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Design of High-Performance MIMO PI-Type Compensators

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Abstract

In this paper, we consider linear time-invariant multi-input multi-output (MIMO) systems that are stabilizable by proportional-integral (PI) compensators. Such systems are also stabilizable by a PI-type compensator to be denoted by PI/P. The PI/P compensator consists of a PI compensator in the feedforward loop preceding the plant to be controlled, and a proportional gain in a feedback loop around the plant. Systems that are controlled by the PI/P compensators achieve a better performance. We present an efficient and easy-to-implement algorithm for computing the optimal gains of the PI and PI/P compensators.

1. Introduction

The proportional-integral (PI) and the proportional-integral-derivative (PID) compensators are the most widely used compensators in industry. These compensators are simple and perform fairly well for large classes of systems when they are tuned properly. Thus, the tuning of the PI and PID compensators has received considerable attention by researchers and process control designers (see, e.g., [Åst.1, 2], [Koi.1], [Tan.1]).

Although the PI and PID compensators can be easily tuned and implemented, they may not achieve high performance. Therefore, it is desirable to slightly modify these compensators in order to improve their performance. One such modification of the PI compensator is the two-input PI-type compensator shown in Figure 1. In this figure, $K_{p1} + K_i/s$ is a PI compensator, K_{p2} is a proportional gain, and H is the plant to be controlled. The PI-type compensator in Figure 1 has been studied by several researchers for single-input single-output (SISO) systems (see, e.g., [Åst.1], [Kan.1], [Sha.1]). We adopt the notation in [Sha.1] to denote the compensator in Figure 1 by PI/P and the closed-loop system by $S(PI/P, H)$. Since the PI/P compensator has three gains, intuitively, it can outperform the PI compensators. In [Sha.1], it is shown that properly tuned PI/P compensators can achieve the following for linear SISO plants: (i) excellent tracking of step inputs with small or no overshoot; (ii) small control efforts to the plant H; (iii) a reliable closed-loop system which remains stable in case of some loops failures.

In this paper, we consider the problem of tuning MIMO PI/P compensators in order to achieve a better performance. The paper is presented in short form. The entire paper can be found in [Sha.2].

2. The Closed-Loop System

In this section, we first study the stabilizability of the closed-loop system $S(PI/P, H)$ which incorporates the PI/P compensator and a linear plant H. Then, we derive some of the transfer functions corresponding to the closed-loop system.

Consider the system $S(PI/P, H)$ in Figure 1. In this system, let H be a linear time-invariant, strictly proper, and square MIMO plant with a minimal state-space realization

$$\dot{x}(t) = A\, x(t) + B\, u(t), \quad x(0) = \theta_n, \tag{2.1a}$$

$$y(t) = C\, x(t), \tag{2.1b}$$

for all $t \geq 0$. In (2.1), the state vector $x(t) \in \mathbb{R}^n$, the input vector $u(t) \in \mathbb{R}^m$, and the output vector $y(t) \in \mathbb{R}^m$; the coefficient matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $C \in \mathbb{R}^{m \times n}$; the vector θ_n denotes the zero vector in $\mathbb{R}^n$. We denote the $m \times m$ transfer function of the plant by $H(s)$.

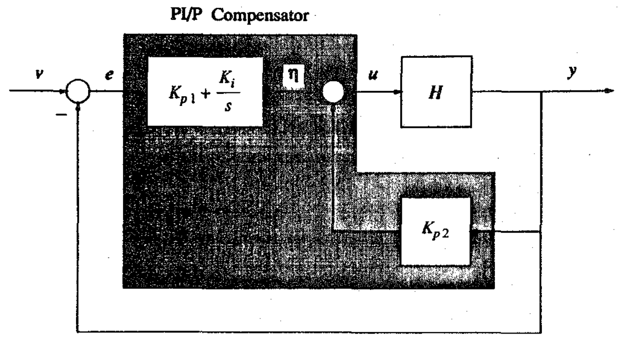

Figure 1. The feedback system $S(PI/P, H)$.

In $S(PI/P, H)$, the PI/P compensator consists of the proportional gain matrix $K_{p2} \in \mathbb{R}^{m \times m}$ in a feedback loop around the plant H, and the PI compensator $K_{p1} + K_i/s$ in the feedforward loop. A state-space realization of the PI compensator is

$$\dot{\xi}(t) = e(t), \quad \xi(0) = \theta_m, \tag{2.2a}$$

$$\eta(t) = K_i\, \xi(t) + K_{p1}\, e(t), \tag{2.2b}$$

for all $t \geq 0$. In (2.2), the state vector $\xi(t) \in \mathbb{R}^m$, the input vector $e(t) \in \mathbb{R}^m$, and the output vector $\eta(t) \in \mathbb{R}^m$; the gain matrices K_{p1}, $K_i \in \mathbb{R}^{m \times m}$.

Having the plant H and the PI/P compensator, we obtain the state-space realization of the closed-loop system $S(PI/P, H)$ as

$$\dot{X}(t) = A_c\, X(t) + B_c\, v(t), \tag{2.3a}$$

$$e(t) = C_e\, X(t) + D_e\, v(t), \tag{2.3b}$$

$$u(t) = C_u\, X(t) + D_u\, v(t), \tag{2.3c}$$

$$y(t) = C_y\, X(t), \tag{2.3d}$$

for all $t \geq 0$, where

$$X(t) := [\, x^T(t) \quad \xi^T(t)\,]^T, \tag{2.4}$$

and A_c, B_c, C_e, D_e, C_u, D_u, and C_y are the coefficient matrices that depend on the gain matrices K_{p1}, K_{p2}, and K_i.

We *assume* that:

(A1) The plant H is stabilizable by a PI compensator.

An easy-to-check necessary condition for the stabilizability of the plant H by a PI compensator is:

Assertion 2.1: The plant H is stabilizable by a PI compensator only if $H(0) \in \mathbb{R}^{m \times m}$ is nonsingular. □

We next show that the stabilizability of the plant H by a PI/P compensator is equivalent to its stabilizability by a PI compensator.

Assertion 2.2: The plant H is stabilizable by a PI/P compensator if and only if it is stabilizable by a PI compensator. □

Assertion 2.2 allows us to use a PI/P compensator instead of a PI compensator for plants which are stabilizable by PI compensators. The reason we wish to use a PI/P compensator is to achieve a better performance.

We will tune the PI/P compensator so that $e(\cdot)$ in (2.3b) is small and $u(\cdot)$ in (2.3c) is of a reasonable size. Thus, we should know the transfer functions from v to e and u, denoted by $H_{ev}(s)$ and $H_{uv}(s)$, respectively.

Assertion 2.3: Consider the system $S(PI/P, H)$. The error e and the input u to the plant H are related to the exogenous input v according to

$$e(s) = H_{ev}(s)\, v(s), \qquad (2.5a)$$

$$u(s) = H_{uv}(s)\, v(s), \qquad (2.5b)$$

where

$$H_{ev}(s) = [I_m + H(s)(K_{p1} + K_{p2}) + \frac{H(s)}{s} K_i]^{-1} [I_m + H(s) K_{p2}], \qquad (2.6a)$$

$$H_{uv}(s) = [I_m + (K_{p1} + K_{p2}) H(s) + K_i \frac{H(s)}{s}]^{-1} (K_{p1} + \frac{K_i}{s}), \qquad (2.6b)$$

where I_m denotes the $m \times m$ identity matrix. □

3. Optimal Tuning of the Gain Matrices

In this section, we devise an algorithm for computing the gain matrices of the PI and PI/P compensators. Our methodology is suitable for implementation in computer-aided design environments.

Our goal is to have the outputs of the system $S(PI/P, H)$ track step inputs. Thus, we will consider a transfer function slightly different from $H_{ev}(s)$ in (2.6a), in order to take step inputs into account. Let the vector of exogenous inputs to the system be $v(t) = \bar{v}\, \mathbf{1}(t) \in \mathbb{R}^m$, $t \geq 0$, where $\mathbf{1}(t)$ denotes the unit step function, and $\bar{v} = [\bar{v}_1 \ \bar{v}_2 \ldots \bar{v}_m]^T \in \mathbb{R}^m$ is the vector of the amplitudes. Since $v(s) = \bar{v}/s$, the error due to v is

$$e(s) = \frac{H_{ev}(s)}{s} \bar{v}, \qquad (3.1)$$

where $H_{ev}(s)$ is given in (2.6a). The transfer function we will be using to develop our algorithm for computing the gain matrices of the compensators is $H_{ev}(s)/s$. We denote this transfer function by $G_{ev}(s)$. Clearly,

$$G_{ev}(s) = \frac{1}{s} [I_m + H(s)(K_{p1} + K_{p2}) + \frac{H(s)}{s} K_i]^{-1} [I_m + H(s) K_{p2}]. \qquad (3.2)$$

A state-space realization of $G_{ev}(s)$ is (see [Sha.2])

$$\dot{\tilde{X}}(t) = \tilde{A}_c\, \tilde{X}(t) + \tilde{B}_c\, w(t), \qquad (3.3a)$$

$$e(t) = \tilde{C}_e\, \tilde{X}(t), \qquad (3.3b)$$

for all $t \geq 0$, where

$$\tilde{X}(t) := [X^T(t) \quad v^T(t)]^T, \qquad (3.4a)$$

$$w(t) = \bar{v}\, \delta(t), \qquad (3.4b)$$

in which $X(t)$ is that given in (2.4), $\delta(t)$ is the unit impulse function, and

$$\tilde{A}_c := \begin{bmatrix} A_c & B_c \\ 0 & 0 \end{bmatrix}, \quad \tilde{B}_c := \begin{bmatrix} 0 \\ I_m \end{bmatrix}, \qquad (3.5a)$$

$$\tilde{C}_e := [C_e \quad D_e]. \qquad (3.5b)$$

It can be easily checked that the system (3.3) has m unobservable modes corresponding to $s = 0$ [Sha.2]. Hence, (3.3) is not a minimal state-space realization of $G_{ev}(s)$. We obtain a minimal realization by using a program from MATLAB, which computes a minimal realization of uncontrollable and/or unobservable systems. We input $[\tilde{A}_c, \tilde{B}_c, \tilde{C}_e, 0]$ to the program to obtain a minimal realization denoted by

$$R_{\min} = [A_{\min}, B_{\min}, C_{\min}, D_{\min}]. \qquad (3.6)$$

In the following, we will use the H_∞-norm of transfer functions, say $P(s)$, given by $\|P\|_\infty := \sup_{\omega \in \mathbb{R}} \bar{\sigma}(P(j\omega))$, where $\bar{\sigma}(M)$ denotes the largest singular value of a matrix M.

With this preliminary, we cast the problem of choosing the gain matrices K_{p1}, K_{p2}, and K_i as an optimization problem.

Problem 3.1: Consider the system (2.3a) - (2.3b), and the system (3.3) whose minimal realization is (3.6). Let a scalar-valued cost function J be defined as

$$J = q\, \|G_{ev}\|_\infty + r\, \|H_{uv}\|_\infty, \qquad (3.7)$$

where the constants $q > 0$ and $r > 0$ are weighting factors, and

$$G_{ev}(s) = \left[\begin{array}{c|c} A_{\min} & B_{\min} \\ \hline C_{\min} & D_{\min} \end{array}\right], \quad H_{uv}(s) = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_u & D_u \end{array}\right]. \qquad (3.8)$$

Let $k_{p1}^* > 0$, $k_{p2}^* > 0$, and $k_i^* > 0$ be given constant real numbers. Determine the gains matrices K_{p1}, K_{p2}, and K_i of the compensator PI/P, subject to $\bar{\sigma}(K_{p1}) \leq k_{p1}^*$, $\bar{\sigma}(K_{p2}) \leq k_{p2}^*$, and $\bar{\sigma}(K_i) \leq k_i^*$, such that J is minimized. □

Problem 3.1 can be solved efficiently when MATLAB is used. An easy-to-implement algorithm is given in [Sha.2]. In [Sha.2], for a two-input two-output plant the optimal gain matrices of the PI and PI/P compensators are computed. Excellent performance is achieved when the PI/P compensators is used.

References

[Åst.1] Åström, K. J. and Hägglund, T., 1988, *Automatic Tuning of PID Controllers*, Instrument Society of America, Research Triangle Park, North Carolina.

[Åst.2] Åström, K. J., Hägglund, T., Hang, C. C., and Ho, W. K., 1993, "Automatic Tuning and Adaptation for PID Controllers - A Survey," in *Adaptive Systems in Control and Signal Processing in 1992*, Dugard, L., M'Saad, M., and Landua, I. D. (Eds.), 4th IFAC Symposium on Adaptive Systems in Control and Signal Processing, Grenoble, France, June 1992, pp. 371-376, Pergamon Press, New York, New York.

[Kan.1] Kanda, M., Iino, Y., and Shigemasa, T., July 1990, "Two Degrees of Freedom PID Auto-Tuning Controller with Supervisor Function," in *Proceedings of the 29th SICE Annual Conference*, pp. 813-816, The Society of Instrument and Control Engineers (SICE), Tokyo, Japan.

[Koi.1] Koivo, H. N. and Tanttu, J. T., January 1991, "Tuning of PID Controllers: Survey of SISO and MIMO techniques," in *Intelligent Tuning and Adaptive Control*, IFAC Symposia Series, Number 7, Devanathan, R. (Ed.), IFAC Symposium on Intelligent Tuning and Adaptive Control, Singapore, pp. 75-80, Pergamon Press, New York, New York.

[Sha.1] Shahruz, S. M., June 1994, "Design of Optimal and Reliable PI-Type Compensators for Linear Single-Input Single-Output Systems," *Proceedings of the 1994 American Control Conference*, pp. 2722-2723.

[Sha.2] Shahruz, S. M., Godbole, D. N., and Gündes, A. N., February 1994, "Design of High-Performance MIMO PI-Type Compensators," *BERI Memorandum*, No. BERI M94/1, Berkeley Engineering Research Institute, Berkeley, California.

[Tan.1] Tanttu, J. T., and Lieslehto, J., January 1991, "A Comparative Study of Some Multivariable PI Controller Tuning Methods," in *Intelligent Tuning and Adaptive Control*, IFAC Symposia Series, Number 7, Devanathan, R. (Ed.), IFAC Symposium on Intelligent Tuning and Adaptive Control, Singapore, pp. 357-362, Pergamon Press, New York, New York.