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Stabilizing controller design for linear systems with nonlinear sensor or actuator failures

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Abstract

For the linear, time-invariant, multi-input multi-output unity-feedback system under a class of nonlinear, time-varying stable perturbations, we obtain conditions for stability and develop a controller design method which ensures stability in the presence of one arbitrary sensor or actuator failure.

1. Introduction

We consider the stability of the linear, time-invariant (LTI), multi-input multi-output (MIMO) unity-feedback system $\mathcal{S}(P,C)$ (Figure 1) under a class of nonlinear, time-varying (NLTV) stable perturbations. This is a generalisation of the standard integrity problem in which the outputs corresponding to the failed sensors or actuators are multiplied by zero. We refer to post-multiplicative diagonal perturbations on the plant as sensor-failures and pre-multiplicative ones as actuator-failures.

We obtain necessary and sufficient conditions for stability and develop a controller design method to ensures stability in the presence of one arbitrary sensor or actuator failure. Due to the input-output approach used, the setting can be continuous-time or discrete-time.

2. Main Results

Unless stated otherwise, all maps are causal, MIMO and LTI. An LTI map and its associated transfer function representation are used interchangeably. LTI maps admit coprime factorisations in terms of $\mathcal{R}_{\mathcal{U}}$-stable maps defined as follows: $\mathcal{U}$ is a subset of the field $\mathbf{C}$ of complex numbers; $\mathcal{U}$ is closed and symmetric about the real axis, $\pm\infty \in \mathcal{U}$; $\mathbf{C} \setminus \mathcal{U}$ is nonempty. $\mathcal{R}_{\mathcal{U}}$ and $\mathbb{R}_p(s)$ are the ring of proper rational functions which have no poles in $\mathcal{U}$ and the ring of proper rational functions, (with real coefficients). $\mathcal{M}(\mathcal{R}_{\mathcal{U}})$ is the set of matrices whose entries are in $\mathcal{R}_{\mathcal{U}}$; $M \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$ is $\mathcal{R}_{\mathcal{U}}$-unimodular iff $M^{-1} \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$. (N_P, D_P) denotes a right-coprime-factorization (RCF) and $(\tilde{D}_P, \tilde{N}_P)$ denotes a left-coprime-factorization (LCF) of $P \in \mathbb{R}_p(s)^{n_o \times n_i}$, where $N_P, D_P, \tilde{N}_P, \tilde{D}_P \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$, $P = N_P D_P^{-1} = \tilde{D}_P^{-1}\tilde{N}_P$. Similarly, (N_C, D_C) and $(\tilde{D}_C, \tilde{N}_C)$ denote a RCF and LCF of $C \in \mathbb{R}_p(s)^{n_i \times n_o}$. All causal NLTV maps are defined over appropriate products of an extended space $\mathcal{L}_e$ [1]. The set of bounded signals is denoted $\mathcal{L}$, where the bound is determined by the associated norm $\|\cdot\|$. A causal NLTV map $\mathcal{H} : \mathcal{L}_e^{n_i} \to \mathcal{L}_e^{n_o}$ is said to be $\mathcal{L}$-stable iff there exists a continuous nondecreasing $\phi_{\mathcal{H}} : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\|\mathcal{H}u\| \le \phi_{\mathcal{H}}(\|u\|)$ for all $u \in \mathcal{L}^{n_i}$. A well-posed NLTV interconnection is said to be $\mathcal{L}$-stable iff the map from the exogenous inputs to closed-loop signals is $\mathcal{L}$-stable. The notion of $\mathcal{L}$-stability is introduced only in the case of NLTV interconnections and analyses thereof. When the interconnections are LTI, we will use a more stringent condition of $\mathcal{R}_{\mathcal{U}}$-stability. Note that an $\mathcal{R}_{\mathcal{U}}$-stable LTI map is $\mathcal{L}$-stable; however, the converse is not true in general.

System descriptions: The systems with possible sensor-failures and possible actuator-failures are $\mathcal{S}(F_S,P,C)$ and $\mathcal{S}(P,F_A,C)$ (Figures 2, 3). The plant $P \in \mathbb{R}_p(s)^{n_o \times n_i}$ and the controller $C \in \mathbb{R}_p(s)^{n_i \times n_o}$; $\mathcal{S}(P,C)$, $\mathcal{S}(F_S,P,C)$ and $\mathcal{S}(P,F_A,C)$ are well-posed; P and C have no hidden modes associated with eigenvalues in $\mathcal{U}$. $\mathcal{S}(P,C)$ is said to be $\mathcal{R}_{\mathcal{U}}$-stable iff the closed-loop map H from (u_P,u_C) to (y_P,y_C) is $\mathcal{R}_{\mathcal{U}}$-stable. Similarly, when F_S (F_A) is $\mathcal{L}$-stable, $\mathcal{S}(F_S,P,C)$ ($\mathcal{S}(P,F_A,C)$) is said to be $\mathcal{L}$-stable iff the closed-loop map from (u_P,u_C) to (y_P,y_C) is $\mathcal{L}$-stable. C is said to be an $\mathcal{R}_{\mathcal{U}}$-stabilising controller for P in $\mathcal{S}(P,C)$ iff $C \in \mathbb{R}_p(s)^{n_i \times n_o}$ and $\mathcal{S}(P,C)$ is $\mathcal{R}_{\mathcal{U}}$-stable. It is well-known that C is an $\mathcal{R}_{\mathcal{U}}$-stabilising controller for P if and only if there is an RCF (N_C, D_C) and an LCF $(\tilde{D}_C, \tilde{N}_C)$ such that

$$\begin{bmatrix} \tilde{D}_C & \tilde{N}_C \\ -\tilde{N}_P & \tilde{D}_P \end{bmatrix} \begin{bmatrix} D_P & -N_C \\ N_P & D_C \end{bmatrix} = \begin{bmatrix} I_{n_i} & 0 \\ 0 & I_{n_o} \end{bmatrix}. \quad (1)$$

We consider sensor (actuator) failures where at most one of any of the n_o sensors (n_i actuators) fails. The failure is represented by a NLTV stable perturbation of the identity map; $\mathcal{F}_{S1}$ denotes the class of sensor-failures, where $\mathcal{F}_{S1} := \{I_{n_o} - e_j q e_j^T \mid q : \mathcal{L}_e \to \mathcal{L}_e$, NLTV $\mathcal{L}$-stable, $j = 1,\ldots,n_o\}$ and $\mathcal{F}_{A1}$ denotes the class of actuator-failures, where $\mathcal{F}_{A1} := \{I_{n_i} - e_j q e_j^T \mid q : \mathcal{L}_e \to \mathcal{L}_e$, NLTV $\mathcal{L}$-stable, $j = 1,\ldots,n_i\}$, where e_j denotes the jth column of the identity of appropriate dimension. The failure is a disconnection in the failure sub-classes $\mathcal{F}_{S1}^0 := \{I_{n_o} - e_j e_j^T \mid j = 1,\ldots,n_o\}$, and $\mathcal{F}_{A1}^0 := \{I_{n_i} - e_j e_j^T \mid j = 1,\ldots,n_i\}$.

Theorem 1 (*conditions for $\mathcal{L}$-stability*): Let C be an $\mathcal{R}_{\mathcal{U}}$-stabilizing controller for P in $\mathcal{S}(P,C)$; let $(\tilde{D}_C, \tilde{N}_C)$ be an LCF and (N_C, D_C) be an RCF of C satisfying (1). Then **a)** $\mathcal{S}(F_S,P,C)$ is $\mathcal{L}$-stable for all $F_S \in \mathcal{F}_{S1}$ if and only if all diagonal entries of $N_P \tilde{N}_C$ are identically equal to zero; **b)** $\mathcal{S}(P,F_A,C)$ is $\mathcal{L}$-stable for all $F_A \in \mathcal{F}_{A1}$ if and only if all diagonal entries of $N_C \tilde{N}_P$ ar · identically equal to zero. □

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Note that $N_P \tilde{N}_C$ is the closed-loop map $H_{pc} : u_C \mapsto y_P$ and $-N_C \tilde{N}_P$ is the closed-loop map $H_{cp} : u_P \mapsto y_C$ of the nominal system $\mathcal{S}(P, C)$.

Let $(\tilde{D}_P, \tilde{N}_P)$ be any LCF of P. If $\mathcal{S}(F_S, P, C)$ is $\mathcal{R}_\mathcal{U}$-stable for all $F_S \in \mathcal{F}^0_{S1}$, then there exists an $\mathcal{R}_\mathcal{U}$-unimodular matrix $L_1 \in \mathcal{R}_\mathcal{U}^{no \times no}$ such that

$$L_1 \tilde{D}_P = \begin{bmatrix} 1 & \tilde{d}_{1,2} & \dots & \tilde{d}_{1,no} \\ 0 & \tilde{d}_{2,2} & \dots & \tilde{d}_{2,no} \\ & & \ddots & \vdots \\ & O & & \tilde{d}_{no,no} \end{bmatrix}, \tag{2}$$

where, for $j = 1, \dots, n_o - 1$, the pair

$$\left(\begin{bmatrix} \tilde{d}_{1,1+j} \\ \vdots \\ \tilde{d}_{j,1+j} \end{bmatrix}, \tilde{d}_{1+j,1+j} \right) \text{ is right-coprime;} \tag{3}$$

equivalently, for $j = 2, \dots, n_o$, $\ell = 1, \dots, j$, there exist $\tilde{y}_{j,\ell} \in \mathcal{R}_\mathcal{U}$ such that $\sum_{\ell=1}^{j} \tilde{y}_{j,\ell}\, \tilde{d}_{\ell,j} = 1$. Let

$$Y_{S1} := \begin{bmatrix} 1 & 0 & & O \\ \tilde{y}_{2,1} & \tilde{y}_{2,2} & & \\ \vdots & \vdots & \ddots & \\ \tilde{y}_{no,1} & \tilde{y}_{no,2} & \dots & \tilde{y}_{no,no} \end{bmatrix} L_1.$$ Then the diagonal entries of $Y_{S1} \tilde{D}_P$ are all equal to one. Let C_{SD} be *any* $\mathcal{R}_\mathcal{U}$-stabilising controller for $\mathcal{S}(P, C)$ such that the map $H_{pc} = PC(I_{no} + PC)^{-1}$ of $\mathcal{S}(P, C)$ is *diagonal* for the given plant P. Similarly, let (N_P, D_P) be any RCF of P. If $\mathcal{S}(P, F_A, C)$ is $\mathcal{R}_\mathcal{U}$-stable for all $F_A \in \mathcal{F}^0_{A1}$, then there exists an $\mathcal{R}_\mathcal{U}$-unimodular matrix $R_1 \in \mathcal{R}_\mathcal{U}^{ni \times ni}$ such that

$$D_P R_1 = \begin{bmatrix} 1 & 0 & & O \\ d_{2,1} & d_{2,2} & & \\ \vdots & \vdots & \ddots & \\ d_{ni,1} & d_{ni,2} & \dots & d_{ni,ni} \end{bmatrix}, \tag{4}$$

where, for $j = 1, \dots, n_i - 1$, the pair

$$(d_{j+1,j+1}, [d_{1+j,1} \dots d_{1+j,j}]) \tag{5}$$

is left-coprime ; equivalently, for $j = 2, \dots, n_i$, $\ell = 1, \dots, j$, there exist $y_{\ell,j} \in \mathcal{R}_\mathcal{U}$ such that $\sum_{\ell=1}^{j} d_{j,\ell}\, y_{\ell,j} = 1$. Let $Y_{A1} := R_1 \begin{bmatrix} 1 & y_{1,2} & \dots & y_{1,ni} \\ 0 & y_{2,2} & \dots & y_{2,ni} \\ & & \ddots & \vdots \\ & O & & y_{ni,ni} \end{bmatrix}$. Then the diagonal entries of $D_P Y_{A1}$ are all equal to one. Let C_{AD} be *any* $\mathcal{R}_\mathcal{U}$-stabilising controller for $\mathcal{S}(P, C)$ such that the map $H_{cp} = -CP(I_{ni} + CP)^{-1}$ of $\mathcal{S}(P, C)$ is *diagonal* for the given plant P.

Let $\mathcal{U}$ be the closed right-half-plane (continuous-time) or the complement of the open unit-disk (discrete-time); then conditions (2)-(3) on the denominator-matrix $\tilde{D}_P$ are necessary if $\mathcal{S}(F_S, P, C)$ is $\mathcal{L}$-stable for all $F_S \in \mathcal{F}_{S1}$ since $\mathcal{F}^0_{S1} \subset \mathcal{F}_{S1}$. Similarly, conditions (4)-(5) on D_P are necessary for $\mathcal{L}$-stability of $\mathcal{S}(P, F_A, C)$ for all $F_A \in \mathcal{F}_{A1}$.

Proposition 1 (*a set of controllers*): **a)** Let P be such that conditions (2)-(3) hold. Let C_{SD} be any $\mathcal{R}_\mathcal{U}$-stabilising controller for P such that the transfer-function H_{pc} of $\mathcal{S}(P, C)$ is diagonal. Let $(\tilde{D}_{SD}, \tilde{N}_{SD})$ be an LCF and (N_{SD}, D_{SD}) be an RCF of C_{SD} such that $\tilde{D}_{SD} D_P + \tilde{N}_{SD} N_P = I$, $\tilde{N}_P N_{SD} + \tilde{D}_P D_{SD} = I$. Then $C = \tilde{D}_C^{-1} \tilde{N}_C = (\tilde{D}_{SD} + \tilde{N}_{SD} Y_{S1} \tilde{N}_P)^{-1} (\tilde{N}_{SD} - \tilde{N}_{SD} Y_{S1} \tilde{D}_P) = N_C D_C^{-1} = (N_{SD} - D_P \tilde{N}_{SD} Y_{S1})(D_{SD} + N_P \tilde{N}_{SD} Y_{S1})^{-1} = C_{SD}(I_{no} + Y_{S1} \tilde{N}_P C_{SD})^{-1}(I_{no} - Y_{S1} \tilde{D}_P)$ is a controller such that $\mathcal{S}(F_S, P, C)$ is $\mathcal{L}$-stable for all $F_S \in \mathcal{F}_{S1}$.
b) Let P be such that conditions (4)-(5) hold. Let C_{AD} be any $\mathcal{R}_\mathcal{U}$-stabilising controller for P such that the transfer-function H_{cp} of the nominal system $\mathcal{S}(P, C)$ is diagonal. Let $(\tilde{D}_{AD}, \tilde{N}_{AD})$ be an LCF and (N_{AD}, D_{AD}) be an RCF of C_{AD} such that $\tilde{D}_{AD} D_P + \tilde{N}_{AD} N_P = I$, $\tilde{N}_P N_{AD} + \tilde{D}_P D_{AD} = I$. Then $C = \tilde{D}_C^{-1} \tilde{N}_C = (\tilde{D}_{AD} + Y_{A1} N_{AD} \tilde{N}_P)^{-1} (\tilde{N}_{AD} - Y_{A1} N_{AD} \tilde{D}_P) = N_C D_C^{-1} = (N_{AD} - D_P Y_{A1} N_{AD})(D_{AD} + N_P Y_{A1} N_{AD})^{-1} = (I_{ni} - D_P Y_{A1})(I_{ni} + C_{AD} N_P Y_{A1})^{-1} C_{AD}$ is a controller such that $\mathcal{S}(P, F_A, C)$ is $\mathcal{L}$-stable for all $F_A \in \mathcal{F}_{A1}$. □

It is possible to choose C_{SD}, Y_{S1} and similarly, C_{AD}, Y_{A1} in Proposition 1 so that the controller C is proper.

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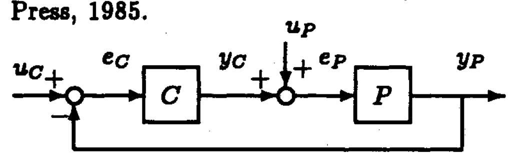

Figure 1: The system $\mathcal{S}(P, C)$

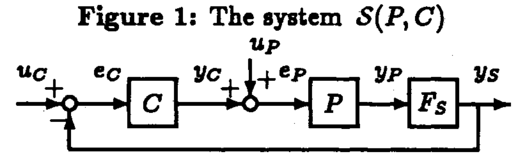

Figure 2: The system $\mathcal{S}(F_S, P, C)$

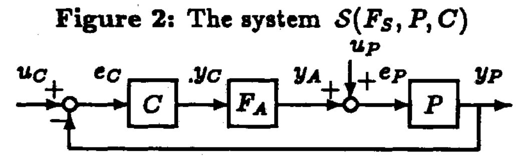

Figure 3: The system $\mathcal{S}(P, F_A, C)$