Stabilizing Controllers for Systems with Sensor or Actuator Failures

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Abstract
We parametrize the set of all controllers such that the standard unity-feedback system is stable when sensors or actuators fail. We consider two classes of failures: the failure of one connection and the failure of any number of connections provided that at least one connection does not fail.

1. Introduction
In this paper we parametrize the set of all controllers such that the standard unity-feedback system is stable in the presence of arbitrary sensor or actuator failures. The characterization of controllers in this parametrization is not independent of the failures.

We consider the linear, time-invariant, multi-input multi-output feedback systems $S(F_p, P, C)$ and $S(P, F_A, C)$ (Fig. 1, 2), where $P$ represents the plant and $C$ represents the controller transfer functions, $F_p$ represents the sensor-connections and $F_A$ represents the actuator-connections. $F_p$ and $F_A$ are stable diagonal matrices whose entries are nominally 1; if the $j$-th sensor (actuator) fails, the $j$-th entry is no longer 1 and becomes any stable perturbation including 0. We consider two classes of failures. The main results are the parametrizations of all stabilizing controllers for these systems (Theorems 3.1 and 3.2).

2. Preliminaries
Let $U$ be a subset of the field $C$ of complex numbers; $U$ is closed and symmetric about the real axis, $\pm z \in U$ and $C \setminus U$ is nonempty. Let $R_U, \mathbb{R}_U(s), \mathbb{I}_U(s), \mathbb{H}_U(s)$ be the ring of proper rational functions which have no poles in $U$, the ring of proper rational functions, the set of strictly proper rational functions and the field of rational functions of $s$, respectively. Let $F$ be the group of units of $R_U$ and let $F := R_U \setminus \{0\}$. The set of matrices whose entries are in $R_U$ is $M(R_U)$. $M$ is a controller if $M \in R_U$-unimodular iff $det M \neq 0$.

Let $\mathcal{F}_k$ denote the class of sensor failures defined as follows: If $F_k$ in $\mathcal{F}_k$, then $F_k = \text{diag} [f_1, \ldots, f_m]$, where, for $j = 1, \ldots, n$, $f_j \in R_U$ and at least $n_r - k$ of the entries $f_j = 1$; $k$ is the maximum number of sensor failures and $f_j = 0$ if the $j$-th sensor is disconnected. We are interested in the classes $\mathcal{F}_k$ (the arbitrary failure of at most one the $n_r$ sensors) and $\mathcal{F}_{(n_r)}$ (arbitrary failures of at most $n_r$ of the $n_r$ sensors). Similarly, $\mathcal{F}_{(n_r)}$ denotes the class of actuator-connection failures defined as $\mathcal{F}_{(n_r)} = \{\text{diag} [f_1, \ldots, f_m] \mid f_1 = 1, \ldots, f_m = 1, \forall j \in \{1, \ldots, m\} \}$, where, for $j = 1, \ldots, n_a$, $f_j \in R_U$ and at least $n_a - m$ of the entries $f_j = 1$; $m$ is the maximum number of actuator failures and $f_j = 0$ if the $j$-th actuator is disconnected. Again the classes of interest here are $\mathcal{F}_k$ and $\mathcal{F}_{(n_r)}$ defined similarly.


Assumptions: i) The plant $P \in \mathbb{R}_U(s)$ is $\mathbb{R}_U(s)$-minimal. ii) The controller $C$ is $\mathbb{R}_U(s)$-minimal. iii) The systems $S(F_p, P, C)$ and $S(P, F_A, C)$ are well-posed; equivalently, $H_y \in M(R_U(s))$ and $H_u \in M(R_U(s))$.

Let $p = N_p D_p^{-1}$ denote any right-coprime-factorization (rcf) and $p = D_p^{-1} N_p$ denote any left-coprime-factorization (lcf) of $P \in \mathbb{R}_U(s)$, where $N_p \in R_U^{n_r \times m_r}$, $D_p \in R_U^{m_r \times m_r}$; det $D_p \in \mathbb{C}$ (equivalently, det $D_p \in \mathbb{C}$) if and only if $P \in M(\mathbb{R}_U(s))$. There exist $V_F, U_F$, $V_F = U_F \in M(R_U)$ such that $V_F D_F + U_F N_F = I_n$, $D_F V_F + N_F U_F = I_m$, $V_F U_F = V_F U_F$.

Definitions: a) $S(F_p, P, C)$ is said to be $\mathcal{R}_u$-stable if $H_y \in M(R_U)$. b) $S(F_p, P, C)$ has $k$-sensor-integrity, if for $k = 1, \ldots, n_r$, $S(F_p, P, C)$ is said to have $k$-sensor-integrity if it is $\mathcal{R}_u$-stable for all $F_k \in \mathcal{F}_k$. c) $P$ is said to have no $k$-sensor-failure hidden $U$-modes if for all $F_k \in \mathcal{F}_k$, $\text{rank } D_F = n_r$, for all $s \in U$. iv) $C$ is a controller with $k$-sensor-integrity iff $C \in \mathbb{R}_U(s)$ and $S(F_p, P, C)$ has $k$-sensor-integrity; the set $S_k(P) := \{C \in \mathbb{R}_U(s) \mid S(F_p, P, C)$ has $k$-sensor-integrity$\}$ is called the set of all controllers with $k$-sensor-integrity.

3. Main Results
Consider $S(F_p, P, C)$. If $S(F_p, P, C)$ has $k$-sensor-integrity, then $P$ has no $k$-sensor-failure hidden $U$-modes. Let $F_k \in \mathcal{F}_k$; $P$ has no 1-sensor-failure hidden $U$-modes if and only if there is an $\mathcal{R}_u$-unimodular matrix $L_1$ such that $L_1 D_F = \begin{bmatrix} \hat{d}_{11} & \ldots & \hat{d}_{1n_c} \\ \vdots & \ddots & \vdots \\ \hat{d}_{n_1} & \ldots & \hat{d}_{n_1n_c} \end{bmatrix}$, where \( \hat{d}_{i,j} \neq 0 \) if the $j$-th sensor is disconnected.


Assumptions: i) The plant $P \in \mathbb{R}_U(s)$ is $\mathbb{R}_U(s)$-minimal. ii) The controller $C$ is $\mathbb{R}_U(s)$-minimal. iii) The systems $S(F_p, P, C)$ and $S(P, F_A, C)$ are well-posed; equivalently, $H_y \in M(R_U(s))$ and $H_u \in M(R_U(s))$. iv) $P$ and $C$ have no hidden $U$-modes.

Let $p = N_p D_p^{-1}$ denote any right-coprime-factorization (rcf) and $p = D_p^{-1} N_p$ denote any left-coprime-factorization (lcf) of $P \in \mathbb{R}_U(s)$, where $N_p \in R_U^{n_r \times m_r}$, $D_p \in R_U^{m_r \times m_r}$.

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Let \( M(n-1) := \hat{Y}_{n-1} D_P + (I_{m}-\hat{Y}_{n-1} D_P) F_S = I_{m} - (I_{m} - \hat{Y}_{n-1} D_P) F_S \); then for all \( F_S \in F_{SA}(n-1) \), \( M(n-1) \) is \( R_U \)-unimodular. If \( k = 1 \) or \( k = (n-1) \), for the right-coprime pair \( (F_S, N_P, D_P) \) the following Bezout-identity holds for all \( F_S \in F_{SA} \) (where \( F_S \) is either \( F_{SA} \) or \( F_{SA}(1-1) \)):

\[
\begin{align*}
V_P + U_P M(n-1) &\cong Y_P N_P \\
-U_P M(n-1) &\cong (I_{m} - \hat{Y}_{n-1} D_P) Y_P, \\
D_P &\cong Y_P + F_S V_D(I_{m} - \hat{Y}_{n-1} D_P) Y_P.
\end{align*}
\]

Now consider \( S(P, F_S, C) \). If \( S(P, F_S, C) \) has m-actuator-integrity, then for all \( F_S \in F_{SA}(1-1) \), \( M(n-1) \) is \( R_U \)-unimodular. If \( k = 1 \) or \( k = (n-1) \), for the right-coprime pair \( (F_S, N_P, D_P) \) the following Bezout-identity holds for all \( F_S \in F_{SA} \) (where \( F_S \) is either \( F_{SA} \) or \( F_{SA}(1-1) \)):

\[
\begin{align*}
V_P + U_P M(n-1) &\cong Y_P N_P \\
-U_P M(n-1) &\cong (I_{m} - \hat{Y}_{n-1} D_P) Y_P, \\
D_P &\cong Y_P + F_S V_D(I_{m} - \hat{Y}_{n-1} D_P) Y_P.
\end{align*}
\]

3.2 Theorem (all controllers with m-actuator-integrity): Consider \( S(P, F_S, C) \). If \( F_S \in F_{SA}(1-1) \), let \( P \) have no 1-actuator-failure hidden \( U \)-modes; let \( Y_P \) be \( \hat{Y}_{(n-1)} \) and let \( M \) be \( M_{(n-1)} \). If \( F_S \in F_{SA}(1-1) \), let \( P \) have no \((n-1)-1\)-actuator-failure hidden \( U \)-modes; let \( Y_P \) be \( \hat{Y}_{(n-1)} \) and let \( M \) be \( M_{(n-1)} \). Then the set \( S_{Am}(P) \) of all controllers with m-actuator-integrity \((m = 1 \text{ or } (n-1)) \) is:

\[
S_{Am}(P) = \{ C = \bar{D}_C^{-1} \bar{N}_C = N_C D_C^{-1} | \bar{D}_C = V_P + U_P M(n-1) Y_P N_P \\
\quad -Q(I_{m} - \hat{Y}_{n-1} D_P) M_{n-1}^{-1} \bar{N}_P, \bar{N}_C = U_P M_{n-1}^{-1} (I_{m} - \hat{Y}_{n-1} D_P) \\
\quad +Q D_P(I_{m} - \hat{Y}_{n-1} D_P) Y_P, N = U_P(I_{m} - \hat{Y}_{n-1} D_P) Y_P + D_P Q, \bar{D}_C = Y_P + F_S V_D(I_{m} - \hat{Y}_{n-1} D_P) Y_P + F_S N_P Q, Q \in R_{U}^{mtimes} \\
\quad \text{det}(V_P + U_P M_{n-1}^{-1} Y_P N_P - Q(I_{m} - \hat{Y}_{n-1} D_P) M_{n-1}^{-1} \bar{N}_P) \sim \text{det}(Y_P + F_S V_D(I_{m} - \hat{Y}_{n-1} D_P) Y_P + F_S N_P Q) , \in \overline{I} \} \]