s)\Delta(s) = 0$, and this sign is positive when P or Δ have real blocking $\mathcal{U}$-zeros □

2.5. Corollary (*Conditions when P is scalar*): Let $P \in \mathrm{R_p}$. Let $\Delta \in \mathcal{R}$. Then P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable if and only if **i)** $\Delta(s) \neq -1$ at all $\mathcal{U}$-poles of P, *and* **ii)** $(1 + \Delta(s))$ has the same sign at all real $\mathcal{U}$-poles of P, and this sign is positive when when P or Δ have real $\mathcal{U}$-zeros. □

2.6. Corollary (*Simultaneously stabilizing controllers for special* Δ): Let (N, D) be an RCF and $(\widetilde{D}, \widetilde{N})$ be an LCF of $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta = F\widetilde{D}$, where $F \in \mathcal{M}(\mathcal{R})$. Let U, V, $\widetilde{U}$,$\widetilde{V}$ be as in (2). Then P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable. Furthermore, a simultaneously $\mathcal{R}$-stabilizing controller C is given by (1), with $Q \in \mathcal{M}(\mathcal{R})$ chosen as $Q = U\,F\widetilde{N} \sum_{\ell=2}^{k} \frac{r_\ell}{k^\ell} (DUF\widetilde{N})^{\ell-2}\widetilde{U}$, where r_ℓ are the binomial coefficients and k is any integer such that $k > \|\, UF\widetilde{N}D \,\|$. □

2.7. Corollary (*Sufficient conditions when P has no unstable pole-zero coincidences*): Let $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\operatorname{rank} P = n_o$. Let $\Delta \in \mathcal{M}(\mathcal{R})$. Let P have no common $\mathcal{U}$-poles and $\mathcal{U}$-zeros. Let $((I+\Delta), \widetilde{D})$ be right-coprime. If $\det(I_{n_o} + \Delta(s))$ has the same sign at all real $\mathcal{U}$-poles of P, and this sign is positive when ΔP has real blocking $\mathcal{U}$-zeros, then P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable. □

2.8. Corollary (*Diagonal Δ case*): Let $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\operatorname{rank} P := r$. Let $\Delta = \delta I_{n_o}$, $\delta \in \mathcal{R}$. If **i)** $\delta(s) \neq -1$ at all $\mathcal{U}$-poles of P, *and* **ii)** $(1 + \delta(s))$ has the same sign at all real $\mathcal{U}$-zeros of the smallest denominator-invariant-factor of P and at all real $\mathcal{U}$-poles of P coinciding with its (transmission) zeros, and this sign is positive when δ has real $\mathcal{U}$-zeros or when P has real blocking $\mathcal{U}$-zeros, then P and $(1+\delta)P$ are simultaneously $\mathcal{R}$-stabilizable. □

2.9. Corollary (*Constant Δ case*): Let $P \in \mathrm{R_p}^{n_o \times n_i}$. Let $\Delta = KI_{n_o}$, $K \in \mathbb{R}$. If $K > -1$, then P and $(1+K)P$ are simultaneously $\mathcal{R}$-stabilizable. If $K = -1$, then P and $(1+K)P$ are simultaneously $\mathcal{R}$-stabilizable if and only if $P \in \mathcal{M}(\mathcal{R})$. □

2.10. Example: Let $P = \frac{(-3s^2+10s+1)}{s(s-1)(s+7)}$. Let $\mathcal{U}$ be the closed right-half-plane. An RCF (N, D) of P is $N = \widetilde{N} = \frac{-3s^2+10s+1}{(s+1)^3}$, $D = \widetilde{D} = \frac{s(s-1)(s+7)}{(s+1)^3}$; a solution for (2) is $U = 1$, $V = 1$. By (1), C is an $\mathcal{R}$-stabilizing controller for P if and only if $C = (1 - QN)^{-1}(1 + QD)$, where $Q \in \mathcal{R}$. Since P has a pole at $s = 0$, $((U + Q\widetilde{D})\Delta N)(0) = \Delta(0)$; the gain $\|\,(U + Q\widetilde{D})\Delta N\,\| \geq |\,\Delta(0)\,|$ for any $Q \in \mathcal{R}$. The existence of $Q \in \mathcal{M}(\mathcal{R})$ satisfying (3) cannot be concluded using the small-gain condition since there is no restriction on the "gain" of Δ. Now apply the conditions of Corollary 2.5: Since P is strictly-proper, P and $(I+\Delta)P$ are simultaneously $\mathcal{R}$-stabilizable for any $\Delta \in \mathcal{R}$ such that $\Delta(0) > -1$, and $\Delta(1) > -1$.

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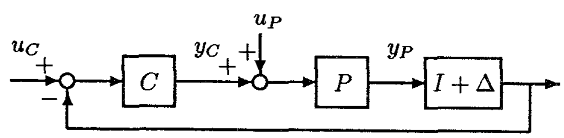

Figure 1: The system $\mathcal{S}(\Delta, P, C)$.