STABILITY UNDER SENSOR OR ACTUATOR FAILURES

A. Nazli Gündeş[1]
Department of Electrical Engineering and Computer Science
University of California, Davis, CA 95616

M. Güntekin Kabuli
Integrated Systems, Inc.
3260 Jay Street, Santa Clara, CA 95054-3309

Abstract

We consider the standard linear, time-invariant, multi-input multi-output unity-feedback system with possible failures in the sensor or actuator-connections. We parametrize the set of all controllers such that the closed-loop system is stable when sensors or actuators fail. We consider two classes of failures: the failure of one connection and the failure of any number of connections provided that at least one connection does not fail. The general parametrization requires knowledge of the failure to update the stabilizing controller. For plants which can be decoupled, we also give a class of controllers which ensure stability under any sensor or actuator failures without updating for different perturbations.

1. Introduction

A multivariable feedback system is said to have integrity if it remains stable in the presence of arbitrary failures of the sensor or actuator-connections. The problem of integrity is a robust stability problem. Designing controllers such that the system remains stable for a class of failures is equivalent to simultaneous stabilization of the nominal plant and the plant multiplied by different failure-matrices.

In this paper we consider the stability of the standard linear, time-invariant, multi-input multi-output unity-feedback system under possible failures in either the sensor or actuator-connections. In the standard problem of integrity, the failure of a sensor or actuator means that the corresponding connection is disconnected and hence, the appropriate output is multiplied by zero. Here we use a more general description for the failure of a connection and allow the connection to be multiplied by any arbitrary stable rational function (including zero) in case of failure.

Let P and C denote the transfer-functions of the plant and the controller. If we consider the possibility of failures in the sensor-connections, then the system we consider is $\mathcal{S}(F_S, P, C)$ (shown in Figure 1), where the outputs of P are multiplied by arbitrary stable transfer functions. The sub-block F_S represents the sensor-connections; it is a stable diagonal matrix, whose diagonal entries are nominally equal to 1 (meaning that the corresponding sensor-connection did not fail). The matrix F_S belongs to the class $\mathcal{F}_{Sk}$, which denotes the set of all possible failures of at most k of the sensor-connections, where the maximum number of failures k is between 1 and the number of outputs n_o. Similarly, the system with possible actuator-connection failures is $\mathcal{S}(P, F_A, C)$ (shown in Figure 2), where F_A represents the actuator-connections. If the j-th actuator-connection fails, then the j-th output of the controller is multiplied by a stable transfer-function, which is different than 1. The diagonal matrix F_A belongs to the class $\mathcal{F}_{Am}$, which denotes the set of all possible failures of at most m of the actuator-connections, where the maximum number of failures m is between 1 and the number of inputs n_i. If $k = n_o$ or $m = n_i$, then the plant and the controller must be stable; in this case the problem becomes the *complete* integrity problem. Otherwise, the plant and the controller need not be stable and instead of *complete* integrity, we deal with *k-sensor-integrity* or *m-actuator-integrity*.

The purpose of this paper is to develop a controller design methodology such that the systems $\mathcal{S}(F_S, P, C)$ and $\mathcal{S}(P, F_A, C)$ are stable. We consider two classes of failures: **1)** There exists at most one failure; **2)** there exists at least one channel without failure. The controller characterizations in the general parametrizations (Theorems 3.1 and 3.2) are not independent of the failures. But for plants which can be decoupled in some sense, we also find a class of fixed controllers that ensure stability of $\mathcal{S}(F_S, P, C)$ and $\mathcal{S}(P, F_A, C)$ without updating the controller when the failure-matrices change (Propositions 3.3 and 3.4).

2. Preliminaries

Let $\mathcal{U}$ be a subset of the field $\mathbb{C}$ of complex numbers; $\mathcal{U}$ is closed and symmetric about the real axis, $\pm\infty \in \mathcal{U}$ and $\mathbb{C} \setminus \mathcal{U}$ is nonempty. Let $\mathcal{R}_{\mathcal{U}}, \mathbb{R}_p(s), \mathbb{R}_{sp}(s), \mathbb{R}(s)$ be the ring of proper rational functions which have no poles in $\mathcal{U}$, the ring of proper rational functions, the set of strictly proper rational functions and the field of rational functions of s (with real coefficients), respectively. Let $\mathcal{J}$ be the group of units of $\mathcal{R}_{\mathcal{U}}$ and let $\mathcal{I} := \mathcal{R}_{\mathcal{U}} \setminus \mathbb{R}_{sp}(s)$. The set of matrices whose entries are in $\mathcal{R}_{\mathcal{U}}$ is $\mathcal{M}(\mathcal{R}_{\mathcal{U}})$. $M \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$

[1]Research supported by the National Science Foundation Grant ECS-9010996

0-7803-0593-0/92 $3.00 1992 IEEE

is $\mathcal{R}_{\mathcal{U}}$-unimodular iff $\det M \in \mathcal{J}$.

Let $\mathcal{F}_{Sk}$ denote the class of sensor failures defined as follows: If $F_S \in \mathcal{F}_{Sk}$, then $F_S = \mathrm{diag}[\, f_1 \ \dots \ f_{no} \,]$, where, for $j = 1, \dots, n_o$, $f_j \in \mathcal{R}_{\mathcal{U}}$ and at least $(n_o - k)$ of the entries $f_j = 1$; k is the maximum number of sensor failures and $f_j = 0$ if the j-th sensor is disconnected. We are interested in the classes $\mathcal{F}_{S1}$ (the arbitrary failure of at most one the n_o sensors) and $\mathcal{F}_{S(no-1)}$ (arbitrary failures of at most $(n_o - 1)$ of the n_o sensors). Similarly, $\mathcal{F}_{Am}$ denotes the class of actuator-connection failures defined by $\mathcal{F}_{Am} := \{\mathrm{diag}[\, f_1 \ \dots \ f_{ni} \,]\}$, where, for $j = 1, \dots, n_i$, $f_j \in \mathcal{R}_{\mathcal{U}}$ and at least $(n_i - m)$ of the entries $f_j = 1$; m is the maximum number of actuator failures and $f_j = 0$ if the j-th actuator is disconnected. Again the classes of interest here are $\mathcal{F}_{A1}$ and $\mathcal{F}_{A(ni-1)}$, defined similarly.
In $\mathcal{S}(F_S, P, C)$, $[\, y_P \ \ y_C \,]^T = H_S [\, u_P \ \ u_C \,]^T$ and in $\mathcal{S}(P, F_A, C)$, $[\, y_P \ \ y_C \,]^T = H_A [\, u_P \ \ u_C \,]^T$.

Assumptions: i) The plant $P \in \mathbb{R}_p(s)^{no \times ni}$. **ii)** The controller $C \in \mathbb{R}_p(s)^{ni \times no}$. **iii)** The systems $\mathcal{S}(F_S, P, C)$ and $\mathcal{S}(P, F_A, C)$ are well-posed; equivalently, $H_S \in \mathcal{M}(\mathbb{R}_p(s))$ and $H_A \in \mathcal{M}(\mathbb{R}_p(s))$. **iv)** P and C have no hidden-$\mathcal{U}$-modes. □
Let $P = N_P D_P^{-1}$ denote any right-coprime-factorization **(rcf)** and $P = \widetilde{D}_P^{-1} \widetilde{N}_P$ denote any left-coprime-factorization **(lcf)** of $P \in \mathbb{R}_p(s)^{no \times ni}$, where $N_P \in \mathcal{R}_{\mathcal{U}}{}^{no \times ni}$, $D_P \in \mathcal{R}_{\mathcal{U}}{}^{ni \times ni}$, $\widetilde{N}_P \in \mathcal{R}_{\mathcal{U}}{}^{no \times ni}$, $\widetilde{D}_P \in \mathcal{R}_{\mathcal{U}}{}^{no \times no}$; $\det D_P \in \mathcal{I}$ (equivalently, $\det \widetilde{D}_P \in \mathcal{I}$) if and only if $P \in \mathcal{M}(\mathbb{R}_p(s))$. There exist V_P, U_P, $\widetilde{V}_P$, $\widetilde{U}_P \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$ such that $V_P D_P + U_P N_P = I_{ni}$, $\widetilde{D}_P \widetilde{V}_P + \widetilde{N}_P \widetilde{U}_P = I_{no}$, $V_P \widetilde{U}_P = U_P \widetilde{V}_P$.

Definitions: a) i) $\mathcal{S}(F_S, P, C)$ is said to be *$\mathcal{R}_{\mathcal{U}}$-stable* iff $H_S \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$. **ii)** For $k = 1, \dots, n_o$, $\mathcal{S}(F_S, P, C)$ is said to have *k-sensor-integrity* iff it is $\mathcal{R}_{\mathcal{U}}$-stable for *all* $F_S \in \mathcal{F}_{Sk}$. **iii)** P is said to have no *k-sensor-failure hidden-$\mathcal{U}$-modes* iff for all $F_S \in \mathcal{F}_{Sk}$, *rank* $\begin{bmatrix} \widetilde{D}_P \\ F_S \end{bmatrix} = n_o$, for all $s \in \mathcal{U}$. **iv)** C is a *controller with k-sensor-integrity* iff $C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(F_S, P, C)$ has k-sensor-integrity; the set $\mathbf{S}_{Sk}(P) := \{\, C \mid C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(F_S, P, C)$ has k-sensor-integrity $\}$ is called the *set of all controllers with k-sensor-integrity.* **b) i)** $\mathcal{S}(P, F_A, C)$ is said to be *$\mathcal{R}_{\mathcal{U}}$-stable* iff $H_A \in \mathcal{M}(\mathcal{R}_{\mathcal{U}})$. **ii)** For $m = 1, \dots, n_i$, $\mathcal{S}(P, F_A, C)$ is said to have *m-actuator-integrity* iff it is $\mathcal{R}_{\mathcal{U}}$-stable for *all* $F_A \in \mathcal{F}_{Am}$. **iii)** P is said to have no *m-actuator-failure hidden-$\mathcal{U}$-modes* iff for all $F_A \in \mathcal{F}_{Am}$, *rank* $[\, D_P \ \ F_A \,] = n_i$, for all $s \in \mathcal{U}$. **iv)** C is a *controller with m-actuator-integrity* iff $C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(P, F_A, C)$ has m-actuator-integrity; the set $\mathbf{S}_{Am}(P) := \{\, C \mid C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(P, F_A, C)$ has m-actuator-integrity$\}$ is called the *set of all controllers with m-actuator-integrity.* □

3. Main Results

Consider $\mathcal{S}(F_S, P, C)$. If it has k-sensor-integrity, then P has no k-sensor-failure hidden-$\mathcal{U}$-modes [3]. Let $F_S \in \mathcal{F}_{S1}$; P has no 1-sensor-failure hidden-$\mathcal{U}$-modes if and only if there is an $\mathcal{R}_{\mathcal{U}}$-unimodular matrix L_1 such that $L_1 \widetilde{D}_P = \begin{bmatrix} 1 & \tilde{d}_{1,2} & \dots & \tilde{d}_{1,no} \\ 0 & \tilde{d}_{2,2} & \dots & \tilde{d}_{2,no} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \tilde{d}_{no,no} \end{bmatrix}$, where $\left(\begin{bmatrix} \tilde{d}_{1,1+j} \\ \vdots \\ \tilde{d}_{j,1+j} \end{bmatrix}, \ \tilde{d}_{1+j,1+j} \right)$ is right-coprime for $j = 1, \dots, n_o - 1$. For $j = 2, \dots, n_o$, $\ell = 1, \dots, j$, there exist $\tilde{y}_{j,\ell} \in \mathcal{R}_{\mathcal{U}}$ such that $\sum_{\ell=1}^{j} \tilde{y}_{j,\ell}\, \tilde{d}_{\ell,j} = 1$. Let $\widetilde{Y}_1 :=$ $\begin{bmatrix} 1 & 0 & \dots & 0 \\ \tilde{y}_{2,1} & \tilde{y}_{2,2} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{y}_{no,1} & \tilde{y}_{no,2} & \dots & \tilde{y}_{no,no} \end{bmatrix} L_1$. Let $\widetilde{M}_1 := \widetilde{Y}_1 \widetilde{D}_P + (I_{no} - \widetilde{Y}_1 \widetilde{D}_P) F_S = I_{no} - (I_{no} - \widetilde{Y}_1 \widetilde{D}_P)(I_{no} - F_S)$; then for all $F_S \in \mathcal{F}_{S1}$, $\widetilde{M}_1$ is $\mathcal{R}_{\mathcal{U}}$-unimodular.

Let $F_S \in \mathcal{F}_{S(no-1)}$; P has no $(n_o - 1)$-sensor-failure hidden-$\mathcal{U}$-modes if and only if there is an $\mathcal{R}_{\mathcal{U}}$-unimodular matrix $L_{(n_o-1)}$ such that $L_{(n_o-1)} \widetilde{D}_P =$ $\begin{bmatrix} 1 & 0 & \dots & \tilde{d}_{1,no} \\ 0 & 1 & & \tilde{d}_{2,no} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \tilde{d}_{no,no} \end{bmatrix}$, where $(\, \tilde{d}_{j,no} \,, \ \tilde{d}_{no,no} \,)$ is coprime for $j = 1, \dots, n_o - 1$. For $j = 1, \dots, n_o - 1$, there exist $\tilde{v}_j$, $\tilde{u}_j \in \mathcal{R}_{\mathcal{U}}$ such that $\tilde{v}_j \tilde{d}_{no,no} + \tilde{u}_j \tilde{d}_{j,no} = 1$. Let $\widetilde{Y}_{(no-1)} :=$

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -\tilde{d}_{2,no}\tilde{u}_1 & 1 & 0 & \dots & 0 & -\tilde{d}_{2,no}\tilde{v}_1 \\ 0 & -\tilde{d}_{3,no}\tilde{u}_2 & 1 & & 0 & -\tilde{d}_{3,no}\tilde{v}_2 \\ \vdots & & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & & & 1 & -\tilde{d}_{no-1,no}\tilde{v}_{no-2} \\ 0 & 0 & \dots & & \tilde{x}_{no-1} & \tilde{x}_{no-1,no-1} \end{bmatrix} L_{(no-1)} .$$

Let $\widetilde{M}_{(no-1)} := \widetilde{Y}_{(no-1)} \widetilde{D}_P + (I_{no} - \widetilde{Y}_{(no-1)} \widetilde{D}_P) F_S = I_{no} - (I_{no} - \widetilde{Y}_{(no-1)} \widetilde{D}_P)(I_{no} - F_S)$; then for all $F_S \in \mathcal{F}_{S(no-1)}$, $\widetilde{M}_{(no-1)}$ is $\mathcal{R}_{\mathcal{U}}$-unimodular. If $k = 1$ or $(n_o - 1)$, for the right-coprime pair $(F_S N_P, D_P)$ the following (Bezout-identity) holds for all $F_S \in \mathcal{F}_{Sk}$ ($\mathcal{F}_{Sk}$ is either $\mathcal{F}_{S1}$ or $\mathcal{F}_{S(no-1)}$):

$$\begin{bmatrix} V_P + U_P \widetilde{M}_k^{-1} \widetilde{Y}_k \widetilde{N}_P & U_P \widetilde{M}_k^{-1} (I_{no} - \widetilde{Y}_k \widetilde{D}_P) \\ -(I_{no} - \widetilde{D}_P (I_{no} - F_S) \widetilde{M}_k^{-1} \widetilde{Y}_k) \widetilde{N}_P & \widetilde{D}_P (F_S + (I_{no} - F_S) \widetilde{Y}_k \widetilde{D}_P)^{-1} \end{bmatrix}$$

$$\cdot \begin{bmatrix} D_P & -\widetilde{U}_P (I_{no} - \widetilde{D}_P \widetilde{Y}_k) \\ F_S N_P & \widetilde{Y}_k + F_S \widetilde{V}_P (I_{no} - \widetilde{D}_P \widetilde{Y}_k) \end{bmatrix} = I_{ni+no} .$$

Now consider $\mathcal{S}(P, F_A, C)$. If it has m-actuator-integrity, then P has no m-actuator-failure hidden-$\mathcal{U}$-modes [3]. Let $F_A \in \mathcal{F}_{A1}$; P has no 1-actuator-failure hidden-$\mathcal{U}$-modes if and only if there is an $\mathcal{R}_{\mathcal{U}}$-unimodular matrix R_1 such that $D_P R_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ d_{2,1} & d_{2,2} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ d_{ni,1} & d_{ni,2} & \dots & d_{ni,ni} \end{bmatrix}$, where $(\, d_{1+j,1+j} \,, \ [\, d_{1+j,1} \ \ d_{1+j,2} \ \dots \ d_{1+j,j} \,] \,)$ is left-coprime for $j = 1, \dots, n_i - 1$. For $j = 2, \dots, n_i$, $\ell = 1, \dots, j$, there exist $y_{\ell,j} \in \mathcal{R}_{\mathcal{U}}$ such that $\sum_{\ell=1}^{j} d_{j,\ell}\, y_{\ell,j} = 1$.

Let $Y_1 := R_1 \begin{bmatrix} 1 & y_{1,2} & \cdots & y_{1,ni} \\ 0 & y_{2,2} & \cdots & y_{2,ni} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & y_{ni,ni} \end{bmatrix}$. Let $M_1 := D_P Y_1 + F_A(I_{ni} - D_P Y_1) = I_{ni} - (I_{ni} - F_A)(I_{ni} - D_P Y_1)$; then for all $F_A \in \mathcal{F}_{A1}$, M_1 is $\mathcal{R_U}$-unimodular.

Let $F_A \in \mathcal{F}_{A(ni-1)}$; P has no (n_i-1)-actuator-failure hidden-$\mathcal{U}$-modes if and only if there is an $\mathcal{R_U}$-unimodular matrix $R_{(n_i-1)}$ such that $D_P R_{(n_i-1)} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ d_{ni,1} & d_{ni,2} & \cdots & d_{ni,ni} \end{bmatrix}$, where $(d_{ni,ni}, d_{ni,j})$ is coprime for $j = 1, \ldots, n_i - 1$. For $j = 1, \ldots, n_i - 1$, there exist $v_j, u_j \in \mathcal{R_U}$ such that $d_{ni,ni} v_j + d_{ni,j} u_j = 1$. Let $\widetilde{Y}_{(ni-1)} :=$

$$R_{(ni-1)} \begin{bmatrix} 1 & -u_1 d_{ni,2} & 0 & \cdots & 0 & 0 \\ 0 & 1 & -u_2 d_{ni,3} & & 0 & 0 \\ \vdots & & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & u_{ni-1} \\ 0 & -v_1 d_{ni,2} & -v_2 d_{ni,3} & \cdots & -v_{ni-2} d_{ni,ni-1} & v_{ni-1,ni-1} \end{bmatrix}.$$

Let $M_{(ni-1)} := D_P \widetilde{Y}_{(ni-1)} + F_A(I_{ni} - D_P \widetilde{Y}_{(ni-1)}) = I_{ni} - (I_{ni} - F_A)(I_{ni} - D_P \widetilde{Y}_{(ni-1)})$; then for all $F_A \in \mathcal{F}_{A(ni-1)}$, $M_{(ni-1)}$ is $\mathcal{R_U}$-unimodular. If $m = 1$ or $(n_i - 1)$, for the left-coprime pair $(\widetilde{D}_P, \widetilde{N}_P F_A)$ the following (Bezout-identity) holds for all $F_A \in \mathcal{F}_{Am}$ ($\mathcal{F}_{Am}$ is either $\mathcal{F}_{A1}$ or $\mathcal{F}_{A(ni-1)}$):

$$\begin{bmatrix} Y_m + (I_{ni} - Y_m D_P) V_P F_A & (I_{ni} - Y_m D_P) U_P \\ -\widetilde{N}_P F_A & \widetilde{D}_P \end{bmatrix} \cdot$$

$$\begin{bmatrix} (F_A + D_P Y_m (I_{ni} - F_A))^{-1} D_P & -(I_{ni} - D_P Y_m) M_m^{-1} \widetilde{U}_P \\ N_P (I_{ni} - Y_m M_m^{-1} (I_{ni} - F_A D_P) & \widetilde{V}_P + N_P Y_m M_m^{-1} \widetilde{U}_P \end{bmatrix}$$
$$= I_{ni+no} \ .$$

In the following results, we use the well-known fact that $C \in \mathbb{R}_p(s)^{ni \times no}$ is an $\mathcal{R_U}$-stabilizing controller for the (nominal) plant P if and only if $C = \widetilde{D}_C^{-1} \widetilde{N}_C \;\; N_C D_C^{-1} = (V_P - Q_n \widetilde{N}_P)^{-1}(U_P + Q_n \widetilde{D}_P) = (\widetilde{U}_P + D_P Q_n)(\widetilde{V}_P - N_P Q_n)^{-1}$, where $Q_n \in \mathcal{R_U}^{ni \times no}$ such that $\det(V_P - Q_n \widetilde{N}_P) \in \mathcal{I}$.

3.1. Theorem (*all controllers with k-sensor-integrity*): Consider $\mathcal{S}(F_S, P, C)$. Let $\widetilde{D}_C^{-1} \widetilde{N}_C = N_C D_C^{-1}$ be an $\mathcal{R_U}$-stabilizing controller for the (nominal) plant P. If $F_S \in \mathcal{F}_{S1}$, let P have no 1-sensor-failure hidden-$\mathcal{U}$-modes; let $\widetilde{Y}_k$ be $\widetilde{Y}_1$ and let $\widetilde{M}_k$ be $\widetilde{M}_1$. If $F_S \in \mathcal{F}_{S(no-1)}$, let P have no (n_o-1)-sensor-failure hidden-$\mathcal{U}$-modes; let $\widetilde{Y}_k$ be $\widetilde{Y}_{(no-1)}$ and let $\widetilde{M}_k$ be $\widetilde{M}_{(no-1)}$. The set $\mathbf{S}_{Sk}(P)$ of all controllers with k-sensor-integrity ($k = 1$ or $(n_o - 1)$) is: $\mathbf{S}_{Sk}(P) = \{ C =$

$$(\widetilde{D}_C + \widetilde{N}_C \widetilde{M}_k^{-1} \widetilde{Y}_k \widetilde{N}_P - Q(I_{no} - \widetilde{D}_P (I_{no} - F_S) \widetilde{M}_k^{-1} \widetilde{Y}_k) \widetilde{N}_P)^{-1}$$
$$(\widetilde{N}_C \widetilde{M}_k^{-1} (I_{no} - \widetilde{Y}_k \widetilde{D}_P) + Q \widetilde{D}_P (F_S + (I_{no} - F_S) \widetilde{Y}_k \widetilde{D}_P)^{-1}) =$$
$$(N_C (I_{no} - \widetilde{D}_P \widetilde{Y}_k) + D_P Q)(\widetilde{Y}_k + F_S D_C (I_{no} - \widetilde{D}_P \widetilde{Y}_k) + F_S N_P Q)^{-1}$$
$$\mid \; Q \in \mathcal{R_U}^{ni \times no},$$
$$\det(\widetilde{D}_C + \widetilde{N}_C \widetilde{M}_k^{-1} \widetilde{Y}_k \widetilde{N}_P - Q(I_{no} - \widetilde{D}_P (I_{no} - F_S) \widetilde{M}_k^{-1} \widetilde{Y}_k) \widetilde{N}_P)$$
$$\sim \det(\widetilde{Y}_k + F_S D_C (I_{no} - \widetilde{D}_P \widetilde{Y}_k) + F_S N_P Q) \in \mathcal{I} \} . \quad \square$$

If P is strictly proper, then for any $Q \in \mathcal{R_U}^{ni \times no}$, $\det(\widetilde{D}_C + \widetilde{N}_C \widetilde{M}_k^{-1} \widetilde{Y}_k \widetilde{N}_P - Q(I_{no} - \widetilde{D}_P (I_{no} - F_S) \widetilde{M}_k^{-1} \widetilde{Y}_k) \widetilde{N}_P) \sim \det(\widetilde{Y}_k + F_S D_C (I_{no} - \widetilde{D}_P \widetilde{Y}_k) + F_S N_P Q) \in \mathcal{I}$. $\square$

Figure 3 shows the block-diagram of the $\mathcal{R_U}$-stable system $\mathcal{S}(F_S, P, C)$, where $C \in \mathbf{S}_{Sk}(P)$.

3.2 Theorem (*all controllers with m-actuator-integrity*): Consider $\mathcal{S}(P, F_A, C)$. Let $\widetilde{D}_C^{-1} \widetilde{N}_C = N_C D_C^{-1}$ be an $\mathcal{R_U}$-stabilizing controller for the (nominal) plant P. If $F_A \in \mathcal{F}_{A1}$, let P have no 1-actuator-failure hidden-$\mathcal{U}$-modes; let Y_m be Y_1 and let M_m be M_1. If $F_A \in \mathcal{F}_{A(ni-1)}$, let P have no $(n_i - 1)$-actuator-failure hidden-$\mathcal{U}$-modes; let Y_m be $\widetilde{Y}_{(ni-1)}$ and let M_m be $M_{(ni-1)}$. The set $\mathbf{S}_{Am}(P)$ of all controllers with m-actuator-integrity ($m = 1$ or $(n_i - 1)$) is: $\mathbf{S}_{Am}(P) = \{ C =$

$$((I_{ni} - D_P Y_m) M_m^{-1} N_C + (F_A + D_P Y_m (I_{ni} - F_A))^{-1} D_P Q)$$
$$(D_C + N_P Y_m M_m^{-1} N_C - N_P (I_{ni} - Y_m M_m^{-1} (I_{ni} - F_A) D_P) Q)^{-1} =$$
$$(\widetilde{Y}_k + (I_{no} - \widetilde{Y}_k D_P) \widetilde{D}_C F_A - Q \widetilde{N}_P F_A)^{-1} ((I_{no} - \widetilde{Y}_k D_P) \widetilde{N}_C + Q \widetilde{D}_P)$$
$$\mid \; Q \in \mathcal{R_U}^{ni \times no},$$
$$\det(D_C + N_P Y_m M_m^{-1} N_C - N_P (I_{ni} - Y_m M_m^{-1} (I_{ni} - F_A) D_P))$$
$$\sim \det(\widetilde{Y}_k + (I_{no} - \widetilde{Y}_k D_P) \widetilde{D}_C F_A - Q \widetilde{N}_P F_A) \in \mathcal{I} \} . \quad \square$$

If P is strictly proper, then for any $Q \in \mathcal{R_U}^{ni \times no}$, $\det(D_C + N_P Y_m M_m^{-1} N_C - N_P (I_{ni} - Y_m M_m^{-1} (I_{ni} - F_A) D_P)) \sim \det(\widetilde{Y}_k + (I_{no} - \widetilde{Y}_k D_P) \widetilde{D}_C F_A - Q \widetilde{N}_P F_A) \in \mathcal{I}$.

Figure 4 shows the block-diagram of the $\mathcal{R_U}$-stable system $\mathcal{S}(P, F_A, C)$, where $C \in \mathbf{S}_{Am}(P)$.

In Proposition 3.3 below, assume that the transfer-function $H_{pc} : u_C \mapsto y_P$ can be made diagonal using an $\mathcal{R_U}$-stabilizing controller; i.e., there exists C_{SD} such that $H_{pc} = P C_{SD} (I_{no} + P C_{SD})^{-1}$ is diagonal for the given (nominal) plant P. Then $C_{SD} = (V_P - Q_{SD} \widetilde{N}_P)^{-1} (U_P + Q_{SD} \widetilde{D}_P) = (\widetilde{U}_P + D_P Q_{SD})(\widetilde{V}_P - N_P Q_{SD})^{-1}$, where $Q_{SD} \in \mathcal{R_U}^{ni \times no}$ is such that $H_{pc} = N_P (U_P + Q_{SD} \widetilde{D}_P)$ is diagonal. The class of plants for which the transfer-function H_{pc} can be diagonalized is not empty; a sufficient condition for the existence of such controllers is that the plant P is full row-rank and does not have any $\mathcal{U}$-poles coinciding with zeros (in this case the transfer-function H_{pc} can be made diagonal and *nonsingular*). Let $C_{SD} = \widetilde{D}_{SD}^{-1} \widetilde{N}_{SD} = N_{SD} D_{SD}^{-1}$ be any controller which diagonalizes H_{pc}; the parametrization of all such decoupling controllers is given in [5] for full row-rank plants without pole-zero coincidences in $\mathcal{U}$.

Similarly, in Proposition 3.4, assume that the transfer-function $H_{cp} : u_P \mapsto y_C$ can be made diagonal using an $\mathcal{R_U}$-stabilizing controller; i.e., there exists $C_{AD} \in \mathbf{S}(P)$ such that $H_{cp} = -C_{AD}(I_{no} + P C_{AD})^{-1} P$ is diagonal for the given (nominal) plant P. Then $C_{AD} = (V_P - Q_{AD} \widetilde{N}_P)^{-1} (U_P + Q_{AD} \widetilde{D}_P) = (\widetilde{U}_P + D_P Q_{AD})(\widetilde{V}_P - N_P Q_{AD})^{-1}$, where $Q_{AD} \in \mathcal{R_U}^{ni \times no}$ is such that $H_{cp} = -(\widetilde{U}_P + D_P Q_{AD}) \widetilde{N}_P$ is diagonal. The class of plants for which the transfer-function H_{pc} can be diagonalized is not empty; a sufficient condition for the existence of such controllers is that the plant P is full column-rank and does not have any $\mathcal{U}$-poles coinciding with zeros (in this case the transfer-function H_{cp} can be made diagonal and *nonsingular*). Let $C_{AD} = \widetilde{D}_{AD}^{-1} \widetilde{N}_{AD} = N_{AD} D_{AD}^{-1}$ be any controller which diagonalizes H_{cp}.

3.3. Proposition (*controllers with k-sensor-integrity for decoupled plants*): Consider $\mathcal{S}(F_S, P, C)$. Assume that the conditions of Theorem 3.1 hold. Suppose that there exists an $\mathcal{R}_\mathcal{U}$-stabilizing decoupling controller C_{SD} for the (nominal) plant P, such that the transfer-function H_{pc} is diagonal. Under these conditions, a class of controllers with k-sensor-integrity ($k = 1$ or $(n_o - 1)$) is given by

$$\{ C = (\widetilde{D}_{SD} + \widetilde{N}_{SD}\tilde{Y}_k\widetilde{N}_P)^{-1}\widetilde{N}_{SD}(I_{no} - \tilde{Y}_k\widetilde{D}_P)$$
$$= (N_{SD} - D_P\widetilde{N}_{SD}\tilde{Y}_k)(D_{SD} + N_P\widetilde{N}_{SD}\tilde{Y}_k)^{-1} \} ,$$

where $C_{SD} = \widetilde{D}_{SD}^{-1} \widetilde{N}_{SD} = N_{SD} D_{SD}^{-1}$ is any $\mathcal{R}_\mathcal{U}$-stabilizing decoupling controller for the (nominal) plant P such that $\det(\widetilde{D}_{SD} + \widetilde{N}_{SD}\tilde{Y}_k\widetilde{N}_P) \sim \det(D_{SD} + N_P\widetilde{N}_{SD}\tilde{Y}_k) \in \mathcal{I}$. □ Figure 5 shows the block-diagram of the $\mathcal{R}_\mathcal{U}$-stable system $\mathcal{S}(F_S, P, C)$ using the controller in Proposition 3.3.

3.4. Proposition (*controllers with m-actuator-integrity for decoupled plants*): Consider $\mathcal{S}(P, F_A, C)$. Assume that the conditions of Theorem 3.2 hold. Suppose that there exists an $\mathcal{R}_\mathcal{U}$-stabilizing decoupling controller C_{AD} for the (nominal) plant P, such that the transfer-function H_{cp} is diagonal. Under these conditions, a class of controllers with m-actuator-integrity ($m = 1$ or $(n_i - 1)$) is given by

$$\{ C = (\widetilde{D}_{AD} + Y_m N_{AD}\widetilde{N}_P)^{-1}(\widetilde{N}_{AD} - Y_m N_{AD}\widetilde{D}_P)$$
$$= (I_{ni} - D_P Y_m) N_{AD}(D_{AD} + N_P Y_m N_{AD})^{-1} \} ,$$

where $C_{SD} = \widetilde{D}_{SD}^{-1} \widetilde{N}_{SD} = N_{SD} D_{SD}^{-1}$ is any $\mathcal{R}_\mathcal{U}$-stabilizing decoupling controller for the (nominal) plant P such that $\det(\widetilde{D}_{AD} + Y_m N_{AD}\widetilde{N}_P) \sim \det(D_{AD} + N_P Y_m N_{AD}) \in \mathcal{I}$. □ Figure 6 shows the block-diagram of the $\mathcal{R}_\mathcal{U}$-stable system $\mathcal{S}(P, F_A, C)$ using the controller in Proposition 3.4.

References

[1] M. Fujita and E. Shimemura, "Integrity against arbitrary feedback-loop failure in linear multivariable control systems," *Automatica*, Vol. 24, 765, 1988.

[2] A. N. Gündeş and M. G. Kabuli, "Conditions for stability of feedback systems under sensor failures," *Proc. 28th Conference on Decision and Control,* pp. 1688-1689, 1989.

[3] A. N. Gündeş, "Stability of feedback systems with sensor or actuator failures: Analysis," *International Journal of Control*, to appear.

[4] A. N. Gündeş and M. G. Kabuli, "Stabilizing controllers for systems with sensor or actuator failures," *Proc. 30 Conference on Decision and Control,* pp. 81-82, 1991.

[5] A. N. Gündeş, "Parametrization of decoupling controllers in the unity-feedback system," *IIEEE Transactions on Automatic Control,* to appear.

[6] M. Vidyasagar and N. Viswanadham, "Reliable stabilization using a multi-controller configuration," *Automatica*, Vol. 21, no. 5, pp. 599-602, 1985.

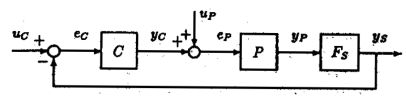

Figure 1: The system $\mathcal{S}(F_S, P, C)$

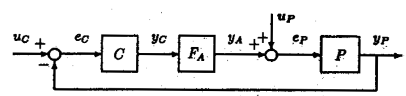

Figure 2: The system $\mathcal{S}(P, F_A, C)$

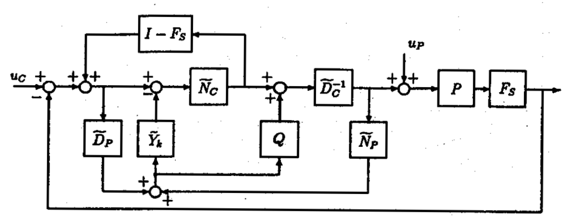

Figure 3: $\mathcal{R}_\mathcal{U}$-stable $\mathcal{S}(F_S, P, C)$ for $k = 1$ or $k = n_o - 1$

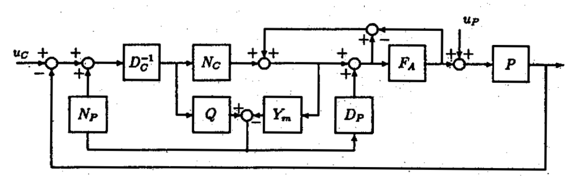

Figure 4: $\mathcal{R}_\mathcal{U}$-stable $\mathcal{S}(P, F_A, C)$ for $m = 1$ or $m = n_i - 1$

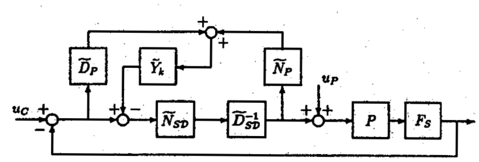

Figure 5: $\mathcal{S}(F_S, P, C)$ with $\widetilde{D}_{SD}^{-1}\widetilde{N}_{SD}$ diagonalizing H_{pc} for the nominal P

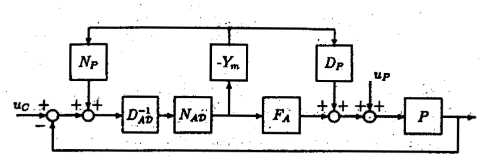

Figure 6: $\mathcal{S}(P, F_A, C)$ with $N_{AD} D_{AD}^{-1}$ diagonalizing H_{cp} for the nominal P