Let

\[ x = \begin{bmatrix} x_p - x_c \\ x_c \end{bmatrix}, \quad w = \begin{bmatrix} w_p \\ v \end{bmatrix}. \]

We can get the closed-loop system in the form of (1) with 2 input and 3 output. If we choose an identity weight matrix \( W \) and the given matrix \( Q \)

\[ Q = \begin{bmatrix} 1 & 1 \\ 1 & 100 \end{bmatrix} \]

the bound on the \( L_\infty \) norm is

\[ \sigma[Y] \text{tr } QW^{-1} = 12.3327. \]

Using the continuous time algorithm given, the optimal \( \hat{W} \) is

\[ \hat{W} = \begin{bmatrix} 6.7853 & 7.8153 \\ 7.8153 & 125.1410 \end{bmatrix} \]

Using \( \hat{W} \), the bound is

\[ \sigma[Y] \text{tr } Q\hat{W}^{-1} = 3.1112 \]

an improvement of a factor of four. Discretizing the closed-loop system with sampling time \( T = 0.1 \) we can get the discrete closed-loop system in the form of (36). With the same \( Q \) matrix, if we choose identity weight matrix \( W_d \), the bound on the \( L_\infty \) norm is

\[ \sigma[Y] \text{tr } QW_d^{-1} = 1.2323. \]

If we use the weight matrix

\[ \hat{W}_d = \begin{bmatrix} 6.7845 & 7.8125 \\ 7.8125 & 125.1410 \end{bmatrix} \]

obtained by the discrete algorithm, the bound becomes

\[ \sigma[Y] \text{tr } Q\hat{W}_d^{-1} = 0.3109 \]

a factor of four improvement. \( \square \)

For this example, the initial weight matrix is identity. With the error tolerance \( 10^{-12} \), the algorithm converges in seven iterations.

III. CONCLUSION

The optimal \( L_\infty \) bounds are given for disturbance rejection. The algorithm which produces these bounds is iterative and several examples show that the algorithm converges very quickly. These bounds are explicit in the output covariance matrix and may have a use in synthesizing covariance controllers to achieve a specified degree of disturbance rejection. This is an interesting problem for future work.

REFERENCES

be the ring of proper rational functions which have no poles in \( \mathbb{Z} \), the ring of proper rational functions, and the field of rational functions of \( s \) (with real coefficients). The group of units of \( \mathcal{R}_p \) is \( \mathcal{F} = \mathcal{R}_p \setminus \{0\} \). The set of matrices whose entries are in \( \mathcal{R}_p \) is denoted \( \mathcal{A}(\mathcal{R}_p) \). A matrix \( M \) is called \( \mathcal{R}_p \)-stable iff \( M \in \mathcal{A}(\mathcal{R}_p) \), \( M \in \mathcal{A}(\mathcal{R}_p) \) is \( \mathcal{R}_p \)-unimodular iff det \( M \in \mathcal{F} \), where det \( M \) denotes the determinant of the matrix \( M \). If \( p, q \in \mathcal{R}_p \), then \( p \sim q \) iff \( p = mq \) for some \( m \in \mathcal{F} \). The identity matrix of size \( n \) is denoted \( I_n \). The diagonal matrix, whose entries are \( a_1, a_2, \ldots, a_m \) is denoted \( \text{diag}(a_1, a_2, \ldots, a_m) \).

II. MAIN RESULTS

A. The System \( S(P, C) \)

Consider the LTI, MIMO feedback system \( S(P, C) \) shown in Fig. 1, where \( P: \varepsilon_j \rightarrow \gamma_j \) and \( C: \varepsilon_j \rightarrow \gamma_j \) represent the plant and the controller transfer functions. We assume that: 1) the plant and the controller have no hidden modes associated with eigenvalues of \( \mathbb{Z} \); 2) \( \varepsilon \) and \( C \) are proper (\( P \in \mathcal{R}_a(s)^{p \times m} \) and \( C \in \mathcal{R}_a(s)^{p \times m} \)); and 3) the system \( S(P, C) \) is well posed, i.e., \( \text{rank} P = \min\{n, n_o\} \). The closed loop input–output transfer function \( H_{pc} : u \rightarrow y_p \) of \( S(P, C) \) is

\[
H_{pc} = PH_{cc} = PC(I_n + P C)^{-1}
\]

where \( H_{cc} : u \rightarrow y_p \) is given by \( H_{cc} = (I_n + P C)^{-1} \).

**Definition 2.1.1:** The system \( S(P, C) \) is said to be \( \mathcal{R}_a \)-stable iff \( H_{pc} : u \rightarrow y_p \in \mathcal{A}(\mathcal{R}_a) \); \( S(P, C) \) is said to be decoupled iff it is \( \mathcal{R}_a \)-stable and the input–output transfer function \( H_{pc} : u \rightarrow y_p \) is diagonal and nonsingular.

The set \( \mathcal{A}(P) := \{C \in \mathcal{R}_a(s)^{p \times m} \mid S(P, C) \text{ is } \mathcal{R}_a \text{-stabile} \} \) is called the set of all \( \mathcal{R}_a \)-stabilizing controllers for \( P \). The set \( \mathcal{A}(P) := \{H_{pc} : u \rightarrow y_p \mid C \in \mathcal{A}(P) \} \) is called the set of all achievable input–output transfer functions \( u \) to \( y_p \) from the input \( u \) to the output \( y_p \). The set \( \mathcal{A}(P) := \{C \in \mathcal{A}(P) \mid H_{pc} \text{ is diagonal and nonsingular} \} \) is called the set of \( \mathcal{R}_a \)-stabilizing controllers for \( P \). The set \( \mathcal{A}(P) := \{H_{pc} : C \in \mathcal{A}(P) \} \) is called the set of all achievable input–output transfer functions \( H_{pc} \) from the input \( u \) to the output \( y_p \).

To achieve decoupling in the system \( S(P, C) \), the plant's transfer function \( P \) must be full (normal) row rank (Lemma 2.1.2). If \( P \in \mathcal{R}_a(s)^{n \times m} \) has full-row rank (i.e., rank \( P = n_o \)), then a sufficient condition for the existence of decoupling controllers in the system \( S(P, C) \) is that the full-row rank plant \( P \) does not have any \( \mathbb{Z} \)-poles coinciding with \( \mathbb{Z} \)-zeros. In Section II-B, we parameterize the class of all decoupling controllers \( \mathcal{A}(P) \) and all achievable decoupled transfer functions \( \mathcal{A}(P) \) for plants which satisfy this sufficient condition. Co-prime factorizations of the plant derived from its Smith–McMillan form are used in this parameterization; we discuss these briefly in Fact 2.1.3.

**Lemma 2.1.2 (Necessary Condition for Decoupling):** Let \( P \in \mathcal{R}_a(s)^{n \times m} \). If the system \( S(P, C) \) is decoupled, then rank \( P = n_o \leq n_i \).

**Proof:** If \( S(P, C) \) is decoupled, then the \( n_o \times n_o \) transfer function \( H_{pc} \) is nonsingular, where \( H_{pc} = PH_{cc} \), therefore,

\[
\text{rank} H_{pc} = n_o = \text{rank} (PH_{cc}) \leq \text{rank} P \leq \min\{n_o, n_i\}.
\]

Consequently, rank \( P = n_o \), moreover, \( n_o \leq n_i \).

**Facts 2.1.3** [8], [3]: Let \( P \in \mathcal{R}_a(s)^{n \times m} \) and let rank \( P = n_o \). Under these assumptions, the following holds:

i) **Smith–McMillan Form of \( P \):** There exist \( \mathcal{R}_a \)-unimodular matrices \( L \in \mathcal{R}_a(s)^{n \times n} \) and \( R \in \mathcal{R}_a(s)^{m \times m} \) such that

\[
P = L[\Lambda : 0_{n \times (n - n_o)}] \quad \Psi^{-1} \quad I_{(n - n_o)} \quad R
\]

\[
= L \Psi^{-1} [\Lambda : 0_{n \times (n - n_o)}] R
\]

\[
\Lambda := \text{diag} [\lambda_1, \ldots, \lambda_{n_o}], \quad \Psi := \text{diag} [\psi_1, \ldots, \psi_{m_o}].
\]

For \( j = 1, \ldots, n_o \), let \( u_j \in \mathbb{R}_a(s) \) and \( v_j \in \mathbb{R}_a(s) \) satisfy (2.3). Let

\[
U := \text{diag} [u_1, \ldots, u_{n_o}] \quad V := \text{diag} [v_1, \ldots, v_{m_o}].
\]

therefore, \( U + \Psi = \Lambda U \) and \( V = \Psi V = \Lambda V \). Let \( N_p := L[\Lambda : 0_{n \times (n - n_o)}] \quad D_p := R^{-1} \quad \Psi^{-1} \quad I_{(n - n_o)} \)

\[
\bar{D}_p := \Psi L^{-1} \quad \bar{N}_p := [\Lambda : 0_{n \times (n - n_o)}] R
\]

\[
U_p := \left[ \begin{array}{c} U \\ \varepsilon_{n_o} \end{array} \right] \quad \bar{U}_p := R^{-1} U_p L,
\]

\[
V_p := \text{diag} \left[ V_{1:\varepsilon_{n_o}}, \bar{N}_p \right] R \quad \bar{V}_p := \Lambda V_p L.
\]

Then, \( P = N_p \bar{D}_p^{-1} \) is a right-co-prime factorization (rcf) and \( P = \bar{D}_p \bar{N}_p \) is a left-co-prime factorization (lcf) of \( P \), i.e., \( N_p \in \mathcal{R}_a(s)^{n \times n} \), \( D_p \in \mathcal{R}_a(s)^{m \times m} \), \( \bar{N}_p \in \mathcal{R}_a(s)^{n \times m} \), \( \bar{D}_p \in \mathcal{R}_a(s)^{m \times n} \), det \( \bar{D}_p \) \neq 0. \( V_p \in \mathcal{R}_a(s)^{m \times n}, U_p \in \mathcal{R}_a(s)^{p \times m}, V_p \in \mathcal{R}_a(s)^{n \times m} \) are given by (2.7).

ii) **All \( \mathcal{R}_a \)-Stabilizing Controllers and All Achievable Input–Output Transfer Functions:** The set \( \mathcal{A}(P) \) of all \( \mathcal{R}_a \)-stabilizing controllers is

\[
\mathcal{A}(P) = \left\{ C = (V_p - \bar{N}_p \bar{Q})^{-1} (U_p + \bar{Q}) \right\}
\]

\[
= (\bar{U}_p + D_p \bar{Q}) (V_p - \bar{N}_p \bar{Q})^{-1}
\]

\[
= R^{-1} [U + \Psi \bar{Q}] \quad \Psi^{-1} L^{-1}
\]

\[
\bar{Q} = \left[ \begin{array}{c} \bar{Q} \\ \varepsilon_{m_o} \end{array} \right] \quad \psi_{m_o} \in \mathcal{R}_a(s)^{m \times m}, Q \in \mathcal{R}_a(s)^{m \times m}, \quad \text{det} (\bar{V}_p - \bar{N}_p \bar{Q}) \approx \text{det} (V_p - \bar{Q}) \quad \sim \text{det} (V - \Lambda \bar{Q}) \quad \text{in } \mathcal{F}.
\]
The set \(\mathcal{S}(P)\) of all achievable input–output transfer functions is
\[
\mathcal{S}(P) = \left\{ H_P = N_P(U_P + \hat{Q}D_P) = I_{n_0} - (\hat{V}_P - N_P \hat{Q})D_P \right\}.
\]

If \(P\) is strictly proper, then \(\det(\hat{V}_P - N_P \hat{Q}) \in \mathcal{S}(\hat{Q})\).

If \(P\) is not strictly proper, then \(\det[(\hat{V}_P - N_P \hat{Q})^\dagger] \in \mathcal{S}(\hat{Q})\).

If \(P\) is strictly proper, then \(\det(\hat{V}_P - N_P \hat{Q}) \in \mathcal{S}(\hat{Q})\).

If \(P\) is not strictly proper, then \(\det[(\hat{V}_P - N_P \hat{Q})^\dagger] \in \mathcal{S}(\hat{Q})\).

Theorem 2.2.2: Let \(P \in \mathcal{S}(\hat{Q})\) and let \(Q = 0\). Let \(m_i/d_i \in \mathcal{S}(\hat{Q})\) denote the \(i, j\)-entry of \(P^j\), where the pair \((m_i, d_i)\) is coprime, \(n_i, d_i \in \mathcal{S}(\hat{Q})\), \(d_i \neq 0\). Therefore, the plant \(P\) does not have any \(m\)-poles coinciding with \(m\)-zeros if and only if \(A_{\mathcal{S}(\hat{Q})}\) is a coprime pair or equivalently, there exist \(\mathcal{S}(\hat{Q})\)-stable matrix \(W\) if and only if \(W_R = \Delta Q_R\), where \(Q_R \in \mathcal{S}(\hat{Q})\) is diagonal, nonsingular; and \(\Delta Q_R\)-stable matrix \(W\) if and only if \(W_L = \Delta Q_L\), where \(Q_L \in \mathcal{S}(\hat{Q})\) is diagonal, nonsingular; and \(\Delta Q_R\)-stable matrix \(W\) if and only if \(W_L = \Delta Q_L\), where \(Q_L \in \mathcal{S}(\hat{Q})\) is diagonal, nonsingular.

Proof: We only prove part (ii); the proof of (ii) is similar.

Let \(\Delta Q_L = \text{diag} [\delta_{11}, \ldots, \delta_{m_n}]\) be a gcd of all of the entries in the \(j\)-th column of \(Q_L\) and define \(\Delta_L := \text{diag} [\delta_{11}, \ldots, \delta_{m_n}]\). Let \(\delta_{ij} \in \mathcal{S}(\hat{Q})\) be a gcd of the entries in the \(j\)-th column of \(Q_L\) and define \(\theta_{ij} \in \mathcal{S}(\hat{Q})\)

B. Parameterization of Decoupling Controllers

In this section, we assume that rank \(P = n_0\) and that \(P\) does not have any \(m\)-poles coinciding with \(m\)-zeros. Under these assumptions, it is possible to find decoupling controllers. In Theorem 2.2.4, we parameterize: 1) the class of all decoupling controllers for \(P\); and 2) the class of all achievable decoupled input–output transfer functions \(H_{\mathcal{S}}\).
would have diagonal entries differing only by units in 
\( Q \), equivalently, \( (A_{1},...) \) is coprime, i.e., (2.11) holds for some \( P \). By assumption, \( A \) has no \( \bar{z} \)-poles coinciding with \( \bar{z} \)-zeros, then the coupled Maps)

for some diagonal, nonsingular \( Q_{R} \). Under these assumptions:

Moreover, \( \Lambda^{s} \) is \( \mathcal{R}_{D} \)-stable. By (2.19), \( (\psi_{2}^{(s)}\Delta^{s} - I) \) is diagonal, nonsingular. By Lemma 2.2.2(ii) \( (\psi_{2}^{(s)}\Delta^{s} - I) \) is diagonal for some \( Q_{R} \), \( \in \mathcal{R}_{D} \), i.e.,

Similarly, \( \Psi^{s} - I \) is \( \mathcal{R}_{D} \)-stable. By (2.19), \( (\psi_{2}^{(s)}\psi_{2}^{(s)}\Delta^{s} - I) \) is diagonal, nonsingular, \( \mathcal{R}_{D} \)-stable matrix \( Q_{R} \) if and only if \( (\psi_{2}^{(s)}\Delta^{s} - I) \) is diagonal, nonsingular, \( \mathcal{R}_{D} \)-stable matrix \( Q_{R} \). Therefore, if \( W = \Delta_{R}Q_{R}\Theta_{R} \) for some diagonal, nonsingular, \( \mathcal{R}_{D} \)-stable matrix \( Q_{R} \), then \( N^{-1}WD^{-1} = N^{-1}\Delta_{R}Q_{R}\Theta_{R} \).

\( \text{Corollary 2.2.3: Under the assumption of Lemma 2.2.2, if } P \text{ does not have any } \bar{z} \text{-poles coinciding with } \bar{z} \text{-zeros, equivalently, } \Lambda_{na}, \psi_{2} \text{ is coprime, i.e., (2.11) holds for some } \alpha, \beta \in \mathcal{R}_{D}. \) By (2.21), (2.22), and (2.11)

where all of the \( \mathcal{R}_{D} \) matrices in (2.23) are diagonal, \( \mathcal{R}_{D} \)-stable and hence, \( (\Theta_{R}, \Theta_{R}, \Theta_{R}) \) is right coprime and \( (\Theta_{R}, \Theta_{R}, \Theta_{R}) \) is left coprime.

\( i) \text{ the set } \mathcal{P}(P) \text{ of all decoupled input-output transfer functions } H_{\mu} \text{ is given by}

\[
\mathcal{P}(P) = \left\{ H_{\mu} = \alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu} \right\}
\]

\( j \text{ the set } \mathcal{P}(P) \text{ of all decoupling controllers for } P \text{ is given by}

\[
\mathcal{P}(P) = \left\{ C = \frac{\alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu}}{Q_{\mu}} \right\}
\]

\( \text{Comments: In (2.24) and (2.25), for } j = 1, \ldots, n_{o}, \text{ } q_{j} \in \mathcal{R}_{D} \text{ satisfies}

\[
q_{j}(\infty) = \frac{\beta q_{j} - 1}{\delta_{j} \delta_{j} \delta_{j} \delta_{j}}(\infty).
\]

\( \text{Condition (2.26) guarantees that the decoupling controllers are proper. If } P \text{ is strictly proper, (2.26) is satisfied for any } q_{j} \in \mathcal{R}_{D}. \) To ensure the nonsingularity of the achieved decoupled transfer functions, \( H_{\mu} = \alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu} \), \( j = 1, \ldots, n_{o}, \text{ } q_{j} \in \mathcal{R}_{D} \text{ must also satisfy}

\[
q_{j}(\infty) = \frac{\beta q_{j} - 1}{\delta_{j} \delta_{j} \delta_{j} \delta_{j}}(\infty).
\]

However, as shown in the proof of Lemma 2.2.2, (2.22) implies that \( (\Theta_{R}, \Theta_{R}, \Theta_{R}) \) is coprime with \( \psi_{2} \), except when \( \psi_{2} = 1 \) (equivalently, \( P \in \mathcal{R}_{D} \)). Therefore, (2.27) is automatically satisfied for all \( q_{j} \in \mathcal{R}_{D} \) except when \( P \in \mathcal{R}_{D} \), in which case, the additional condition that \( q_{j} \neq 0 \) should be included in the parameterizations as shown in Corollary 2.2.5 below.

\( \text{Proof of Theorem 2.2.4: By assumption, } \alpha_{\mu}, \psi_{2} \text{ is coprime, hence, by Fact 2.1.3-ii) } H_{\mu} = \alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu} \text{ is diagonal if and only if}

\( H_{\mu} = \alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu} \) is diagonal for some \( \alpha_{\mu}, \psi_{2} \in \mathcal{R}_{D}. \) Therefore, \( Q_{\mu} \in \mathcal{R}_{D} \). By Corollary 2.2.3, \( W = \Delta_{R}Q_{R}\Theta_{R} \) is diagonal if and only if \( Q_{\mu} \in \mathcal{R}_{D} \) is of the form

\[
Q_{\mu} = \frac{\alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu}}{Q_{\mu}}.
\]

for some diagonal \( Q_{\mu} \in \mathcal{R}_{D} \). With \( Q_{\mu} = \text{diag}[q_{1}, \ldots, q_{n_{o}}] \), \( H_{\mu} = \alpha_{\mu} I_{m_{0}} + \Delta_{\mu} Q_{\mu} Q_{\mu} \) is nonsingular if and only if \( \alpha_{\mu} \neq 1 \) (equivalently, \( P \in \mathcal{R}_{D} \)), then \( H_{\mu} \) is nonsingular if only if \( q_{j} \neq 0 \) (see Corollary 2.2.5).
The parameterization (2.25) is obtained from (2.15) by letting $Q$ be as in (2.28). Since $L$ is $\mathcal{F}$-unimodular, in the parameterization (2.15), $\det(\beta_1 H_0) = -\Delta Q^T \Psi \in \mathcal{F}$ if and only if $\det(L(\beta_1 H_0 - \Delta Q^T \Psi)) = \det((I_0 - L P) - \Delta Q^T \Psi^T) \in \mathcal{F}$; equivalently $(\beta_1 H_0 - \Delta Q^T \Psi^T)$ has at least one pole at $\infty$. 

**Corollary 2.2.5 (Decoupling Controllers for $\mathcal{F}$-Stable Plants):** Let $P \in \mathcal{F}$ and let rank $P = n_0$. Then the sets $\mathcal{A}_c(P)$ and $\mathcal{S}_c(P)$ become

$$\mathcal{A}_c(P) = \left\{ H_{pc} = \Delta_1 \Delta_2 Q_{DP} : q_j \in \mathcal{F}, q_j(\omega) = \frac{1}{\beta_{ij}} \right\},$$

$$\mathcal{S}_c(P) = \left\{ C = R^{-1} \left[ N^{-1} \Delta_1 Q D 0 \Delta_2^{-1} \right] \right\},$$

where $Q_A \in \mathcal{F}_{\Psi(\infty) \rightarrow \Psi(\infty)}$, $Q_D = \text{diag}[q_1, \ldots, q_{n_0}]$, and $q_j(\omega) \neq 0, j = 1, \ldots, n_0$.

**Proof:** When $P \in \mathcal{F}$-stable, without loss of generality, $\Psi = L_0$ and $\Psi = 1$. Then $\Psi = I_{n_0} = \Theta_{P}$ and $D = L = D_{0}$. The parameterizations (2.29) and (2.30) follow by substituting these values of $\Theta_{P}$, $\Theta_{P}$, $D$ into (2.24) and (2.25), where the additional constraint that $q_j \neq 0$ is imposed to ensure $H_{pc}$ is nonsingular.

### III. Conclusions

For LTI, MIMO, full-row rank plants which do not have any undesirable poles coinciding with zeros, we parameterized the class of all controllers such that the unity-feedback system is internally stable and the closed-loop transfer function from the command-input $u_c$ to the plant-output $y_p$ is diagonal and nonsingular. If the plant had undesirable poles coinciding with zeros, this class of controllers could not be used, in that case, two-degrees-of-freedom stabilizing decoupling controllers would be more useful since any full-row rank plant (which does not have any undesirable hidden modes) can be decoupled using two-parameter controllers [1], [3].

### References


### Vibrational Control of Linear Time Lag Systems with Arbitrarily Large but Bounded Delays

Brad Lehman and Joseph Bentsman

**Abstract**—This note shows that vibrational stabilization can be effective for linear systems with large bounded delays. Theorems are given that define the procedures for the search of the stabilizing vibrations. Robust oscillatory stabilization insensitive to the delay size is also shown to take place for some classes of systems.

### I. Introduction

A number of practically important systems, such as chemical reactors [1] and combustion systems [2] are best described by including time delays in their states. Feedback stabilization of such systems [3], [4] is usually not an easy task, especially if the delays are significant and there are restrictions on sensing and actuation. In recent papers [5], [6] an open-loop vibrational control technique introduced in [7] was shown to be effective for a class of systems with small delays in the states. While the results of [5] and [6] demonstrated the viability of the technique as a possible alternative to feedback for time lag systems as well as provided the tools for the synthesis of fast periodic feedback for this class of systems, the restriction that the delay size be small limited their practical utility. The purpose of the present note is to remove this restriction on the delay size. This is not a trivial task and it partially motivated development of new averaging theorems for differential delay equations [8]. The techniques presented can also be used for the synthesis of fast periodic feedback controllers for systems with large bounded time lags.

The present note gives the conditions for the existence of the stabilizing vibrations for a class of linear time lag systems with arbitrary fixed bounded delays and presents the procedure for the search of the parameters of the stabilizing vibrations (Section II). The calculation formula for the choice of the parameters of the stabilizing vibrations, and the conditions for the vibrational stabilization to be insensitive to the delay size are also given for specific classes of systems (Section III). The results are supported by the numerical examples (Section IV). Conclusions are given in Section V.

Manuscript received August 16, 1990; revised July 5, 1991. This work was supported in part by the National Science Foundation under Grant MSS-8957198.

B. Lehman is with the School of Electrical Engineering, Georgia Institute of Technology, Atlanta, GA 30332. J. Bentsman is with the Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801.

IEEE Log Number 9201935.