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Reliable Decentralized Control

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Abstract

We study reliable stabilization of linear, time-invariant, multi-input multi-output, two-channel decentralized control systems. We develop necessary and sufficient conditions for reliable decentralized stabilizability under sensor or actuator failures and present reliable decentralized controller design methods for strongly stabilizable plants.

1. Introduction

We consider the reliable stabilization problem using the linear, time-invariant (LTI), multi-input, multi-output (MIMO), two-channel decentralized system configuration $\mathcal{S}(P, C_d)$ (Figure 1). The reliable stabilization problem aims to find two controllers such that the system $\mathcal{S}(P, C_d)$ is stable when both controllers are acting together (normal mode) and when each controller is acting alone (failure mode). The failure of a controller is modeled by setting its transfer-function equal to zero.

A multi-controller system configuration achieving reliable stabilization was introduced in [5], [6]. Factorization methods were used in [2], [3], [9] to study reliable stability with two full-feedback controllers; it was shown that a given plant can be reliably stabilized with two full-feedback controllers if and only if it is strongly stabilizable (i.e., it can be stabilized using a stable controller) in the standard unity-feedback system. Reliable stabilization using a two-channel decentralized control system was considered in [7].

In this paper, we develop necessary and sufficient conditions for existence of decentralized controllers, which achieve reliable stability. For certain classes of plants we present decentralized controller design methods. Due to the algebraic methods used here, the results apply to continuous-time as well as discrete-time systems.

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2. Main Results

Notation: • Let $\mathcal{M}(\mathcal{R})$ be the set of matrices whose entries are in $\mathcal{R} \subset \mathcal{R}_p$, where $\mathcal{R}_p$ denotes proper rational functions with real coefficients and $\mathcal{R}$ denotes proper rational functions which do not have any poles in the region of instability $\mathcal{U}$; here $\mathcal{U}$ contains the extended closed right-half-plane (for continuous-time systems) or the complement of the open unit-disk (for discrete-time systems). A map M is called $\mathcal{R}$-stable iff $M \in \mathcal{M}(\mathcal{R})$; An $\mathcal{R}$-stable map M is $\mathcal{R}$-unimodular iff M^{-1} is also $\mathcal{R}$-stable. • Let the norm of an $\mathcal{R}$-stable map $M \in \mathcal{M}(\mathcal{R})$ be defined as $\| M \| = \sup_{s \in \partial\mathcal{U}} \bar{\sigma}(M(s))$, where $\bar{\sigma}$ denotes the maximum singular value and $\partial\mathcal{U}$ denotes the boundary of $\mathcal{U}$. • A right-coprime-factorization (RCF) and a left-coprime-factorization (LCF) of $P \in \mathcal{R}_p{}^{n_o \times n_i}$ are denoted by (N_P, D_P) and $(\tilde{D}_P, \tilde{N}_P)$, where $N_P, D_P, \tilde{N}_P, \tilde{D}_P \in \mathcal{M}(\mathcal{R})$, D_P and $\tilde{D}_P$ are biproper and $P = N_P D_P^{-1} = \tilde{D}_P^{-1} \tilde{N}_P$.

2.1. System description

Consider the LTI, MIMO, two-channel decentralized control system $\mathcal{S}(P, C_d)$ shown in Figure 1: $\mathcal{S}(P, C_d)$ is a well-posed system, where

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \in \mathcal{R}_p{}^{n_o \times n_i},$$

$$C_d = \operatorname{diag}[C_1, C_2] \in \mathcal{R}_p{}^{n_i \times n_o},$$

$C_j \in \mathcal{R}_p{}^{n_{ij} \times n_{oj}}$, $j = 1, 2$; P and C_d represent the plant and the decentralized controller, respectively. It is assumed that P and C_d do not have any hidden modes associated with eigenvalues in $\mathcal{U}$. For $j = 1, 2$, F_{Sj} and F_{Aj} are $\mathcal{R}$-stable maps representing sensor and actuator failures in the first and second channels, respectively. Under normal operation, $F_{Sj} = I$ and $F_{Aj} = I$; the (complete disconnection) failure of the j-th channel is represented by setting the corresponding F_{Sj} or F_{Aj} equal to zero.

Using an RCF (N_P, D_P) of P and an LCF $(\widetilde{D}_{Cj}, \widetilde{N}_{Cj})$ of C_j, with $D_P \xi_P = e_P$ and $\widetilde{D}_{Cj} y_{Cj} = \widetilde{N}_{Cj} e_{Cj}$, $j = 1, 2$, $\widetilde{D}_C = \mathrm{diag}[\widetilde{D}_{C1}, \widetilde{D}_{C2}]$, $\widetilde{N}_C = \mathrm{diag}[\widetilde{N}_{C1}, \widetilde{N}_{C2}]$, $F_S = \mathrm{diag}[F_{S1}, F_{S2}]$, $F_A = \mathrm{diag}[F_{A1}, F_{A2}]$, $u_P = (u_{P1}, u_{P2})$, $u_C = (u_{C1}, u_{C2})$, $y_P = (y_{P1}, y_{P2})$, $y_C = (y_{C1}, y_{C2})$, the system $\mathcal{S}(P, C_d)$ is described as follows:

$$\begin{bmatrix} D_P & -F_A \\ \widetilde{N}_C F_S N_P & \widetilde{D}_C \end{bmatrix} \begin{bmatrix} \xi_P \\ y_C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \widetilde{N}_C \end{bmatrix} \begin{bmatrix} u_P \\ u_C \end{bmatrix},$$

$$\begin{bmatrix} y_P \\ y_C \end{bmatrix} = \begin{bmatrix} N_P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \xi_P \\ y_C \end{bmatrix}. \qquad (1)$$

Equation (1) is of the form $D_H \xi = u$, $y = N \xi$. The system $\mathcal{S}(P, C_d)$ is well-posed if and only if the map D_H is biproper, equivalently, the closed-loop map $H : (u_P, u_C) \mapsto (y_P, y_C)$ is proper; $\mathcal{S}(P, C_d)$ is automatically well-posed if P or C_d is strictly proper.

2.2. Conditions for stability

Following standard definitions, with F_S and F_A $\mathcal{R}$-stable, the system $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable iff the closed-loop map H from (u_P, u_C) to (y_P, y_C) is $\mathcal{R}$-stable. From the system description (1), $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable if and only if the map D_H is $\mathcal{R}$-unimodular. The decentralized controller C_d is called an $\mathcal{R}$-stabilizing controller for P iff C_d is proper and $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable.

We now investigate $\mathcal{R}$-stability of the system $\mathcal{S}(P, C_d)$ under various failure cases. Without loss of generality, we assume that the RCF (N_P, D_P) and the LCF $(\widetilde{D}_P, \widetilde{N}_P)$ of P have a lower-triangular denominator matrix D_P and an upper-triangular denominator matrix $\widetilde{D}_P$ [1]; i.e.,

$$P = N_P D_P^{-1} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} D_{11} & 0 \\ D_{21} & D_{22} \end{bmatrix}^{-1}$$

$$= \widetilde{D}_P^{-1} \widetilde{N}_P = \begin{bmatrix} \widetilde{D}_{11} & \widetilde{D}_{12} \\ 0 & \widetilde{D}_{22} \end{bmatrix}^{-1} \begin{bmatrix} \widetilde{N}_{11} & \widetilde{N}_{12} \\ \widetilde{N}_{21} & \widetilde{N}_{22} \end{bmatrix}. \qquad (2)$$

Then from (1), the nominal system $\mathcal{S}(P, C_d)$ without failure is $\mathcal{R}$-stable if and only if D_H is $\mathcal{R}$-unimodular, equivalently,

$$\begin{bmatrix} \widetilde{D}_{C1} D_{11} + \widetilde{N}_{C1} N_{11} & \widetilde{N}_{C1} N_{12} \\ \widetilde{D}_{C2} D_{21} + \widetilde{N}_{C2} N_{21} & \widetilde{D}_{C2} D_{22} + \widetilde{N}_{C2} N_{22} \end{bmatrix}$$

$$\text{is } \mathcal{R}\text{-unimodular.} \qquad (3)$$

Case 1: Sensor failure in the first channel: Suppose that $F_{A1} = I$, $F_{A2} = I$ and $F_{S2} = I$. Then $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{S1} = 0$ if and only if

$$D_{11} = I \ , \quad \widetilde{D}_{C2} D_{22} + \widetilde{N}_{C2} N_{22} = I \qquad (4)$$

and

$$\widetilde{D}_{C1} = I \ . \qquad (5)$$

Therefore, there exists an $\mathcal{R}$-stabilizing decentralized controller C_d for the plant P if and only if P is of the form

$$P = N_P D_P^{-1} = \begin{bmatrix} N_{11} & N_{12} \\ \widetilde{V}_{22} \widetilde{N}_{21} & N_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ -\widetilde{U}_{22} \widetilde{N}_{21} & D_{22} \end{bmatrix}^{-1},$$

$$(N_{22}, D_{22}) \text{ right} - \text{coprime} \qquad (6)$$

and $\widetilde{U}_{22}$, $\widetilde{V}_{22}$ are $\mathcal{R}$-stable matrices satisfying the following identity for the RCF $N_{22} D_{22}^{-1}$ of P_{22} for some $\mathcal{R}$-stable matrices U_{22}, V_{22} [1]:

$$\begin{bmatrix} V_{22} & U_{22} \\ -\widetilde{N}_{22} & \widetilde{D}_{22} \end{bmatrix} \begin{bmatrix} D_{22} & -\widetilde{U}_{22} \\ N_{22} & \widetilde{V}_{22} \end{bmatrix} = I. \qquad (7)$$

By (4), $C_d \in \mathcal{M}(\mathcal{R}_p)$ is a decentralized $\mathcal{R}$-stabilizing controller for P if and only if C_1 is $\mathcal{R}$-stable and C_2 is and $\mathcal{R}$-stabilizing controller for P_{22}, i.e.,

$$C_d = \mathrm{diag}[\, C_1 \, , \, C_2 \,] \ , \quad C_1 \in \mathcal{R}^{n_{o1} \times n_{i1}} \ ,$$

$$C_2 = (V_{22} - Q_2 \widetilde{N}_{22})^{-1} (U_{22} + Q_2 \widetilde{D}_{22})$$
$$= (\widetilde{U}_{22} + D_{22} Q_2)(\widetilde{V}_{22} + N_{22} Q_2)^{-1} \qquad (8)$$

for some $\mathcal{R}$-stable Q_2 such that

$$(V_{22} - Q_2 \widetilde{N}_{22}) \text{ is biproper;} \qquad (9)$$

note that (9) automatically holds for all $Q_2 \in \mathcal{M}(\mathcal{R})$ when P_{22} is strictly proper [8].

Now $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with the controller C_d in (8) assuming that $F_{S1} = 0$; but the controller should be designed to ensure $\mathcal{R}$-stability for the nominal system as well. From (2), (6) and (8), $D_{21} = -\widetilde{U}_{22} \widetilde{N}_{21}$, $N_{21} = \widetilde{V}_{22} \widetilde{N}_{21}$ and $(\widetilde{D}_{C2} D_{21} + \widetilde{N}_{C2} N_{21}) = Q_2 \widetilde{N}_{21}$ [1]. The decentralized controller C_d is an $\mathcal{R}$-stabilizing controller for P for both of the possibilities of $F_{S1} = I$ and $F_{S1} = 0$ if and only if C_2 is the same as in (8) but C_1 is given by

$$C_1 = (I - Q_1(N_{11} - N_{12}Q_2\tilde{N}_{21}))^{-1}Q_1 \, , \quad (10)$$

where the $Q_1 \in \mathcal{R}^{n_{i1}\times n_{o1}}$ is such that

$$\tilde{D}_{C1} = I - Q_1(N_{11} - N_{12}Q_2\tilde{N}_{21}) \qquad (11)$$

is $\mathcal{R}$-unimodular.

Case 2: Actuator failure in the first channel: Suppose that $F_{S1} = I$, $F_{S2} = I$ and $F_{A2} = I$. Then $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{A1} = 0$ if and only if (4)-(5) hold, i.e., if and only if $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{S1} = 0$. Therefore, there exists an $\mathcal{R}$-stabilizing decentralized controller C_d for the plant P if and only if P is of the form in (6) and $C_d \in \mathcal{M}(\mathcal{R}_p)$ is a decentralized $\mathcal{R}$-stabilizing controller for P if and only if it is of the form given by (8). As in the sensor failure case above, the decentralized controller C_d is an $\mathcal{R}$-stabilizing controller for P for both of the possibilities of $F_{A1} = I$ and $F_{A1} = 0$ if and only if C_2 is the same as in (8) but C_1 is given by (10), where Q_1 is such that (11) is $\mathcal{R}$-unimodular.

Case 3: Simultaneous sensor and actuator failure in the first channel: Suppose that $F_{S2} = I$ and $F_{A2} = I$. Then $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{S1} = 0$ and $F_{A1} = 0$ if and only if (4) holds. In this case (5) is not needed because when both the sensors and actuators of the first channel fail, $y_{C1} = 0$; since C_1 is no longer taken into account, it need not be $\mathcal{R}$-stable. Therefore, there exists an $\mathcal{R}$-stabilizing decentralized controller C_d for the plant P if and only if P is of the form in (2) and $C_d \in \mathcal{M}(\mathcal{R}_p)$ is a decentralized $\mathcal{R}$-stabilizing controller for P if and only if C_2 is of the form given by (8). Now the same controller will also $\mathcal{R}$-stabilize the nominal system $\mathcal{S}(P, C_d)$ without failure if and only if the additional constraint (10) is put on C_1, except that the $\mathcal{R}$-stable matrix Q_1 is chosen such that the matrix in (11) biproper since C_1 need not be $\mathcal{R}$-stable in this case; note that (11) is biproper for any strictly proper Q_1.

Case 4: Sensor failure in the second channel: Suppose that $F_{A1} = I$, $F_{A2} = I$ and $F_{S1} = I$. Then $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{S2} = 0$ if and only if

$$\begin{bmatrix} \tilde{D}_{C1}D_{11} + \tilde{N}_{C1}N_{11} & \tilde{N}_{C1}N_{12} \\ D_{21} & D_{22} \end{bmatrix} \text{ is } \mathcal{R}\text{-unimodular} \qquad (12)$$

and

$$\tilde{D}_{C2} = I \, . \qquad (13)$$

Therefore, C_2 is necessarily $\mathcal{R}$-stable.

Now the system $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable for either $F_{S2} = I$ or $F_{S2} = 0$ (i.e., with or without sensor failure of the second channel) if and only if both (3) and (12) hold.

Case 5: Actuator failure in the second channel: Suppose that $F_{S1} = I$, $F_{S2} = I$ and $F_{A1} = I$. Then $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{A2} = 0$ if and only if (12) holds, i.e., if and only if $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{S2} = 0$. As in the sensor failure case above, $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable for either $F_{A2} = I$ or $F_{A2} = 0$ (i.e., with or without actuator failure of the second channel) if and only if both (3) and (12) hold.

Case 6: Simultaneous sensor and actuator failure in the second channel: Suppose that $F_{S1} = I$ and $F_{A1} = I$. Then $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable with $F_{S2} = 0$ and $F_{A2} = 0$ if and only if (12) holds. In this case, (13) is not needed because when both sensors and actuators of the second channel fail, $y_{C2} = 0$; since C_2 is no longer taken into account, it need not be $\mathcal{R}$-stable. Now the nominal system $\mathcal{S}(P, C_d)$ is also $\mathcal{R}$-stable if and only if (3) also holds in addition to condition (12).

Case 7: Simultaneous sensor and actuator failure in either the first or the second channel: We now investigate $\mathcal{R}$-stability of the system $\mathcal{S}(P, C_d)$ under simultaneous sensor and actuator failure in either the first or the second channel; this is the same as the reliable decentralized stabilization problem studied in [7]. We study this case in detail in section 2.3 below.

2.3. Reliable decentralized stabilizability

The system $\mathcal{S}(P, C_d)$ is said to be reliably stabilized iff $\mathcal{S}(P, C_d)$ is $\mathcal{R}$-stable under any of the following three conditions:

> *i)* The nominal system $\mathcal{S}(P, C_d)$ is stable, i.e., $F_S = I$ and $F_A = I$
> *ii)* the system $\mathcal{S}(P, C_d)$ is stable with simultaneous sensor and actuator failure in the first channel, i.e., $F_{S1} = 0$, $F_{A1} = 0$, $F_{S2} = I$, $F_{A2} = I$
> *iii)* the system $\mathcal{S}(P, C_d)$ is stable with simultaneous sensor and actuator failure in the second channel, i.e., $F_{S2} = 0$, $F_{A2} = 0$, $F_{S1} = I$, $F_{A1} = I$.

Now the first of these conditions is satisfied if and only if (3) holds. The second condition was explained in case 3 of section 2.2 above; it is satisfied if and only if (4) holds. The third condition was explained in case 6 of section 2.2 above; it is satisfied if and only if (12) holds. Note that P is of the form given by (6) for conditions 2 and 3 to hold. Putting (3), (4) and (12) together with the necessary form of P in (6), we conclude that the system $\mathcal{S}(F_S, P, F_A, C)$ is reliably stabilized if and only if

$$D_{22} - N_{C2}\,\widetilde{N}_{21}\,Q_1\,N_{12} \quad \text{is } \mathcal{R}\text{-unimodular.} \tag{14}$$

Furthermore, $C_d = \mathrm{diag}[\,C_1\,,\,C_2\,]$ is a decentralized $\mathcal{R}$-stabilizing controller such that $\mathcal{S}(\,P, C_d\,)$ is reliably stabilized if and only if C_1 and C_2 are given by (10) and (8), respectively, for some $\mathcal{R}$-stable Q_1 and Q_2 (of appropriate sizes) satisfying (11) and (9) and are such that (14) holds, i.e.,

$$D_{22} + (\,\widetilde{U}_{22} + D_{22}\,Q_2\,)\,\widetilde{N}_{21}\,Q_1\,N_{12}$$
$$\text{is } \mathcal{R}\text{-unimodular.} \tag{15}$$

Although condition (15) characterizes all parameter matrices Q_1 and Q_2 that achieve reliable stabilization of $\mathcal{S}(\,P, C_d\,)$, it does not explicitly describe how to choose them in order to make the matrix in (15) $\mathcal{R}$-unimodular. However, from (14) and equivalently (15), the conditions in Theorem 2.3.1 below on the plant P are *necessary* for existence of decentralized controllers which reliably stabilize $\mathcal{S}(F_S, P, F_A, C)$. By Theorem 2.3.1, to achieve reliable decentralized stabilization, P_{12} and P_{21} are necessarily strongly $\mathcal{R}$-stabilizable. An LTI system $\hat{P}$ is said to be strongly $\mathcal{R}$-stabilizable if there is an $\mathcal{R}$-stable $\mathcal{R}$-stabilizing controller $C \in \mathcal{M}(\mathcal{R})$ for $\hat{P}$ (in the standard full-feedback system). If $\mathcal{U} = \mathbf{C}_+$, $\hat{P}$ is strongly $\mathcal{R}$-stabilizable if and only if it satisfies the *parity interlacing property*, i.e., $\hat{P}$ has an even number of poles between pairs of blocking zeros on the positive real-axis ([8], [9]). From a coprime factorizations view-point, $\hat{P}$ is strongly $\mathcal{R}$-stabilizable if and only if, for any RCF $(\,N_P\,,\,D_P\,)$ of $\hat{P}$, there exists an $\mathcal{R}$-unimodular $\widetilde{D}$ such that $\widetilde{D}\,D_P + \widetilde{N}\,N_P$ is $\mathcal{R}$-unimodular for some $\mathcal{R}$-stable $\widetilde{N}$.

2.3.1. Theorem (*Necessary conditions for reliable decentralized stabilizability*): Let $P \in \mathcal{R}^{n_o \times n_i}$ be as in (2). If there exists a decentralized controller C_d such that $\mathcal{S}(F_S, P, F_A, C)$ is reliably stabilized, then *i)* $(\,N_{12}\,,\,D_{22}\,)$ is an RCF of P_{12} and P_{12} is strongly $\mathcal{R}$-stabilizable, and *ii)* $(\,\widetilde{D}_{22}\,,\,\widetilde{N}_{21}\,)$ is an LCF of P_{21} and P_{21} is strongly $\mathcal{R}$-stabilizable. □

From Theorem 2.3.1, reliable decentralized stabilization may not always be possible to achieve. We now study special cases where there exist decentralized controllers achieving reliable stabilization.

Reliable decentralized stabilization for stable plants: Let the plant P be $\mathcal{R}$-stable; then an RCF of P is given by $(\,P\,,\,I\,)$. The decentralized controller C_d achieves reliable stabilization if and only if $C_d = \mathrm{diag}\,[\,C_1\,,\,C_2\,]$, with

$$C_1 = (\,I - Q_1\,(\,P_{11} - P_{12}\,Q_2\,P_{21}\,)\,)^{-1}\,Q_1\,,$$
$$C_2 = (\,I - Q_2\,P_{22}\,)^{-1}\,Q_2\,, \tag{16}$$

where $Q_1\,,\ Q_2 \in \mathcal{M}(\mathcal{R})$ are such that

$$I + Q_2\,P_{21}\,Q_1\,P_{12} \quad \text{is } \mathcal{R}\text{-unimodular,} \tag{17}$$

$$(\,I - Q_1\,(\,P_{11} - P_{12}\,Q_2\,P_{21}\,)\,) \quad \text{is biproper,}$$
$$(\,I - Q_2\,P_{22}\,) \quad \text{is biproper} \tag{18}$$

Reliable decentralized stabilization for lower- or upper-triangular plants: From (2), the plant P is lower-triangular (upper-triangular) if and only if $N_{12} = 0$ ($\widetilde{N}_{21} = 0$, respectively). In either case, from (14), reliable stabilization can be achieved if and only if D_{22} is $\mathcal{R}$-unimodular, equivalently, P is $\mathcal{R}$-stable. Hence, C_d achieves reliable stabilization if and only if it is given by (16), where $Q_1\,,\ Q_2 \in \mathcal{M}(\mathcal{R})$ are such that (18) holds; note that (17) is automatically satisfied since either $P_{12} = 0$ or $P_{21} = 0$.

Reliable decentralized stabilization when P_{22} is strongly $\mathcal{R}$-stabilizable: Let P_{22} be strictly proper and strongly $\mathcal{R}$-stabilizable. Suppose that P_{12} and P_{21} are square and invertible. A sufficient condition for decentralized reliable stabilization is that P_{22} is strongly $\mathcal{R}$-stabilizable, and in addition, $P_{12}^{-1} = D_{22}\,N_{12}^{-1}$ and $P_{21}^{-1} = \widetilde{N}_{21}^{-1}\,\widetilde{D}_{22}$ are $\mathcal{R}$-stable, i.e., N_{12} and $\widetilde{N}_{21}$ are $\mathcal{R}$-unimodular. In this case, let C_S be any $\mathcal{R}$-stable $\mathcal{R}$-stabilizing controller for P_{22}. Without loss of generality, we can assume that the RCF $(\,N_{22}\,,\,D_{22}\,)$ of P_{22} is such that $D_{22} + C_S\,N_{22} = I_{ni}$ and hence,

$$\begin{bmatrix} I & C_S \\ -N_{22} & I - N_{22}\,C_S \end{bmatrix} \begin{bmatrix} D_{22} & -C_S \\ N_{22} & I \end{bmatrix} = I. \tag{19}$$

Then for some $A \in \mathcal{M}(\mathcal{R})$, $\widetilde{U}_{22}$ in (7) is $\widetilde{U}_{22} = (\,C_S + D_{22}\,A\,)$. A reliable decentralized controller is given by $C_d = \mathrm{diag}\,[\,C_1\,,\,C_2\,]$, where C_2 is given by (8) with $Q_2 = -A$ and C_1 is given by (10) with $Q_1 = \widetilde{N}_{21}{}^{-1}\,N_{22}\,N_{12}^{-1}$.

Now with P_{22} strongly $\mathcal{R}$-stabilizable, suppose that P_{21} is square and invertible and $P_{12} = M\,P_{22}$ for some $\mathcal{R}$-unimodular matrix M. Then $N_{12} = M N_{22}$. These conditions are also sufficient for existence of reliable decentralized controllers. In this case, a controller similar to the one given above can be used, where $Q_2 = -A$ and $Q_1 = \tilde{N}_{21}^{-1} M^{-1}$.

Reliable decentralized stabilization when P_{22}^{-1} is strongly $\mathcal{R}$-stabilizable: Suppose that P_{22}, P_{12} and P_{21} are square and invertible. A sufficient condition for decentralized reliable stabilization is that P_{22}^{-1} is strongly $\mathcal{R}$-stabilizable, and in addition, $P_{12}^{-1} = D_{22} N_{12}^{-1}$ and $P_{21}^{-1} = \tilde{N}_{21}^{-1} \tilde{D}_{22}$ are $\mathcal{R}$-stable, i.e., N_{12} and $\tilde{N}_{21}$ are $\mathcal{R}$-unimodular. Since P_{22}^{-1} is strongly $\mathcal{R}$-stabilizable, there exists a $Q_2 \in \mathcal{M}(\mathcal{R})$ such that $(\tilde{U}_{22} + D_{22} Q_2)$ is $\mathcal{R}$-unimodular, where $\tilde{U}_{22}$ satisfies (7). In this case, a reliable decentralized controller is given by $C_d = \operatorname{diag}[C_1, C_2]$, where C_2 is given by (8) with Q_2 such that $(\tilde{U}_{22} + D_{22} Q_2)$ is $\mathcal{R}$-unimodular and C_1 is given by (10) with $Q_1 = \tilde{N}_{21}^{-1} (\tilde{U}_{22} + D_{22} Q_2)^{-1} [-D_{22} + I] N_{12}^{-1}$.

Reliable decentralized stabilization when $X P_{12} = P_{22}$ and $P_{21} Y = P_{22}$: Let P_{22} be strictly proper and strongly $\mathcal{R}$-stabilizable. Suppose that there exist $\mathcal{R}$-stable matrices X and Y of appropriate dimensions such that $X P_{12} = P_{22}$ (equivalently, $X N_{12} = N_{22}$) and $P_{21} Y = P_{22}$ (equivalently, $\tilde{N}_{21} Y = N_{22}$). These conditions are also sufficient for existence of reliable decentralized controllers. Again, let C_S be any $\mathcal{R}$-stable $\mathcal{R}$-stabilizing controller for P_{22} and assume that the RCF (N_{22}, D_{22}) of P_{22} is such that $D_{22} + C_S N_{22} = I_{n_i}$ and hence, (19) holds. Then for some $A \in \mathcal{M}(\mathcal{R})$, $\tilde{U}_{22}$ in (7) is $\tilde{U}_{22} = (C_S + D_{22} A)$. A reliable decentralized controller is $C_d = \operatorname{diag}[C_1, C_2]$, where C_2 is given by (8) with $Q_2 = -A$ and C_1 is given by (10) with $Q_1 = Y \hat{Q}_1 X$ and $\hat{Q}_1$ is chosen as follows: Let k be any integer larger than $\| C_S N_{22} \|$; then $(I - (C_S N_{22})/k)^k$ is $\mathcal{R}$-unimodular. By the binomial expansion (see for example [9]),

$$(I - (C_S N_{22})/k)^k = I - (C_S N_{22}) + \sum_{\ell=2}^{k} r_\ell (C_S N_{22})^\ell ,$$

where r_ℓ are the binomial coefficients. Choose $\hat{Q}_1$ as

$$\hat{Q}_1 = \sum_{\ell=2}^{k} r_\ell (C_S N_{22})^{\ell-2} C_S . \qquad (20)$$

For this $\hat{Q}_1$, condition (15) is satisfied and hence, the system is reliably stabilized.

3. Conclusions

Reliable decentralized stabilization was considered using a factorization approach. It was shown that reliable stabilization can be achieved using two decentralized controllers only if P_{12} and P_{21} are strongly stabilizable. Decentralized controllers achieving reliable stabilization were proposed for plants, where P_{22} is also strongly stabilizable.

References

[1] C. A. Desoer and A. N. Gündeş, "Algebraic theory of feedback systems with tow-input two-output plant and compensator," *Int. Journal of Control*, vol. 47, no. 1, pp. 33-51, 1988.

[2] A. N. Gündeş, "Reliable stabilization of linear plants using a two-controller configuration," *Systems and Control Letters*, to appear, 1994.

[3] A. N. Gündeş, "Stability of feedback systems with sensor or actuator failures: Analysis," *Int. Journal of Control*, vol. 56, no. 4, pp. 735–753, 1992.

[4] K. D. Minto, K.D. and R. Ravi, New results on the multi-controller scheme for the reliable control of linear plants, *Proc. American Control Conference*, pp. 615-619, 1991.

[5] D. D. Siljak, On reliability of control, *Proc. 17th IEEE Conference on Decision and Control*, pp. 687-694, 1978.

[6] D. D. Siljak, Reliable control using multiple control systems, *Int. Journal of Control*, vol. 31, no. 2, 303-329, 1980.

[7] X. L. Tan, D. D. Siljak and M. Ikeda, Reliable stabilization via factorization methods, *IEEE Trans. Automatic Control*, vol. 37, pp. 1786-1791, 1992.

[8] M. Vidyasagar, *Control System Synthesis: A Factorization Approach*, Cambridge, MA: M.I.T. Press, 1985.

[9] M. Vidyasagar and N. Viswanadham, Algebraic design techniques for reliable stabilization, *IEEE Trans. Automatic Control*, vol. 27, pp. 1085-1095, 1982.

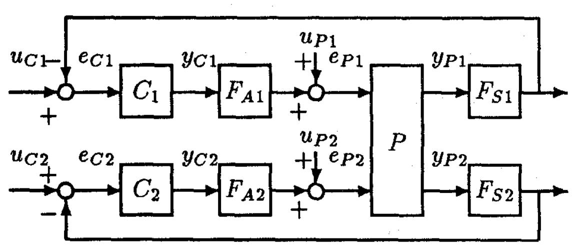

Figure 1: The system $\mathcal{S}(P, C_d)$