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Reliable Control Using Two Controllers

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Abstract

We consider the reliable stabilization of linear, time-invariant, multi-input multi-output control systems using a two-controller configuration. For any given plant, we develop a method of designing two controllers which maintain closed-loop stability both when working together and when acting independently. For stable plants, we develop a decomposition method of a given stabilizing controller into the sum of two controllers which provide reliable stabilization.

1. Introduction

A linear, time-invariant, multi-input, multi-output plant can be reliably stabilized in the configuration of the system $\mathcal{S}(P, C_1, C_2)$ (Figure 2) if and only if it is strongly stabilizable (i.e., it can be stabilized using a stable controller) in the standard unity-feedback configuration of the system $\mathcal{S}(P, C)$ (Figure 1) [1], [4]. In this paper we develop a method of finding two controllers that achieve reliable stabilization, where neither of the controllers is necessarily stable. We also develop a reliable decomposition method of a given stabilizing controller into the sum of two controllers.

We assume that the plant is free of unstable hidden-modes. The results apply to continuous-time as well as discrete-time systems.

2. Preliminaries

Notation: Let $\mathcal{U}$ be a subset of the field $\mathbb{C}$ of complex numbers; $\mathcal{U}$ is closed and symmetric about the real axis, $\pm\infty \in \mathcal{U}$, $\mathbb{C} \setminus \mathcal{U}$ is nonempty. Let $\mathcal{R}_{\mathcal{U}}$, $\mathbb{R}_p(s)$, $\mathbb{R}_{sp}(s)$, $\mathbb{R}(s)$ be the ring of proper rational functions with no poles in $\mathcal{U}$, the ring of proper rational functions, the set of strictly proper rational functions and the field of rational functions of s (with real coefficients), respectively. Let $\mathcal{J}$ be the group of units of $\mathcal{R}_{\mathcal{U}}$ and let $\mathcal{I} := \mathcal{R}_{\mathcal{U}} \setminus \mathbb{R}_{sp}(s)$. The set of matrices whose entries are in $\mathcal{R}_{\mathcal{U}}$ is $\mathcal{M}(\mathcal{R}_{\mathcal{U}})$. A matrix M is called $\mathcal{R}_{\mathcal{U}}$-stable iff $M \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$; $M \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$ is $\mathcal{R}_{\mathcal{U}}$-unimodular iff $\det M \in \mathcal{J}$. The identity matrix of size n is denoted I_n. The norm of a matrix $M \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$ is defined as $\| M \| := \sup_\omega \bar{\sigma}(M(j\omega))$. Let (N_P, D_P) denote a right-coprime-factorization (RCF) and $(\tilde{D}_P, \tilde{N}_P)$ denote a left-coprime-factorization (LCF) of P, where $P = N_P D_P^{-1} = \tilde{D}_P^{-1}\tilde{N}_P$, N_P, D_P, $\tilde{N}_P$, $\tilde{D}_P \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$, $\det D_P$, $\det \tilde{D}_P \in \mathcal{I}$. □

Consider the system $\mathcal{S}(P, C)$ where $P \in \mathbb{R}_p(s)^{no \times ni}$ and $C \in \mathbb{R}_p(s)^{ni \times no}$. The system $\mathcal{S}(P, C)$ is said to be $\mathcal{R}_{\mathcal{U}}$-stable iff the transfer-function $H_{yu}(P, C) : [u_C^T \; u_P^T]^T \mapsto [y_C^T \; y_P^T]^T \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$. The controller C is said to be an $\mathcal{R}_{\mathcal{U}}$-stabilizing controller for P iff C is proper and $H_{yu}(P, C) \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$. C is an $\mathcal{R}_{\mathcal{U}}$-stabilizing controller for P if and only if $\tilde{D}_C D_P + \tilde{N}_C N_P$ is $\mathcal{R}_{\mathcal{U}}$-unimodular, equivalently, $\tilde{D}_P D_C + \tilde{N}_P N_C$ is $\mathcal{R}_{\mathcal{U}}$-unimodular for *any* LCF $(\tilde{D}_C, \tilde{N}_C)$ and *any* RCF (N_C, D_C) of C.

Now consider the system $\mathcal{S}(P, C_1, C_2)$; this system is said to be $\mathcal{R}_{\mathcal{U}}$-stable iff the transfer-function $H_{yu}(P, C_1, C_2) : [u_{C1}^T \; u_{C2}^T \; u_P^T]^T \mapsto [y_{C1}^T \; y_{C2}^T \; y_P^T]^T \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$.

2.1 Lemma: Let (N_P, D_P), $(\tilde{D}_P, \tilde{N}_P)$ be any RCF and any LCF of P; let (N_{Cj}, D_{Cj}) be any RCF and $(\tilde{D}_{Cj}, \tilde{N}_{Cj})$ be any LCF of C_j, $j = 1, 2$. Then the following are equivalent:

i) The system $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R}_{\mathcal{U}}$-stable.

ii) $\begin{bmatrix} \tilde{D}_{C1} D_P + \tilde{N}_{C1} N_P & -\tilde{D}_{C1} \\ \tilde{N}_{C2} N_P & \tilde{D}_{C2} \end{bmatrix}$ is $\mathcal{R}_{\mathcal{U}}$-unimodular.

iii) $\begin{bmatrix} \tilde{D}_P D_{C1} + \tilde{N}_P N_{C1} & \tilde{N}_P N_{C2} \\ -D_{C1} & D_{C2} \end{bmatrix}$ is $\mathcal{R}_{\mathcal{U}}$-unimodular.

2.2 Corollary: **a)** If $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R}_{\mathcal{U}}$-stable, then $(\tilde{D}_{C1}, \tilde{D}_{C2})$ is right-coprime and (D_{C1}, D_{C2}) is left-coprime, where (N_{Cj}, D_{Cj}) is any RCF and $(\tilde{D}_{Cj}, \tilde{N}_{Cj})$ is any LCF of C_j, $j = 1, 2$.

b) Let $(\tilde{D}_{C1}, \tilde{D}_{C2})$ be right-coprime and (D_{C1}, D_{C2}) be left-coprime, where (N_{Cj}, D_{Cj}) is any RCF and $(\tilde{D}_{Cj}, \tilde{N}_{Cj})$ is any LCF of C_j, $j = 1, 2$. Then

i) $C := C_1 + C_2 = \tilde{D}_{C1}^{-1}(\tilde{N}_{C1} D_{C2} + \tilde{D}_{C1} N_{C2}) D_{C2}^{-1}$, where $(\tilde{D}_{C1}, \tilde{N}_{C1} D_{C2} + \tilde{D}_{C1} N_{C2})$ is left-coprime and $(\tilde{N}_{C1} D_{C2} + \tilde{D}_{C1} N_{C2}, D_{C2})$ is right-coprime.

ii) $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R}_{\mathcal{U}}$-stable if and only if $C := C_1 + C_2$ is an $\mathcal{R}_{\mathcal{U}}$-stabilizing controller for P.

3. Main Results

From Lemma 2.1, $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R}_{\mathcal{U}}$-stable with $C_1 = 0$ if and only if C_2 is an $\mathcal{R}_{\mathcal{U}}$-stabilizing controller for P; similarly, it is $\mathcal{R}_{\mathcal{U}}$-stable with $C_2 = 0$ if and only if C_1 is an $\mathcal{R}_{\mathcal{U}}$-stabilizing controller for P.

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In Algorithm 3.4, we develop a design method such that $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R_U}$-stable when C_1 and C_2 work together and when one of them is zero. In Algorithm 3.5, we show a reliable decomposition of a *given* $\mathcal{R_U}$-stabilizing controller C for $\mathcal{R_U}$-stable plants; i.e., for $P \in \mathcal{R_U}^{no \times ni}$, and a given $\mathcal{R_U}$-stabilizing controller $C \in \mathrm{IR_p}(s)^{ni \times no}$, we find two controllers C_1 and C_2 such that $C = C_1 + C_2$, C_1 $\mathcal{R_U}$-stabilizes P, C_2 $\mathcal{R_U}$-stabilizes P and $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R_U}$-stable. Note that $H_{yu}(P, C_1, C_2) \in \mathcal{M}(\mathcal{R_U})$ implies that $C = C_1 + C_2$ $\mathcal{R_U}$-stabilizes P but the converse is not necessarily true.

3.1 Definitions: a) The pair (C_1, C_2) is said to be a *reliable controller pair* for P iff **(i)** C_1 is an $\mathcal{R_U}$-stabilizing controller for P, **(ii)** C_2 is an $\mathcal{R_U}$-stabilizing controller for P and **(iii)** $\mathcal{S}(P, C_1, C_2)$ is $\mathcal{R_U}$-stable.
b) The pair (C_1, C_2) is said to be a *reliable decomposition* of C iff **(i)** $C_1 + C_2 = C$ and **(ii)** (C_1, C_2) is a reliable controller pair for P.

3.2 Lemma: [1] There exists a reliable controller pair (C_1, C_2) for P if and only if there exists an $\mathcal{R_U}$-stabilizing controller for P which is $\mathcal{R_U}$-stable (i.e., P is strongly $\mathcal{R_U}$-stabilizable).

3.3 Corollary: a) If P is $\mathcal{R_U}$-stable, then (C_1, C_2) is a reliable controller pair for P if and only if for $j = 1, 2$, $C_j = (I_{ni} - Q_j P)^{-1} Q_j$, where $Q_j \in \mathcal{R_U}^{ni \times no}$ is such that $(I_{ni} - Q_2 P Q_1 P)$ is $\mathcal{R_U}$-unimodular; additionally, Q_j must satisfy $\det(I_{ni} - Q_j P) \in \mathcal{I}$ (which automatically holds for all $Q_j \in \mathcal{M}(\mathcal{R_U})$ when P is strictly proper).
b) If the pair (C_1, C_2) is a reliable controller pair for P, then $C_2 P (I_{ni} + C_1 P + C_2 P)^{-1} C_1$ is a strongly $\mathcal{R_U}$-stabilizing controller for P. Conversely, if C_1 and C_2 are two $\mathcal{R_U}$-stabilizing controllers such that $C_2 P (I_{ni} + C_1 P + C_2 P)^{-1} C_1$ is a strongly $\mathcal{R_U}$-stabilizing controller for P, then the pair (C_1, C_2) is a reliable controller pair.

3.4 Algorithm (*Reliable controller pair design*): Let $P \in \mathrm{IR_p}(s)^{no \times ni}$ be a given strongly $\mathcal{R_U}$-stabilizable plant. Let (N_P, D_P) and $(\tilde{D}_P, \tilde{N}_P)$ be any RCF and LCF of P.
Method 1: *Step 1:* Find an $\mathcal{R_U}$-stabilizing controller $C_S \in \mathcal{M}(\mathcal{R_U})$ for P. Let $(\tilde{D}_C, \tilde{N}_C)$ be an LCF and (N_C, D_C) be an RCF of C_S such that $\tilde{D}_C D_P + \tilde{N}_C N_P = I_{ni}$ and $\tilde{D}_P D_C + \tilde{N}_P N_C = I_{no}$. *Step 2:* Find $Q_2 \in \mathcal{M}(\mathcal{R_U})$ so that $(I_{ni} - \tilde{N}_C N_P + Q_2 N_P \tilde{N}_C N_P)$ is $\mathcal{R_U}$-unimodular and $\det(I_{ni} - Q_2 N_P) \in \mathcal{I}$. *Step 3:* A reliable controller pair (C_1, C_2) is given by $C_1 := C_S$ and $C_2 := \tilde{D}_C^{-1}(I_{ni} - Q_2 N_P)^{-1} Q_2$.
Method 2: Repeat steps 1 and 2 above. *Step 3:* Find $Q_1 \in \mathcal{M}(\mathcal{R_U})$ such that $(I_{ni} - Q_1 \tilde{N}_P \tilde{D}_C^{-1} Q_2 N_P)$ is $\mathcal{R_U}$-unimodular and $\det(I_{ni} - Q_1 \tilde{N}_P \tilde{D}_C^{-1} N_P) \in \mathcal{I}$. *Step 4:* A reliable controller pair (C_1, C_2) is given by $C_1 := \tilde{D}_C^{-1}(I_{ni} - Q_1 \tilde{N}_P \tilde{D}_C^{-1})^{-1}(\tilde{N}_C + Q_1 \tilde{D}_P)$, $C_2 := \tilde{D}_C^{-1}(I_{ni} - Q_2 N_P)^{-1} Q_2$. □

One of the two controllers in Method 1 of Algorithm 3.4 is always $\mathcal{R_U}$-stable; the second controller is also $\mathcal{R_U}$-stable if and only if Q_2 is such that $(I_{ni} - Q_2 N_P)$ is $\mathcal{R_U}$-unimodular. The two controllers in Method 2 may or may not be $\mathcal{R_U}$-stable.

3.5 Algorithm (*Reliable decomposition*): Let $P \in \mathcal{R_U}^{no \times ni}$. Let $C \in \mathrm{IR_p}(s)^{ni \times no}$ be any given $\mathcal{R_U}$-stabilizing controller for P; let $(\tilde{D}_C, \tilde{N}_C)$ and (N_C, D_C) be any LCF and RCF of C. *Step 1:* Find any $\hat{Q} \in \mathcal{R_U}^{ni \times no}$ such that $(I_{ni} - \hat{Q} P)$ is $\mathcal{R_U}$-unimodular. *Step 2:* Define $\alpha := \| \hat{Q} P \|$, $\beta := \| \hat{Q} D_C P \|$. Choose any $k > \alpha + \beta$. *Step 3:* Define $Q_1 := \hat{Q}/k$. A reliable controller pair (C_1, C_2) is given by $C_1 := (I_{ni} - Q_1 P) Q_1$, $C_2 := C - C_1 = C - (I_{ni} - Q_1 P)^{-1} Q_1$. □

The controller C_1 in the reliable decomposition of Algorithm 3.5 is chosen $\mathcal{R_U}$-stable. A sufficient condition to make $(I_{ni} - \hat{Q} P)$ $\mathcal{R_U}$-unimodular is to choose $\hat{Q}$ so that $\| \hat{Q} \| < 1/\| P \|$. The controller C_2 is $\mathcal{R_U}$-stable if and only if the given controller C is.

References

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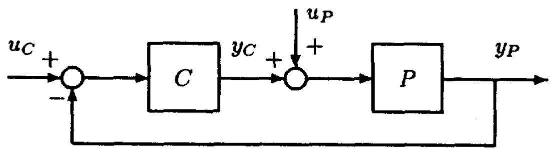

Figure 1: The system $\mathcal{S}(P, C)$

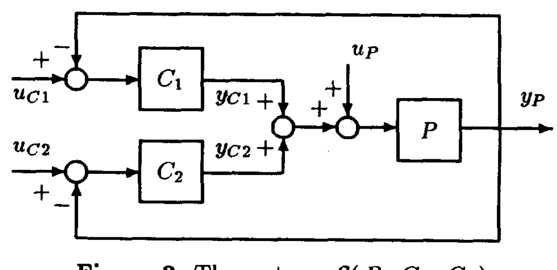

Figure 2: The system $\mathcal{S}(P, C_1, C_2)$