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Stabilizing Controllers for Systems with Sensor or Actuator Failures

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Abstract

We parametrize the set of all controllers such that the standard unity-feedback system is stable when sensors or actuators fail. We consider two classes of failures: the failure of one connection and the failure of any number of connections provided that at least one connection does not fail.

1. Introduction

In this paper we parametrize the set of all controllers such that the standard unity-feedback system is stable in the presence of arbitrary sensor or actuator failures. The characterization of controllers in this parametrization is not independent of the failures.

We consider the linear, time-invariant, multi-input multi-output feedback systems $\mathcal{S}(F_S, P, C)$ and $\mathcal{S}(P, F_A, C)$ (Fig. 1, 2), where P represents the plant and C represents the controller transfer-functions, F_S represents the sensor-connections and F_A represents the actuator-connections. F_S and F_A are stable diagonal matrices whose entries are nominally 1; if the j-th sensor (actuator) fails, the j-the entry is no longer 1 and becomes any stable perturbation including 0. We consider two classes of failures. The main results are the parametrizations of all stabilizing controllers for these systems (Theorems 3.1 and 3.2).

2. Preliminaries

Let $\mathcal{U}$ be a subset of the field $\mathbb{C}$ of complex numbers; $\mathcal{U}$ is closed and symmetric about the real axis, $\pm\infty \in \mathcal{U}$ and $\mathbb{C} \setminus \mathcal{U}$ is nonempty. Let $\mathcal{R}_\mathcal{U}, \mathbb{R}_p(s), \mathbb{R}_{sp}(s), \mathbb{R}(s)$ be the ring of proper rational functions which have no poles in $\mathcal{U}$, the ring of proper rational functions, the set of strictly proper rational functions and the field of rational functions of s (with real coefficients), respectively. Let $\mathcal{J}$ be the group of units of $\mathcal{R}_\mathcal{U}$ and let $\mathcal{I} := \mathcal{R}_\mathcal{U} \setminus \mathbb{R}_{sp}(s)$. The set of matrices whose entries are in $\mathcal{R}_\mathcal{U}$ is $\mathcal{M}(\mathcal{R}_\mathcal{U})$. $M \in \mathcal{M}(\mathcal{R}_\mathcal{U})$ is $\mathcal{R}_\mathcal{U}$-unimodular iff $\det M \in \mathcal{J}$.

Let $\mathcal{F}_{Sk}$ denote the class of sensor failures defined as follows: If $F_S \in \mathcal{F}_{Sk}$, then $F_S = \mathrm{diag}[\, f_1 \ \ldots \ f_{no}\,]$, where, for $j = 1, \ldots, n_o$, $f_j \in \mathcal{R}_\mathcal{U}$ and at least $(n_o - k)$ of the entries $f_j = 1$; k is the maximum number of sensor failures and $f_j = 0$ if the j-th sensor is disconnected. We are interested in the classes $\mathcal{F}_{S1}$ (the arbitrary failure of at most one the n_o sensors) and $\mathcal{F}_{S(no-1)}$ (arbitrary failures of at most $(n_o - 1)$ of the n_o sensors). Similarly, $\mathcal{F}_{Am}$ denotes the class of actuator-connection failures defined by $\mathcal{F}_{Am} := \{\mathrm{diag}[\, f_1 \ \ldots \ f_{ni}\,]\}$, where, for $j = 1, \ldots, n_i$, $f_j \in \mathcal{R}_\mathcal{U}$ and at least $(n_i - m)$ of the entries $f_j = 1$; m is the maximum number of actuator failures and $f_j = 0$ if the j-th actuator is disconnected. Again the classes of interest here are $\mathcal{F}_{A1}$ and $\mathcal{F}_{A(ni-1)}$, defined similarly.

In $\mathcal{S}(F_S, P, C)$, $[\, y_P \ \ y_C\,]^T = H_S [\, u_P \ \ u_C\,]^T$ and in $\mathcal{S}(P, F_A, C)$, $[\, y_P \ \ y_C\,]^T = H_A [\, u_P \ \ u_C\,]^T$.

Assumptions: i) The plant $P \in \mathbb{R}_p(s)^{no \times ni}$. ii) The controller $C \in \mathbb{R}_p(s)^{ni \times no}$. iii) The systems $\mathcal{S}(F_S, P, C)$ and $\mathcal{S}(P, F_A, C)$ are well-posed; equivalently, $H_S \in \mathcal{M}(\mathbb{R}_p(s))$ and $H_A \in \mathcal{M}(\mathbb{R}_p(s))$. iv) P and C have no hidden $\mathcal{U}$-modes. □

Let $P = N_P D_P^{-1}$ denote any right-coprime-factorization (**rcf**) and $P = \widetilde{D}_P^{-1} \widetilde{N}_P$ denote any left-coprime-factorization (**lcf**) of $P \in \mathbb{R}_p(s)^{no \times ni}$, where $N_P \in \mathcal{R}_\mathcal{U}^{no \times ni}$, $D_P \in \mathcal{R}_\mathcal{U}^{ni \times ni}$, $\widetilde{N}_P \in \mathcal{R}_\mathcal{U}^{no \times ni}$, $\widetilde{D}_P \in \mathcal{R}_\mathcal{U}^{no \times no}$; $\det D_P \in \mathcal{I}$ (equivalently, $\det \widetilde{D}_P \in \mathcal{I}$) if and only if $P \in \mathcal{M}(\mathbb{R}_p(s))$. There exist V_P, U_P, $\widetilde{V}_P$, $\widetilde{U}_P \in \mathcal{M}(\mathcal{R}_\mathcal{U})$ such that $V_P D_P + U_P N_P = I_{ni}$, $\widetilde{D}_P \widetilde{V}_P + \widetilde{N}_P \widetilde{U}_P = I_{no}$, $V_P \widetilde{U}_P = U_P \widetilde{V}_P$.

Definitions: **a)** **i)** $\mathcal{S}(F_S, P, C)$ is said to be *$\mathcal{R}_\mathcal{U}$-stable* iff $H_S \in \mathcal{M}(\mathcal{R}_\mathcal{U})$. **ii)** For $k = 1, \ldots, n_o$, $\mathcal{S}(F_S, P, C)$ is said to have *k-sensor-integrity* iff it is $\mathcal{R}_\mathcal{U}$-stable for *all* $F_S \in \mathcal{F}_{Sk}$. **iii)** P is said to have no *k-sensor-failure hidden $\mathcal{U}$-modes* iff for all $F_S \in \mathcal{F}_{Sk}$, $rank \begin{bmatrix} \widetilde{D}_P \\ F_S \end{bmatrix} = n_o$, for all $s \in \mathcal{U}$. **iv)** C is a *controller with k-sensor-integrity* iff $C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(F_S, P, C)$ has k-sensor-integrity; the set $\mathbf{S}_{Sk}(P) := \{\, C \mid C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(F_S, P, C)$ has k-sensor-integrity$\}$ is called the *set of all controllers with k-sensor-integrity.* **b)** **i)** $\mathcal{S}(P, F_A, C)$ is said to be *$\mathcal{R}_\mathcal{U}$-stable* iff $H_A \in \mathcal{M}(\mathcal{R}_\mathcal{U})$. **ii)** For $m = 1, \ldots, n_i$, $\mathcal{S}(P, F_A, C)$ is said to have *m-actuator-integrity* iff it is $\mathcal{R}_\mathcal{U}$-stable for *all* $F_A \in \mathcal{F}_{Am}$. **iii)** P is said to have no *m-actuator-failure hidden $\mathcal{U}$-modes* iff for all $F_A \in \mathcal{F}_{Am}$, $rank\,[\, D_P \ \ F_A\,] = n_i$, for all $s \in \mathcal{U}$. **iv)** C is a *controller with m-actuator-integrity* iff $C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(P, F_A, C)$ has m-actuator-integrity; the set $\mathbf{S}_{Am}(P) := \{\, C \mid C \in \mathbb{R}_p(s)^{ni \times no}$ and $\mathcal{S}(P, F_A, C)$ has m-actuator-integrity$\}$ is called the *set of all controllers with m-actuator-integrity.* □

3. Main Results

Consider $\mathcal{S}(F_S, P, C)$. If $\mathcal{S}(F_S, P, C)$ has k-sensor-integrity, then P has no k-sensor-failure hidden $\mathcal{U}$-modes. Let $F_S \in \mathcal{F}_{S1}$; P has no 1-sensor-failure hidden $\mathcal{U}$-modes if and only if there is an $\mathcal{R}_\mathcal{U}$-unimodular matrix L_1 such that $L_1 \widetilde{D}_P = \begin{bmatrix} 1 & \tilde{d}_{1,2} & \ldots & \tilde{d}_{1,no} \\ 0 & \tilde{d}_{2,2} & \ldots & \tilde{d}_{2,no} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \tilde{d}_{no,no} \end{bmatrix}$, where $\left(\begin{bmatrix} \tilde{d}_{1,1+j} \\ \vdots \\ \tilde{d}_{j,1+j} \end{bmatrix}, \tilde{d}_{1+j,1+j} \right)$ is right-coprime for $j = 1, \ldots, n_o - 1$. For $j = 2, \ldots, n_o$, $\ell = 1, \ldots, j$, there exist $\tilde{y}_{j,\ell} \in \mathcal{R}_\mathcal{U}$ such that $\sum_{\ell=1}^{j} \tilde{y}_{j,\ell}\, \tilde{d}_{\ell,j} = 1$. Let $\widetilde{Y}_1 := \begin{bmatrix} 1 & 0 & \ldots & 0 \\ \tilde{y}_{2,1} & \tilde{y}_{2,2} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{y}_{no,1} & \tilde{y}_{no,2} & \ldots & \tilde{y}_{no,no} \end{bmatrix} L_1$. Let $\widetilde{M}_1 := \widetilde{Y}_1 \widetilde{D}_P + (I_{no} - \widetilde{Y}_1 \widetilde{D}_P) F_S = I_{no} - (I_{no} - \widetilde{Y}_1 \widetilde{D}_P)(I_{no} - F_S)$; then for all $F_S \in \mathcal{F}_{S1}$, $\widetilde{M}_1$ is $\mathcal{R}_\mathcal{U}$-unimodular.

Let $F_S \in \mathcal{F}_{S(no-1)}$; P has no $(n_o - 1)$-sensor-failure hidden $\mathcal{U}$-modes if and only if there is an $\mathcal{R}_\mathcal{U}$-unimodular matrix $L_{(n_o-1)}$ such that $L_{(n_o-1)} \widetilde{D}_P = \begin{bmatrix} 1 & 0 & \ldots & \tilde{d}_{1,no} \\ 0 & 1 & & \tilde{d}_{2,no} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & \tilde{d}_{no,no} \end{bmatrix}$, where $(\tilde{d}_{j,no}, \tilde{d}_{no,no})$ is coprime for $j = 1, \ldots, n_o - 1$. For $j = 1, \ldots, n_o - 1$, there exist $\tilde{v}_j, \tilde{u}_j \in \mathcal{R}_\mathcal{U}$ such that $\tilde{v}_j \tilde{d}_{no,no} + \tilde{u}_j \tilde{d}_{j,no} = 1$. Let $\widetilde{Y}_{(no-1)} :=$

$$\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ -\tilde{d}_{2,no}\tilde{u}_1 & 1 & 0 & \ldots & 0 & -\tilde{d}_{2,no}\tilde{v}_1 \\ 0 & -\tilde{d}_{3,no}\tilde{u}_2 & 1 & & 0 & -\tilde{d}_{3,no}\tilde{v}_2 \\ \vdots & & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & & & 1 & -\tilde{d}_{no-1,no}\tilde{v}_{no-2} \\ 0 & 0 & \ldots & & \tilde{x}_{no-1} & \tilde{x}_{no-1,no-1} \end{bmatrix} L_{(no-1)}.$$

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Let $\widetilde{M}_{(no-1)} := \tilde{Y}_{(no-1)}\widetilde{D}_P + (I_{no} - \tilde{Y}_{(no-1)}\widetilde{D}_P) F_S = I_{no} - (I_{no} - \tilde{Y}_{(no-1)}\widetilde{D}_P)(I_{no} - F_S)$; then for all $F_S \in \mathcal{F}_{S(no-1)}$, $\widetilde{M}_{(no-1)}$ is $\mathcal{R}_{\mathcal{U}}$-unimodular. If $k = 1$ or $k = (n_o - 1)$, for the right-coprime pair $(F_S N_P, D_P)$ the following Bezout-identity holds for all $F_S \in \mathcal{F}_{Sk}$ ($\mathcal{F}_{Sk}$ is either $\mathcal{F}_{S1}$ or $\mathcal{F}_{S(no-1)}$):

$$\begin{bmatrix} V_P + U_P\widetilde{M}_k^{-1}\tilde{Y}_k\widetilde{N}_P & U_P\widetilde{M}_k^{-1}(I_{no} - \tilde{Y}_k\widetilde{D}_P) \\ -(I_{no} - \widetilde{D}_P(I_{no} - F_S)\widetilde{M}_k^{-1}\tilde{Y}_k)\widetilde{N}_P & \widetilde{D}_P(F_S + (I_{no} - F_S)\tilde{Y}_k\widetilde{D}_P)^{-1} \end{bmatrix}$$

$$\cdot \begin{bmatrix} D_P & -\widetilde{U}_P(I_{no} - \widetilde{D}_P\tilde{Y}_k) \\ F_S N_P & \tilde{Y}_k + F_S\widetilde{V}_P(I_{no} - \widetilde{D}_P\tilde{Y}_k) \end{bmatrix} = I_{ni+no} .$$

Now consider $\mathcal{S}(P, F_A, C)$. If $\mathcal{S}(P, F_A, C)$ has m-actuator-integrity, then P has no m-actuator-failure hidden $\mathcal{U}$-modes. Let $F_A \in \mathcal{F}_{A1}$; P has no 1-actuator-failure hidden $\mathcal{U}$-modes if and only if there is an $\mathcal{R}_{\mathcal{U}}$-unimodular matrix R_1 such that $D_P R_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ d_{2,1} & d_{2,2} & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ d_{ni,1} & d_{ni,2} & \dots & d_{ni,ni} \end{bmatrix}$, where $(d_{1+j,1+j}, [d_{1+j,1} \; d_{1+j,2} \; \dots \; d_{1+j,j}])$ is left-coprime for $j = 1, \dots, n_i - 1$. For $j = 2, \dots, n_i$, $\ell = 1, \dots, j$, there exist $y_{\ell,j} \in \mathcal{R}_{\mathcal{U}}$ such that $\sum_{\ell=1}^{j} d_{j,\ell}\, y_{\ell,j} = 1$. Let $Y_1 :=$ $R_1 \begin{bmatrix} 1 & y_{1,2} & \dots & y_{1,ni} \\ 0 & y_{2,2} & \dots & y_{2,ni} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & y_{ni,ni} \end{bmatrix}$. Let $M_1 := D_P Y_1 + F_A(I_{ni} - D_P Y_1)$ $= I_{ni} - (I_{ni} - F_A)(I_{ni} - D_P Y_1)$; then for all $F_A \in \mathcal{F}_{A1}$, M_1 is $\mathcal{R}_{\mathcal{U}}$-unimodular.

Let $F_A \in \mathcal{F}_{A(ni-1)}$; P has no $(n_i - 1)$-actuator-failure hidden $\mathcal{U}$-modes if and only if there is an $\mathcal{R}_{\mathcal{U}}$-unimodular matrix $R_{(n_i-1)}$ such that $D_P R_{(n_i-1)} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & & 0 \\ \vdots & & \ddots & \vdots \\ d_{ni,1} & d_{ni,2} & \dots & d_{ni,ni} \end{bmatrix}$, where $(d_{ni,ni}, d_{ni,j})$ is coprime for $j = 1, \dots, n_i - 1$. For $j = 1, \dots, n_i - 1$, there exist v_j, $u_j \in \mathcal{R}_{\mathcal{U}}$ such that $d_{ni,ni}\, v_j + d_{ni,j}\, u_j = 1$. Let $\widetilde{Y}_{(ni-1)} :=$

$$R_{(ni-1)} \begin{bmatrix} 1 & -u_1 d_{ni,2} & 0 & \dots & 0 & 0 \\ 0 & 1 & -u_2 d_{ni,3} & & 0 & 0 \\ \vdots & & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & u_{ni-1} \\ 0 & -v_1 d_{ni,2} & -v_2 d_{ni,3} & \dots & -v_{ni-2} d_{ni,ni-1} & v_{ni-1,ni-1} \end{bmatrix}.$$

Let $M_{(ni-1)} := D_P\widetilde{Y}_{(ni-1)} + F_A(I_{ni} - D_P\widetilde{Y}_{(ni-1)}) = I_{ni} - (I_{ni} - F_A)(I_{ni} - D_P\widetilde{Y}_{(ni-1)})$; then for all $F_A \in \mathcal{F}_{A(ni-1)}$, $M_{(ni-1)}$ is $\mathcal{R}_{\mathcal{U}}$-unimodular. If $m = 1$ or $m = (n_i - 1)$, for the left-coprime pair $(\widetilde{D}_P, \widetilde{N}_P F_A)$ the following Bezout-identity holds for all $F_A \in \mathcal{F}_{Am}$ ($\mathcal{F}_{Am}$ is either $\mathcal{F}_{A1}$ or $\mathcal{F}_{A(ni-1)}$):

$$\begin{bmatrix} Y_m + (I_{ni} - Y_m D_P)V_P F_A & (I_{ni} - Y_m D_P)U_P \\ -\widetilde{N}_P F_A & \widetilde{D}_P \end{bmatrix} \cdot$$

$$\begin{bmatrix} (F_A + D_P Y_m(I_{ni} - F_A))^{-1} D_P & -(I_{ni} - D_P Y_m)M_m^{-1}\widetilde{U}_P \\ N_P(I_{ni} - Y_m M_m^{-1}(I_{ni} - F_A))D_P) & \widetilde{V}_P + N_P Y_m M_m^{-1}\widetilde{U}_P \end{bmatrix}$$

$$= I_{ni+no} .$$

3.1. Theorem (*all controllers with k-sensor-integrity*): Consider $\mathcal{S}(F_S, P, C)$. If $F_S \in \mathcal{F}_{S1}$, let P have no 1-sensor-failure hidden $\mathcal{U}$-modes; let $\tilde{Y}_k$ be $\tilde{Y}_1$ and let $\widetilde{M}_k$ be $\widetilde{M}_1$. If $F_S \in \mathcal{F}_{S(no-1)}$, let P have no $(n_o - 1)$-sensor-failure hidden $\mathcal{U}$-modes; let $\tilde{Y}_k$ be $\tilde{Y}_{(no-1)}$ and let $\widetilde{M}_k$ be $\widetilde{M}_{(no-1)}$. Then the set $\mathrm{S}_{Sk}(P)$ of all controllers with k-sensor-integrity ($k = 1$ or $(n_o - 1)$) is:

$$\mathrm{S}_{Sk}(P) = \{ C = \widetilde{D}_C^{-1}\widetilde{N}_C = N_C D_C^{-1} \mid \widetilde{D}_C = V_P + U_P\widetilde{M}_k^{-1}\tilde{Y}_k\widetilde{N}_P$$
$$-\hat{Q}(I_{no} - \widetilde{D}_P(I_{no} - F_S)\widetilde{M}_k^{-1}\tilde{Y}_k)\widetilde{N}_P , \; \widetilde{N}_C = U_P\widetilde{M}_k^{-1}(I_{no} - \tilde{Y}_k\widetilde{D}_P)$$
$$+\hat{Q}\widetilde{D}_P(F_S + (I_{no} - F_S)\tilde{Y}_k\widetilde{D}_P)^{-1} , \; N_C = \widetilde{U}_P(I_{no} - \widetilde{D}_P\tilde{Y}_k) + D_P\hat{Q} ,$$
$$D_C = \tilde{Y}_k + F_S\widetilde{V}_P(I_{no} - \widetilde{D}_P\tilde{Y}_k) + F_S N_P\hat{Q} , \quad \hat{Q} \in \mathcal{R}_{\mathcal{U}}^{ni \times no} ,$$
$$\det(V_P + U_P\widetilde{M}_k^{-1}\tilde{Y}_k\widetilde{N}_P - \hat{Q}(I_{no} - \widetilde{D}_P(I_{no} - F_S)\widetilde{M}_k^{-1}\tilde{Y}_k)\widetilde{N}_P)$$
$$\sim \det(\tilde{Y}_k + F_S\widetilde{V}_P(I_{no} - \widetilde{D}_P\tilde{Y}_k) + F_S N_P\hat{Q}) , \in \mathcal{I} \} . \quad \square$$

If P is strictly proper, then for any $\hat{Q} \in \mathcal{R}_{\mathcal{U}}^{ni \times no}$, $\det(V_P + U_P\widetilde{M}_k^{-1}\tilde{Y}_k\widetilde{N}_P - \hat{Q}(I_{no} - \widetilde{D}_P(I_{no} - F_S)\widetilde{M}_k^{-1}\tilde{Y}_k)\widetilde{N}_P) \sim \det(\tilde{Y}_k + F_S\widetilde{V}_P(I_{no} - \widetilde{D}_P\tilde{Y}_k) + F_S N_P\hat{Q}) \in \mathcal{I}$.

3.2 Theorem (*all controllers with m-actuator-integrity*): Consider $\mathcal{S}(P, F_A, C)$. If $F_A \in \mathcal{F}_{A1}$, let P have no 1-actuator-failure hidden $\mathcal{U}$-modes; let Y_m be Y_1 and let M_m be M_1. If $F_A \in \mathcal{F}_{A(ni-1)}$, let P have no $(n_i - 1)$-actuator-failure hidden $\mathcal{U}$-modes; let Y_m be $\widetilde{Y}_{(ni-1)}$ and let M_m be $M_{(ni-1)}$. Then the set $\mathrm{S}_{Am}(P)$ of all controllers with m-actuator-integrity ($m = 1$ or $(n_i - 1)$) is:

$$\mathrm{S}_{Am}(P) = \{ C = N_C D_C^{-1} = \widetilde{D}_C^{-1}\widetilde{N}_C \mid$$
$$N_C = (I_{ni} - D_P Y_m)M_m^{-1}\widetilde{U}_P + (F_A + D_P Y_m(I_{ni} - F_A))^{-1} D_P\hat{Q} ,$$
$$D_C = \widetilde{V}_P + N_P Y_m M_m^{-1}\widetilde{U}_P - N_P(I_{ni} - Y_m M_m^{-1}(I_{ni} - F_A)D_P)\hat{Q} ,$$
$$\widetilde{D}_C = \tilde{Y}_k + (I_{no} - \tilde{Y}_k D_P)V_P F_A - \hat{Q}\widetilde{N}_P F_A ,$$
$$\widetilde{N}_C = (I_{no} - \tilde{Y}_k D_P)U_P + \hat{Q}\widetilde{D}_P , \; \hat{Q} \in \mathcal{R}_{\mathcal{U}}^{ni \times no} ,$$
$$\det(\widetilde{V}_P + N_P Y_m M_m^{-1}\widetilde{U}_P - N_P(I_{ni} - Y_m M_m^{-1}(I_{ni} - F_A)D_P))$$
$$\sim \det(\tilde{Y}_k + (I_{no} - \tilde{Y}_k D_P)V_P F_A - \hat{Q}\widetilde{N}_P F_A) \in \mathcal{I} \} . \quad \square$$

If P is strictly proper, then for any $\hat{Q} \in \mathcal{R}_{\mathcal{U}}^{ni \times no}$, $\det(\widetilde{V}_P + N_P Y_m M_m^{-1}\widetilde{U}_P - N_P(I_{ni} - Y_m M_m^{-1}(I_{ni} - F_A)D_P)) \sim \det(\tilde{Y}_k + (I_{no} - \tilde{Y}_k D_P)V_P F_A - \hat{Q}\widetilde{N}_P F_A) \in \mathcal{I}$.

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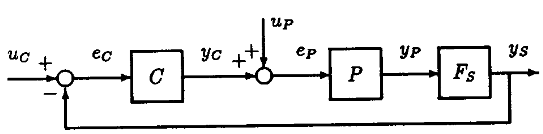

Figure 1: The system $\mathcal{S}(F_S, P, C)$

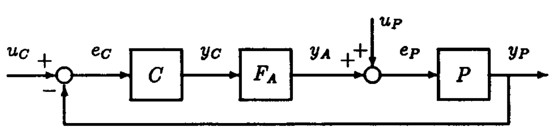

Figure 2: The system $\mathcal{S}(P, F_A, C)$