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Stabilization of Linear Systems Under Nonlinear Stable Diagonal Perturbations

A. N. Gündeş and M. G. Kabuli

***Abstract*—Stability of linear, time-invariant, multi-input multi-output unity-feedback systems is considered under nonlinear, time-varying, stable perturbations. Necessary and sufficient conditions are obtained for stability of the perturbed system and specialized for the case of one arbitrary failure whose location is unknown. Controller design methods are developed ensuring stability under an unknown stable failure of at most one arbitrary sensor or actuator.**

I. Introduction

We consider the stability of the standard linear, time-invariant (LTI), multi-input multi-output (MIMO) unity-feedback system (called $S(P, C)$) under nonlinear, time-varying (NLTV), stable, diagonal perturbations of the plant. We refer to post-multiplicative diagonal perturbations on the plant as sensor failures and pre-multiplicative ones as actuator failures.

The problem studied here is a generalization of the system integrity problem, which requires maintaining closed-loop stability in the presence of disconnection failures of any number of sensor or actuator channels [5]. The standard integrity problem considers a specific failure class, where the sensor or the actuator channel is completely disconnected when it fails. An unrestricted failure description is used here, allowing the corresponding output to be perturbed by any arbitrary stable NLTV map (including zero) in case of failure.

For single-input single-output systems, stability robustness is guaranteed under complete actuator or sensor failure if and only if both the plant and the stabilizing controller are stable. Requiring MIMO systems to have complete integrity against simultaneous failure of all sensors or of all actuators also restricts the plants and the stabilizing controllers to be stable. Motivated by the fact that the plant and the controller need not be stable when all sensor or actuator channels are not expected to fail simultaneously, we examine the case of at most one unknown stable perturbation in any one of the sensors or actuators without restrictions on the nature or location of the failure. For this case of one arbitrary failure whose location is unknown, we obtain necessary and sufficient conditions for stability of the perturbed system; for certain classes of plants, we develop algebraic controller design methods, which ensure simultaneous stabilization of the nominal system and any of the systems resulting from one loop failure.

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Following the system description and stability definitions, necessary and sufficient conditions for stability under failures is given in Theorem 2.2.1. The main analysis result is given in Theorem 2.3.1, where it is shown that closed-loop stability is achieved for all possible unrestricted stable NLTV perturbations of one sensor or actuator if and only if certain maps of the nominal LTI system have zero diagonal entries. Controller design methods are developed for two classes of plants in Section III. Assuming that any one of the sensors or actuators may fail one at a time, without prior knowledge of the failure location, a family of stabilizing controllers is explicitly derived in Proposition 3.2.1. These controllers are LTI and guarantee stability in the presence of NLTV failures.

The following notation is used in this note. Due to the input–output approach adopted, the setting can be continuous-time or discrete-time.

Notation: All NLTV maps considered are causal and are defined over appropriate products of an extended space $\mathcal{L}_e$; see [2] for a thorough study of general extended spaces within the input–output approach to nonlinear systems. Bold-face letters are used to denote nonlinear maps. A nonlinear map is called bicausal if it is causal and has a causal inverse. The set of bounded signals is denoted by $\mathcal{L}$, where the bound is determined by the associated norm $\|\cdot\|$. A causal NLTV map $\mathbf{H}$: $\mathcal{L}_e^{n_i} \rightarrow \mathcal{L}_e^{n_o}$ is said to be $\mathcal{L}$-stable iff there exists a continuous nondecreasing ϕ: $\mathbb{R}_+ \rightarrow \mathbb{R}_+$ such that $\|\mathbf{H}u\| \leq \phi(\|u\|)$ for all $u \in \mathcal{L}^{n_i}$ (see for example [3]). An $\mathcal{L}$-stable NLTV map $\mathbf{H}$ is $\mathcal{L}$-unimodular iff $\mathbf{H}^{-1}$ is also $\mathcal{L}$-stable. Italic letters are used to denote linear time-invariant (LTI) maps that have finite-dimensional state-space representations. With a slight abuse of notation, a causal LTI map and its associated proper transfer-function representation are denoted by the same italic letter. The identity map (of some appropriate dimension) is denoted by I; the j-th column of I is denoted by e_j. For LTI maps, the terms causal and proper, bicausal and biproper are used interchangeably. The set of matrices whose entries are in $\mathcal{R} \subset \mathcal{R}_p$ is denoted by $\mathcal{M}(\mathcal{R})$, where $\mathcal{R}_p$ denotes proper rational functions with real coefficients and $\mathcal{R}$ denotes proper rational functions which do not have any poles in the region of instability $\mathcal{U}$; here $\mathcal{U}$ is the extended closed right-half-plane (for continuous-time systems) or the complement of the open unit-disk (for discrete-time systems). A map M is called $\mathcal{R}$-stable iff $M \in \mathcal{M}(\mathcal{R})$; an $\mathcal{R}$-stable map M is called $\mathcal{R}$-unimodular iff M^{-1} is also $\mathcal{R}$-stable. All LTI maps considered here have coprime factorizations over $\mathcal{R}$. A right-coprime-factorization (RCF) and a left-coprime-factorization (LCF) of $P \in \mathcal{R}_p^{no \times ni}$ are denoted by (N_P, D_P) and $(\tilde{D}_P, \tilde{N}_P)$, respectively; N_P, D_P, $\tilde{N}_P$, $\tilde{D}_P \in \mathcal{M}(\mathcal{R})$, D_P and $\tilde{D}_P$ are biproper and $P = N_P D_P^{-1} = \tilde{D}_P^{-1}\tilde{N}_P$. An RCF and LCF of $C \in \mathcal{R}_p^{ni \times no}$ are denoted by (N_C, D_C) and $(\tilde{D}_C, \tilde{N}_C)$. The composition of two NLTV maps $\mathbf{F}$ and $\mathbf{G}$ is denoted by the map $\mathbf{FG}$. For NLTV maps $\mathbf{A}$ and $\mathbf{B}$ of appropriate dimensions, the augmented map $[\mathbf{A} \quad \mathbf{B}]$ is defined by $[\mathbf{A} \quad \mathbf{B}]\begin{bmatrix} x \\ y \end{bmatrix} := \mathbf{A}(x) + \mathbf{B}(y)$. The notation $a := b$ means a is defined as b.

II. Analysis

A. System Description: $S(\mathbf{F}_S, P, \mathbf{F}_A, C)$

Consider the interconnection $S(\mathbf{F}_S, P, \mathbf{F}_A, C)$ shown in Fig. 1: $S(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is a well-posed system, where $P \in \mathcal{R}_p^{no \times ni}$ and $C \in \mathcal{R}_p^{ni \times no}$ represent the plant and the controller, respectively. It is assumed that P and C do not have any hidden modes associated with eigenvalues in $\mathcal{U}$. Let $\mathbf{F}_S$ and $\mathbf{F}_A$ be NLTV $\mathcal{L}$-stable maps representing sensor and actuator failures, respectively. If $\mathbf{F}_S$ and $\mathbf{F}_A$ are both

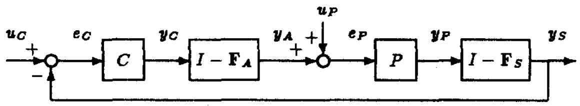

Fig 1. The system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$.

zero, then $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ becomes the standard unity-feedback system denoted by $\mathcal{S}(P, C)$, which we call the nominal system.

The analysis results presented in Sections II-A and II-B do not put restrictions on the NLTV $\mathcal{L}$-stable maps $\mathbf{F}_S$ and $\mathbf{F}_A$. Starting with Section II-C, it is assumed that $\mathbf{F}_S$ and $\mathbf{F}_A$ are diagonal NLTV $\mathcal{L}$-stable maps, where the failure of the j-th channel is represented by a NLTV $\mathcal{L}$-stable perturbation; furthermore, at most one of the n_o sensors (or n_i actuators) may fail and the particular channel which may fail is not known *a priori*. The class of sensor failures and the class of actuator failures corresponding to one failure are denoted by $\mathcal{F}_{S1}$ and $\mathcal{F}_{A1}$

$$\mathcal{F}_{S1} := \Big\{e_j \mathbf{f}_j e_j^T \mid \mathbf{f}_j\colon \mathcal{L}_e \to \mathcal{L}_e, \text{NLTV}, \; \mathcal{L}\text{-stable}, j = 1, \ldots, n_o\Big\},$$
$$\mathcal{F}_{A1} := \Big\{e_j \mathbf{f}_j e_j^T \mid \mathbf{f}_j\colon \mathcal{L}_e \to \mathcal{L}_e, \text{NLTV}, \; \mathcal{L}\text{-stable}, j = 1, \ldots, n_i\Big\}.$$

Under normal operation, $\mathbf{f}_j$ is zero; all other values of the NLTV $\mathcal{L}$-stable map $\mathbf{f}_j$ imply a failure; in particular, when $\mathbf{f}_j$ is the identity map, the corresponding channel is completely disconnected.

Using an RCF (N_P, D_P) of P and an RCF (N_C, D_C) of C, with $D_P \xi_P = e_P$ and $D_C \xi_C = e_C$, the system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is described in the form $\mathbf{D}_H \xi = u, \; y = N\xi$

$$\begin{bmatrix} D_P & -(I - \mathbf{F}_A)N_C \\ (I - \mathbf{F}_S)N_P & D_C \end{bmatrix} \begin{bmatrix} \xi_P \\ \xi_C \end{bmatrix} = \begin{bmatrix} u_P \\ u_C \end{bmatrix},$$
$$\begin{bmatrix} N_P & 0 \\ 0 & N_C \end{bmatrix} \begin{bmatrix} \xi_P \\ \xi_C \end{bmatrix} = \begin{bmatrix} y_P \\ y_C \end{bmatrix}. \tag{1}$$

The system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is well-posed if and only if the map $\mathbf{D}_H$ is bicausal, equivalently, the closed-loop map $\mathbf{H}\colon (u_P, u_C) \mapsto (y_P, y_C)$ exists.

B. Conditions for Stability

A well-posed NLTV interconnection is said to be $\mathcal{L}$-stable iff the map from exogenous inputs to closed-loop signals is $\mathcal{L}$-stable. The notion of $\mathcal{L}$-stability is used only in the case of NLTV interconnections and analyses thereof. When the interconnections are LTI, the equivalent condition of $\mathcal{R}$-stability is used, namely, all closed-loop transfer-functions are in $\mathcal{M}(\mathcal{R})$.

Following standard definitions (see for example [10]), the nominal system $\mathcal{S}(P, C)$ is said to be $\mathcal{R}$-stable iff the closed-loop transfer-function H from $u := (u_P, u_C)$ to $y := (y_P, y_C)$ is $\mathcal{R}$-stable. Similarly, when $\mathbf{F}_S$ and $\mathbf{F}_A$ are $\mathcal{L}$-stable, $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is said to be $\mathcal{L}$-stable iff the NLTV closed-loop map $\mathbf{H}\colon u \mapsto y$ is $\mathcal{L}$-stable (see for example [3]). The controller C is said to be an $\mathcal{R}$-stabilizing controller for P in the nominal system $\mathcal{S}(P, C)$ iff $C \in \mathcal{R}_p^{n_i \times n_o}$ and $\mathcal{S}(P, C)$ is $\mathcal{R}$-stable. The controller C is an $\mathcal{R}$-stabilizing controller for P if and only if there exist an RCF (N_C, D_C) and an LCF $(\tilde{D}_C, \tilde{N}_C)$ of C satisfying the following identity [10], [6]

$$\begin{bmatrix} \tilde{D}_C & \tilde{N}_C \\ -\tilde{N}_P & \tilde{D}_P \end{bmatrix} \begin{bmatrix} D_P & -N_C \\ N_P & D_C \end{bmatrix} = I. \tag{2}$$

It is well known that C is an $\mathcal{R}$-stabilizing controller for P if and only if C is given by [10], [6]

$$C = (V_P - Q\tilde{N}_P)^{-1}(U_P + Q\tilde{D}_P) = (\tilde{U}_P + D_P Q)(\tilde{V}_P - N_P Q)^{-1} \tag{3}$$

for some $\mathcal{R}$-stable Q such that $(V_P - Q\tilde{N}_P)$ is biproper (which holds for all $Q \in \mathcal{M}(\mathcal{R})$ when P is strictly-proper), where $U_P, V_P, \tilde{U}_P, \tilde{V}_P$ are $\mathcal{R}$-stable matrices such that $V_P D_P + U_P N_P = I$, $\tilde{D}_P \tilde{V}_P + \tilde{N}_P \tilde{U}_P = I$, $V_P \tilde{U}_P = U_P \tilde{V}_P$. In fact, a well-posed interconnection $\mathcal{S}(P, \mathbf{C})$, where $\mathbf{C}$ is an NLTV controller, is $\mathcal{L}$-stable if and only if $\mathbf{C} = (\tilde{U}_P + D_P \mathbf{Q})(\tilde{V}_P - N_P \mathbf{Q})^{-1}$ for some NLTV, $\mathcal{L}$-stable map $\mathbf{Q}$ such that $(\tilde{V}_P - N_P \mathbf{Q})$ is bicausal [4].

Theorem 2.2.1—$\mathcal{L}$-Stability Under Both Sensor and Actuator Failures):

a) Let (N_P, D_P) be any RCF and $(\tilde{D}_P, \tilde{N}_P)$ be any LCF of P; let (N_C, D_C) be any RCF and $(\tilde{D}_C, \tilde{N}_C)$ be any LCF of C. Then $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable if and only if

$$\mathbf{D}_H := \begin{bmatrix} D_P & -(I - \mathbf{F}_A)N_C \\ (I - \mathbf{F}_S)N_P & D_C \end{bmatrix} \text{ is } \mathcal{L}\text{-unimodular.} \tag{4}$$

b) Let C be an $\mathcal{R}$-stabilizing controller for P in $\mathcal{S}(P, C)$; let $(\tilde{D}_C, \tilde{N}_C)$ be an LCF and (N_C, D_C) be an RCF of C satisfying (2). Then the following three conditions are equivalent:

i) $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable;

ii) the map $\mathbf{D}_H$ in (4) is $\mathcal{L}$-unimodular;

iii) the map

$$\hat{\mathbf{D}}_H := \begin{bmatrix} I - \tilde{N}_C \mathbf{F}_S N_P & \tilde{D}_C \mathbf{F}_A N_C \\ -\tilde{D}_P \mathbf{F}_S N_P & I - \tilde{N}_P \mathbf{F}_A N_C \end{bmatrix} \text{is } \mathcal{L}\text{-unimodular.} \tag{5}$$

Proof of Theorem 2.2.1:

a) If $\mathbf{D}_H$ is $\mathcal{L}$-unimodular, then by (1), since the map N is $\mathcal{L}$-stable, the closed-loop map $\mathbf{H} = \mathbf{N}\mathbf{D}_H^{-1}$ is $\mathcal{L}$-stable. Conversely, let U_P, V_P, U_C, V_C be $\mathcal{R}$-stable matrices such that $V_P D_P + U_P N_P = I$, $V_C D_C + U_C N_C = I$; such matrices exist since (N_P, D_P) and (N_C, D_C) are right-coprime pairs. Let D, V, U be block-diagonal matrices defined similarly as N and let

$$\mathbf{F} := \begin{bmatrix} \mathbf{0} & -(\mathbf{I} - \mathbf{F_A}) \\ (\mathbf{I} - \mathbf{F_S}) & \mathbf{0} \end{bmatrix}$$

then $\mathbf{D}_H = D + \mathbf{F}N$. If the system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable, then $\mathbf{H} = \mathbf{N}\mathbf{D}_H^{-1}$ $\mathcal{L}$-stable implies that $(U - V\mathbf{F})(N\mathbf{D}_H^{-1}) + V = [(U - V\mathbf{F})N + V\mathbf{D}_H]\mathbf{D}_H^{-1} = \mathbf{D}_H^{-1}$ is $\mathcal{L}$-stable.

b) The equivalence of i) and ii) was shown in part a) above for any RCF of C. To show the equivalence of ii) to iii), define the matrices in (2) as $MM^{-1} = I$. Let

$$\hat{\mathbf{F}} := \begin{bmatrix} \mathbf{0} & \mathbf{F_A} \\ -\mathbf{F_S} & \mathbf{0} \end{bmatrix}$$

then $\mathbf{D}_H = M^{-1} + \hat{\mathbf{F}}N$. Since M is $\mathcal{L}$-unimodular, it follows by composition of these two maps that $\mathbf{D}_H$ is $\mathcal{L}$-unimodular if and only if $M\mathbf{D}_H = (I + M\hat{\mathbf{F}}N) = \hat{\mathbf{D}}_H$ is $\mathcal{L}$-unimodular. □

From Theorem 2.2.1-b), letting either $\mathbf{F}_A$ or $\mathbf{F}_S$ be zero in the map $\hat{\mathbf{D}}_H$ of (5), the following necessary and sufficient conditions are obtained for $\mathcal{L}$-stability under either sensor or actuator failures: When the actuators have no failure $(\mathbf{F}_A = 0)$, $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable if and only if

$$I - \tilde{N}_C \mathbf{F}_S N_P \text{ is } \mathcal{L}\text{-unimodular.} \tag{6}$$

When the sensors have no failure $(\mathbf{F}_S = 0)$, $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable if and only if

$$I - \tilde{N}_P \mathbf{F}_A N_C \text{ is } \mathcal{L}\text{-unimodular.} \tag{7}$$

C. Nonlinear Perturbations of One Sensor or Actuator

In this Section we assume that $\mathbf{F}_S$ and $\mathbf{F}_A$ are diagonal NLTV $\mathcal{L}$-stable maps and at most one of the n_o sensors or one of the n_i actuators may fail, i.e., $\mathbf{F}_S \in \mathcal{F}_{S1}$ and $\mathbf{F}_A \in \mathcal{F}_{A1}$.

Theorem 2.3.1—($\mathcal{L}$-Stability for All Failures Either in $\mathcal{F}_{S1}$ or in $\mathcal{F}_{A1}$): Let the system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ be well posed. Let C be an $\mathcal{R}$-stabilizing controller for P in $\mathcal{S}(P, C)$; let $(\tilde{D}_C, \tilde{N}_C)$ be an LCF and (N_C, D_C) be an RCF of C satisfying (2).

a) Let $\mathbf{F}_A$ be the zero map. Then $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ if and only if all diagonal entries of $(N_P\tilde{N}_C)$ are equal to zero.

b) Let $\mathbf{F}_S$ be the zero map. Then $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_A \in \mathcal{F}_{A1}$ if and only if all diagonal entries of $(N_C\tilde{N}_P)$ are equal to zero.

Proof of Theorem 2.3.1: We prove a); the proof of b) is similar. With $\mathbf{F}_A$ equal to zero, the system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is well-posed if and only if $(I - \tilde{N}_C\mathbf{F}_S N_P)$ is bicausal. By Theorem 2.2.1, $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ if and only if (6) holds for all $\mathbf{F}_S = e_j\mathbf{f}_je_j^T$, $j = 1, \ldots, n_o$; equivalently, $(I - \tilde{N}_Ce_j\mathbf{f}_je_j^TN_P)$ is $\mathcal{L}$-unimodular. Define the jth diagonal entry of $(N_P\tilde{N}_C)$ as $e_j^TN_P\tilde{N}_Ce_j =: h_j$. Necessity: Suppose that $h_j \neq 0$; then we need to show that there exists an $\mathbf{f}_j$ such that $(I - \tilde{N}_Ce_j\mathbf{f}_je_j^TN_P)$ is not $\mathcal{L}$-unimodular. Since the NLTV failure class $\mathcal{F}_{S1}$ includes LTI failures, it suffices to find an LTI failure for $\mathbf{f}_j$. The advantage of $\mathbf{f}_j$ being LTI is that $(I - \mathbf{f}_je_j^TN_P\tilde{N}_Ce_j)^{-1}$ is $\mathcal{L}$-stable if and only if $(I - \tilde{N}_Ce_j\mathbf{f}_je_j^TN_P)^{-1}$ is $\mathcal{L}$-stable (note that this is not necessarily the case if $\mathbf{f}_j$ is not LTI). Now the scalar map $(I - \mathbf{f}_je_j^TN_P\tilde{N}_Ce_j) = (1 - \mathbf{f}_jh_j)^{-1}$ clearly cannot be $\mathcal{L}$-stable for all possible choices of $\mathbf{f}_j$ since h_j is not zero. To see this, let s_o be in undesirable region $\mathcal{U}$ such that $h_j(s_o) \neq 0$; then using standard arguments, it is easy to construct a $\mathcal{R}$-stable LTI $\mathbf{f}_j$ such that $\mathbf{f}_j(s_o) = (h_j(s_o))^{-1}$ (see for example [10], Section 7.4). Since $(1 - \mathbf{f}_jh_j)^{-1}$ has a pole at $s_o \in \mathcal{U}$ by construction, it is not $\mathcal{L}$-stable. Sufficiency: If $h_j = 0$, then the scalar map $I = (I - h_j\mathbf{f}_j) = (I - e_j^TN_P\tilde{N}_Ce_j\mathbf{f}_j)$ is $\mathcal{L}$-unimodular. Define $e_j^TN_P =: A$ and $\tilde{N}_Ce_j\mathbf{f}_j =: \mathbf{B}$. Since A is linear, $(I - A\mathbf{B})^{-1}$ $\mathcal{L}$-stable implies that $(I + \mathbf{B}(I - A\mathbf{B})^{-1}A) = (I + \mathbf{B}A(I - \mathbf{B}A)^{-1}) = (I - \mathbf{B}A)^{-1}$ is $\mathcal{L}$-stable; $(I - \mathbf{B}A)^{-1}$ exists since $(I - \mathbf{B}A)$ is bicausal by the well-posedness assumption on $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$. Therefore, $(I - \tilde{N}_Ce_j\mathbf{f}_je_j^TN_P)$ is $\mathcal{L}$-unimodular and hence, $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable. □

Theorem 2.3.1 establishes that $\mathcal{L}$-stability of $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ under arbitrary failures of one sensor is achievable by the $\mathcal{R}$-stabilizing controller C if and only if C is such that the closed-loop transfer-function H_{pc}: $u_C \mapsto y_P$ of the nominal system $\mathcal{S}(P, C)$ has all zero diagonal entries. Similarly, $\mathcal{L}$-stability of $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ under arbitrary failures of one actuator is achievable by an $\mathcal{R}$-stabilizing controller C which guarantees zero diagonal entries for the transfer-function H_{cp}: $u_C \mapsto y_P$ of $\mathcal{S}(P, C)$. It is clear from the proof that the conditions of zero diagonal entries are still necessary and sufficient even if the failures are restricted to only stable LTI maps.

Using the expressions for $\tilde{N}_C$ and N_C given by (3) in the conditions of Theorem 2.3.1, we observe the following: If the actuators have no failure ($\mathbf{F}_A$ is zero), then $C \in \mathcal{M}(\mathcal{R}_p)$ is a controller such that $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ if and only if C is given by (3) for some $\mathcal{R}$-stable $Q \in \mathcal{R}^{ni \times no}$ such that all diagonal entries of $N_P(U_P + Q\tilde{D}_P)$ are equal to zero. Similarly, if the sensors have no failure ($\mathbf{F}_S$ is zero), then $C \in \mathcal{M}(\mathcal{R}_p)$ is a controller such that $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_A \in \mathcal{F}_{A1}$ if and only if C is given by (3) for some $\mathcal{R}$-stable $Q \in \mathcal{R}^{ni \times no}$ such that all diagonal entries of $(\tilde{U}_P + D_PQ)\tilde{N}_P$ are equal to zero. Note that the controller is proper if and only if $Q \in \mathcal{R}^{ni \times no}$ is such that $(V_P - Q\tilde{N}_P)$ is biproper; this condition holds for all $Q \in \mathcal{M}(\mathcal{R})$ if the plant is strictly-proper.

III. Controller Design

Throughout this section we assume that $\mathbf{F}_S$ and $\mathbf{F}_A$ are diagonal NLTV $\mathcal{L}$-stable maps, which belong to the failure classes $\mathcal{F}_{S1}$ and $\mathcal{F}_{A1}$, respectively. We show controller design methods for the system $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ with possible sensor failures in the class $\mathcal{F}_{S1}$ or actuator failures in the class $\mathcal{F}_{A1}$. Clearly, controllers achieving $\mathcal{L}$-stability under all possible failures of one sensor or one actuator may not exist for some plants. We now describe two classes of plants and associated design methods. These two classes of plants are **1**) $\mathcal{R}$-stable plants (Section III-A) and 2) a class of not necessarily $\mathcal{R}$-stable plants for which certain nominal maps can be decoupled (Section III-B).

A. Controller Design for $\mathcal{R}$-Stable Plants

Let $P \in \mathcal{M}(\mathcal{R})$; by Theorem 2.3.1, the set of all controllers such that $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ (or for all $\mathbf{F}_A \in \mathcal{F}_{A1}$) is given by (8) (or 9), respectively

$$\{C = Q(I - PQ)^{-1} \mid Q \in \mathcal{R}^{ni \times no},\ e_j^TPQe_j = 0,\ j = 1, \ldots, n_o\} \tag{8}$$

$$\{C = (I - QP)^{-1}Q \mid Q \in \mathcal{R}^{ni \times no},\ e_j^TQPe_j = 0,\ j = 1, \ldots, n_i\}. \tag{9}$$

In the sets of all controllers (8) and (9), the $\mathcal{R}$-stable controller parameter should also satisfy the condition that $(I - PQ)$ is biproper so that the controllers are proper (as remarked before, this holds automatically for all Q when the plant is strictly-proper). One method to choose $\mathcal{R}$-stable Q satisfying this condition with all diagonal entries of PQ (or QP) equal to zero is based on performing elementary-column-operations (or elementary-row-operations) over the ring $\mathcal{R}$. The controller $C = Q(I - PQ)^{-1}$ is in the set (8) if Q is chosen as follows:

a) If $n_o = n_i$, then there is an $\mathcal{R}$-unimodular map R such that (PR) is lower-triangular (see Hermite form in [10]); in this case, let $Q = (RQ_T)$, where Q_T is any lower-triangular $\mathcal{R}$-stable map with zero diagonal entries.

b) If $n_o < n_i$, then there is an $\mathcal{R}$-unimodular map R such that $PR = [P_T \vdots 0]$, where P_T is $(n_o \times n_o)$ lower-triangular; in this case, let $Q = R\begin{bmatrix} Q_T \\ Q_A \end{bmatrix}$, where Q_T is any $(n_o \times n_o)$ lower-triangular $\mathcal{R}$-stable map with zero diagonal entries and Q_A is a completely arbitrary $\mathcal{R}$-stable map of suitable size.

c) If $n_o > n_i$, then there is an $\mathcal{R}$-unimodular map R such that $PR = \begin{bmatrix} P_T \\ P_A \end{bmatrix}$, where P_T is $(n_i \times n_i)$ lower-triangular; in this case, let Q be $R[Q_T \vdots 0]$, where Q_T is any $(n_i \times n_i)$ lower-triangular $\mathcal{R}$-stable map with zero diagonal entries.

B. Controller Design for Unstable Plants

Suppose that the plant is not $\mathcal{R}$-stable. For linear, time-invariant failures, if the closed-loop system is $\mathcal{R}$-stable for all failures of one sensor or one actuator, then the denominator matrices of coprime factorizations of the plant must satisfy certain conditions [7]. Since these conditions are necessary for LTI failures, clearly they must be satisfied when we consider the wider class of NLTV failures. These conditions must hold whenever the failure modes include a disconnected channel, where the corresponding sensor output is multiplied by zero; therefore such constraints are not due to considering general nonlinear perturbations in possible failure modes but in fact they would be necessary even for simple LTI failure subclasses. We briefly state these necessary conditions.

Consider the sensor failure case, i.e., let $\mathbf{F}_A$ be zero. Let $(\tilde{D}_P, \tilde{N}_P)$ be any LCF of P. For $j = 1, \cdots, n_o$, let $\tilde{d}_j := \tilde{D}_P e_j$ denote the jth column of $\tilde{D}_P$. If $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$, then each individual column $\tilde{d}_j$ of the denominator map $\tilde{D}_P$ has a (non-unique) left-inverse, denoted by $\tilde{y}_j \in \mathcal{R}^{1\times no}$ (equivalently, $\tilde{d}_j$ is full-rank for all $s \in \mathcal{U}$). Define $Y_{S1} \in \mathcal{R}^{no\times no}$ as the map whose j-th row is $\tilde{y}_j$; then the diagonal entries of $(Y_{S1}\tilde{D}_P)$ are all equal to one.

Similarly, consider now the actuator failure case, i.e., let $\mathbf{F}_S$ be zero. Let (N_P, D_P) be any RCF of P. For $j = 1, \cdots, n_i$, let d_j denote the jth row of D_P. If $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_A \in \mathcal{F}_{A1}$, then each individual row d_j of D_P has a (nonunique) right-inverse, denoted by $y_j \in \mathcal{R}^{no\times 1}$ (equivalently, d_j is full-rank for all $s \in \mathcal{U}$) [7]. Define $Y_{A1} \in \mathcal{R}^{ni\times ni}$ as the map whose jth column is y_j; then the diagonal entries of $(D_P Y_{A1})$ are all equal to one. Since these conditions on the plant's denominator $\tilde{D}_P(D_P)$ are necessary, we assume that they hold whenever sensor (actuator) failures are considered.

The maps Y_{S1} and Y_{A1} described above are clearly nonunique; Y_{S1} is any map whose rows are the nonunique inverses of the columns of $\tilde{D}_P$ and Y_{A1} is any map whose columns are the nonunique inverses of the rows of D_P. In Proposition 3.2.1, we obtain a class of controllers, parametrized by Y_{S1}, which achieve $\mathcal{L}$-stability of $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ under sensor failures for an important class of plants; a dual method is developed for the actuator failure case using Y_{A1}. We now explain the motivation for studying this class of plants:

Recall from Theorem 2.3.1 that $\mathcal{L}$-stability of $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ under arbitrary failures of one sensor (actuator) is achieved if and only if all diagonal entries of the closed-loop transfer-function H_{pc} (H_{cp} for the actuator failure case) of the nominal system $\mathcal{S}(P, C)$ are equal to zero. For the sensor failure case, using the controller parametrization (3), the goal is then to make all diagonal entries of $H_{pc} = N_P\tilde{N}_C = N_P(U_P + Q\tilde{D}_P)$ equal to zero. Observe that if an $\mathcal{R}$-stable U_P could be found such that $(N_P U_P)$ is diagonal, then choosing $Q = -U_P Y_{S1}$, the map $H_{pc} = N_P U_P(I - Y_{S1}\tilde{D}_P)$ has all zero diagonal entries by construction of Y_{S1} . So it is clear that controllers achieving $\mathcal{L}$-stability of $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ under arbitrary failures of one sensor can be constructed by starting with a decoupling controller (i.e., the controller $V_P^{-1}U_P$ such that $(N_P U_P)$ is diagonal) and then using the controller $C = (V_P - (-U_P Y_{S1})\tilde{N}_P)^{-1}(U_P + (-U_P Y_{S1})\tilde{D}_P)$ in the final design to achieve zero diagonal entries for H_{pc}. This is the motivation for considering plants that can be decoupled, i.e., for the sensor failure case, plants for which there exist $\mathcal{R}$-stabilizing controllers such that the map $H_{pc} = PC(I + PC)^{-1}$ of the nominal system $\mathcal{S}(P, C)$ is diagonal and for the dual actuator failure case, plants for which there exist $\mathcal{R}$-stabilizing controllers such that the map $H_{cp} = -CP(I+CP)^{-1}$ of $\mathcal{S}(P, C)$ is diagonal. (These plant classes are nonempty; a sufficient condition for decoupling is that the plant is full row-rank and has no coinciding poles and zeros in $\mathcal{U}$. See [8] for the parametrization of all controllers which achieve decoupling and the set of all achievable diagonal maps H_{pc} or [9] for a decoupling controller design method).

Proposition 3.2.1—A Set of Controllers Achieving $\mathcal{L}$-Stability for One Failure:

a) Let $\mathbf{F}_A$ be the zero map. Let P be such that, for any LCF $(\tilde{D}_P, \tilde{N}_P)$, each column of $\tilde{D}_P$ has a left-inverse in $\mathcal{R}$. Assume that there is an $\mathcal{R}$-stabilizing controller for P such that the map $H_{pc} = PC(I + PC)^{-1}$ is diagonal. Under these assumptions, a class of controllers C such that $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ is given by

$$\begin{aligned}\{C &= (\tilde{D}_{S\mathcal{D}} + \tilde{N}_{S\mathcal{D}}Y_{S1}\tilde{N}_P)^{-1}(\tilde{N}_{S\mathcal{D}} - \tilde{N}_{S\mathcal{D}}Y_{S1}\tilde{D}_P)\\ &= (N_{S\mathcal{D}} - D_P\tilde{N}_{S\mathcal{D}}Y_{S1})(D_{S\mathcal{D}} + N_P\tilde{N}_{S\mathcal{D}}Y_{S1})^{-1}\\ &= C_{S\mathcal{D}}(I + Y_{S1}\tilde{N}_P C_{S\mathcal{D}})^{-1}(I - Y_{S1}\tilde{D}_P)\}\end{aligned} \tag{10}$$

where Y_{S1} is any map such that the diagonal entries of $(Y_{S1}\tilde{D}_P)$ are equal to one; $C_{S\mathcal{D}}$ is any $\mathcal{R}$-stabilizing controller for P, such that the map $H_{pc} = N_P N_{S\mathcal{D}}$ is diagonal; $(\tilde{D}_{S\mathcal{D}}, \tilde{N}_{S\mathcal{D}})$ and $(N_{S\mathcal{D}}, D_{S\mathcal{D}})$ are any LCF and RCF of $C_{S\mathcal{D}}$ satisfying (2); $(\tilde{D}_{S\mathcal{D}} + \tilde{N}_{S\mathcal{D}}Y_{S1}\tilde{N}_P)$ is bicausal.

b) Let $\mathbf{F}_S$ be the zero map. Let P be such that, for any RCF (N_P, D_P), each row of D_P has a right-inverse in $\mathcal{R}$. Assume that there is an $\mathcal{R}$-stabilizing controller for P such that the map $H_{cp} = -CP(I + CP)^{-1}$ is diagonal. Under these assumptions, a class of controllers C such that $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_A \in \mathcal{F}_{A1}$ is given by

$$\begin{aligned}\{C &= (\tilde{D}_{A\mathcal{D}} + Y_{A1}N_{A\mathcal{D}}\tilde{N}_P)^{-1}(\tilde{N}_{A\mathcal{D}} - Y_{A1}N_{A\mathcal{D}}\tilde{D}_P)\\ &= (N_{A\mathcal{D}} - D_P Y_{A1}N_{A\mathcal{D}})(D_{A\mathcal{D}} + N_P Y_{A1}N_{A\mathcal{D}})^{-1}\\ &= (I - D_P Y_{A1})(I + C_{A\mathcal{D}}N_P Y_{A1})^{-1}C_{A\mathcal{D}}\}\end{aligned} \tag{11}$$

where Y_{A1} is any map such that the diagonal entries of $(D_P Y_{A1})$ are equal to one; $C_{A\mathcal{D}}$ is any $\mathcal{R}$-stabilizing controller for P, such that the map $H_{cp} = N_{A\mathcal{D}}\tilde{N}_P$ is diagonal; $(\tilde{D}_{A\mathcal{D}}, \tilde{N}_{A\mathcal{D}})$ and $(N_{A\mathcal{D}}, D_{A\mathcal{D}})$ are any LCF and RCF of $C_{A\mathcal{D}}$ satisfying (2); $(\tilde{D}_{A\mathcal{D}} + Y_{A1}N_{A\mathcal{D}}\tilde{N}_P)$ is bicausal.

Proof of Proposition 3.2.1: We prove a); the proof of b) is similar. The controller C in (10) has an LCF $(\tilde{D}_C, \tilde{N}_C) := ((\tilde{D}_{S\mathcal{D}} - Q\tilde{N}_P), (\tilde{N}_{S\mathcal{D}} + Q\tilde{D}_P))$, where $Q = -\tilde{N}_{S\mathcal{D}}Y_{S1}$. With this $\tilde{N}_C$, the map $(N_P\tilde{N}_C)$ becomes $N_P\tilde{N}_{S\mathcal{D}}(I - Y_{S1}\tilde{D}_P)$; since $(N_P\tilde{N}_{S\mathcal{D}})$ is diagonal and $(I - Y_{S1}\tilde{D}_P)$ has zero diagonal entries, the diagonal entries of $(N_P\tilde{N}_C)$ are all zero, which implies that $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ is $\mathcal{L}$-stable for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ by Theorem 2.3.1.

The controller C in (10) is proper if and only if $\tilde{D}_C^{-1}$ is proper. If either N_P or $\tilde{N}_{S\mathcal{D}}$ is strictly-proper (equivalently, P or $C_{S\mathcal{D}}$ is strictly-proper), $C = C_{S\mathcal{D}}(I + Y_{S1}\tilde{N}_P C_{S\mathcal{D}})^{-1}(I - Y_{S1}\tilde{D}_P)$ is proper. So if P is strictly-proper, the controller C in (10) is proper for any choice of $C_{S\mathcal{D}}$. If P is not strictly-proper, then given any $\mathcal{R}$-stabilizing controller C_d for P such that the map H_{pc} is diagonal, there exists a strictly-proper controller $C_{S\mathcal{D}}$; one way to construct a strictly-proper $C_{S\mathcal{D}}$ is as follows: Let $(\tilde{D}_d, \tilde{N}_d)$ be any LCF of C_d satisfying (2). Let X_d be any diagonal $\mathcal{R}$-stable map such that $X_d(\infty) = -\det\tilde{D}_P^{-1}(\infty)I$, where the determinant $\det\tilde{D}_P(\infty) \neq 0$ since $\tilde{D}_P$ is biproper. Let $X := (\det\tilde{D}_P)(\tilde{N}_d X_d\tilde{D}_P^{-1})$; then X is $\mathcal{R}$-stable since $(\det\tilde{D}_P)\tilde{D}_P^{-1}$ is $\mathcal{R}$-stable. Let $C_{S\mathcal{D}} = \tilde{D}_{S\mathcal{D}}^{-1}\tilde{N}_{S\mathcal{D}}$, where $(\tilde{D}_{S\mathcal{D}}, \tilde{N}_{S\mathcal{D}}) := ((\tilde{D}_d - X\tilde{N}_P), (\tilde{N}_d + X\tilde{D}_P))$ is an LCF of $C_{S\mathcal{D}}$ satisfying (2); then $\tilde{N}_{S\mathcal{D}}$ is strictly-proper and furthermore, $H_{pc} = N_P\tilde{N}_{S\mathcal{D}}$ is diagonal. Therefore, the existence of an $\mathcal{R}$-stabilizing controller C_d diagonalizing H_{pc} implies the existence of a strictly-proper controller $C_{S\mathcal{D}}$ also diagonalizing H_{pc}; this strictly-proper controller $C_{S\mathcal{D}}$ used in (10) guarantees that C is proper. □

Example 3.2.2: Let

$$\begin{aligned}P &= \begin{bmatrix}\frac{s+1}{s-2} & 0\\ \frac{-(s-1)}{s-2} & \frac{s-3}{s+1}\end{bmatrix}\\ &= \begin{bmatrix}1 & 0\\ \frac{-(s-1)}{s+1} & \frac{s-3}{s+1}\end{bmatrix}\begin{bmatrix}\frac{s-2}{s+1} & 0\\ 0 & 1\end{bmatrix}^{-1}\\ &= \begin{bmatrix}\frac{s-2}{s+1} & 0\\ \frac{s-1}{s+1} & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 0\\ 0 & \frac{s-3}{s+1}\end{bmatrix}\\ &= N_P D_P^{-1} = \tilde{D}_P^{-1}\tilde{N}_P.\end{aligned}$$

Each column of $\tilde{D}_P$ is full-rank for all $s \in \mathcal{U}$. One choice for Y_{S1} is $\begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}$. A decoupling controller for P is

$$C_{S\mathcal{D}} = \tilde{D}_{S\mathcal{D}}^{-1}\tilde{N}_{S\mathcal{D}} = \begin{bmatrix} 4 & 0 \\ 0 & \frac{4(s-2)}{s+1} \end{bmatrix}^{-1} \begin{bmatrix} \frac{-3(s-3)}{s+1} & 0 \\ \frac{-3(s-1)}{s+1} & -3 \end{bmatrix}.$$

Note that $(N_P\tilde{N}_{S\mathcal{D}}) = \frac{-3(s-3)}{s+1}I$ is diagonal. The controller $C = (\tilde{D}_{S\mathcal{D}} + \tilde{N}_{S\mathcal{D}}Y_{S1}\tilde{N}_P)^{-1}(\tilde{N}_{S\mathcal{D}} - \tilde{N}_{S\mathcal{D}}Y_{S1}\tilde{D}_P)$ in (10) achieves $\mathcal{L}$-stability of $\mathcal{S}(\mathbf{F}_S, P, \mathbf{F}_A, C)$ for all $\mathbf{F}_S \in \mathcal{F}_{S1}$ since the nominal map

$$\begin{aligned} H_{pc} &= N_P\tilde{N}_C \\ &= N_P(\tilde{N}_{S\mathcal{D}} - \tilde{N}_{S\mathcal{D}}Y_{S1}\tilde{D}_P) \\ &= \frac{-3(s-3)}{s+1}\left(I - \begin{bmatrix} 1 & 3 \\ \frac{s-1}{s+1} & 1 \end{bmatrix}\right) \end{aligned}$$

has zero diagonal entries.

IV. Conclusion

We considered the stability of LTI, MIMO systems under stable NLTV diagonal post-multiplicative perturbations of the plant (sensor failures) and pre-multiplicative perturbations of the plant (actuator failures). Assuming that any one of the sensors or actuators may fail one at a time, without prior knowledge of the NLTV failure and its location, we obtained necessary and sufficient conditions for stabilization of the LTI plant using LTI controllers. We developed controller design methods for two classes of plants and explicitly derived a family of controllers achieving closed-loop stability under an unknown stable NLTV failure of at most one loop.

The closed-loop stability condition for all possible unrestricted stable NLTV perturbations of one sensor or actuator is that all diagonal entries of certain transfer functions of the nominal LTI system are exactly zero. Note that such a strict condition is due to the general unconstrained nature of the NLTV perturbations. If this condition could not be met and the diagonal entries were not zero (but their gains were bounded by sufficiently small ε), then as in standard robustness results based on the small-gain theorem, closed-loop stability is still guaranteed for a restricted class of perturbations (whose gains are bounded by $1/\varepsilon$).

The design goal stated above can easily be expressed as a convex design specification [1]. Despite the obvious advantages of convex problem formulations, optimization based design approaches cannot bring an explicit answer in cases where there does not exist such a controller (in general, not for a particular finite parametrization only) or make explicit use of the plant properties in successive redesigns (each new plant will be treated as a new design problem). Hence, whenever applicable, analytical solutions should be used to weed out infeasible constraints and initialize feasible ones. The results in this note are intended for such complementary utilization. The analysis results provide valuable necessary conditions on plants that admit such controllers. For two classes of plants, the explicit design approach developed here guarantees a desired nominal controller. The particular design approach complements the convex optimization based control design approach since it explicitly generates a family of feasible stable parameters for the controller. The freedom can then be used to satisfy other design specifications.

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Design of Observers for Descriptor Systems

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***Abstract*—In this note simple and straightforward methods to design full- and reduced-order observers for linear time-invariant descriptor systems are presented. The approach for the reduced-order observer design is based on the generalized Sylvester equation. Sufficient conditions for the existence of the observers are given. An illustrative example is included.**

I. Introduction

The problem of designing observers for descriptor systems has received considerable attention in the last two decades [1]–[7]. Many approaches exist to design observers for descriptor systems. In [1], a method based on the singular value decomposition and the concept of matrix generalized inverse to design a reduced-order observer has been proposed. In [2], the generalized Sylvester equation was used to develop a procedure for designing reduced-order observers. In [3], a method based on the generalized inverse, which extend the method developed in [8], was presented. Full- and reduced-order observers for discrete-time descriptor systems have been presented in [5] and [7].

All these works have been done under the assumptions of regularity and generalized observability. These conditions are very restrictive and, as can be seen in [6], the design of the observer can be done under less restrictive conditions. In [6], the conditions of regularity and modal observability have been assumed. In [10], a new method

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