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Parametrization of All Decoupling Compensators and All Achievable Diagonal Maps for the Unity-feedback System

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ABSTRACT

This paper gives a parametrization of all stabilizing compensators which achieve decoupling in the unity-feedback system. It is assumed that the plant transfer-function matrix is full row-rank and does not have unstable poles coinciding with zeros.

1. INTRODUCTION

In the linear, time-invariant, (LTI) multi-input multi-output (MIMO) unity-feedback system, decoupling is achieved if the closed-loop transfer-function H_{pc} from the command-input to the plant-output is diagonalized by using a stabilizing compensator. In this paper we parametrize all decoupling compensators and all achievable diagonal, nonsingular maps when the full row-rank plant does not have coinciding undesirable poles and zeros.

Notation: $\mathcal{U}$ is a subset of $\mathbb{C}$ (the field of complex numbers) such that $\mathcal{U}$ is closed and symmetric about the real axis, $\pm\infty \in \mathcal{U}$ and $\mathbb{C} \setminus \mathcal{U}$ is nonempty. $\mathcal{R_U}$ is the ring of proper rational functions of s (with real coefficients) which have no poles in $\mathcal{U}$; $\mathbb{R}_{\rm p}(s)$ is the ring of proper rational functions, $\mathbb{R}_{\rm sp}(s)$ is the set of strictly proper rational functions and $\mathbb{R}(s)$ is the field of rational functions of s. $\mathcal{J}$ is the group of units of $\mathcal{R_U}$ and $\mathcal{I} = \mathcal{R_U} \setminus \mathbb{R}_{\rm sp}(s)$. The set of matrices whose entries are in $\mathcal{R_U}$ is denoted $\mathcal{M}(\mathcal{R_U})$. A matrix $M \in \mathcal{M}(\mathcal{R_U})$ is $\mathcal{R_U}$-unimodular iff $\det M \in \mathcal{J}$. The identity maps of size n_i and n_o are denoted I_{ni} and I_{no}; n_i and n_o denote the number of inputs and outpus.

2. SYSTEM DESRIPTION AND ANALYSIS

Consider the LTI, MIMO feedback system $S(P, C)$ in Figure 1, where $P : e_p \mapsto y_p$ and $C : e_c \mapsto y_c$ represent the plant and the compensator transfer-functions. The closed-loop input-output map of $S(P, C)$ is denoted $H_{yu} : \begin{bmatrix} u_c \\ u_p \end{bmatrix} \mapsto \begin{bmatrix} y_c \\ y_p \end{bmatrix}$.

2.1 Assumptions: **i)** the plant $P \in \mathbb{R}_{\rm p}(s)^{no\times ni}$; **ii)** the compensator $C \in \mathbb{R}_{\rm p}(s)^{ni\times no}$; **iii)** the system $S(P, C)$ is well-posed; equivalently, $H_{yu} \in \mathcal{M}(\mathbb{R}_{\rm p}(s))$; **iv)** P and C have no hidden-modes associated with eigenvalues in $\mathcal{U}$. □

The closed-loop input-output map is given by $H_{yu} = \begin{bmatrix} H_{cc} & -H_{cc}P \\ PH_{cc} & (I_{no} - PH_{cc})P \end{bmatrix}$, where $H_{cc} : u_c \mapsto y_c$ is $H_{cc} = C(I_{no} + PC)^{-1}$ and the map $H_{pc} : u_c \mapsto y_p$ that we wish to decouple is $H_{pc} = PH_{cc} = PC(I_{no} + PC)^{-1}$.

Let (N_P, D_P) be a right-coprime-fraction representation (**rcfr**) and $(\widetilde{D}_P, \widetilde{N}_P)$ be a left-coprime-fraction representation (**lcfr**) of $P \in \mathbb{R}_{\rm p}(s)^{no\times ni}$, where $N_P \in \mathcal{R_U}^{no\times ni}$, $D_P \in \mathcal{R_U}^{ni\times ni}$, $\widetilde{N}_P \in \mathcal{R_U}^{no\times ni}$, $\widetilde{D}_P \in \mathcal{R_U}^{no\times no}$, $P = N_P D_P^{-1} = \widetilde{D}_P^{-1}\widetilde{N}_P$; $\det D_P \in \mathcal{I}$, ($\det \widetilde{D}_P \in \mathcal{I}$) if and only if $P \in \mathcal{M}(\mathbb{R}_{\rm p}(s))$.

2.2. Definitions: **a)** $S(P, C)$ is said to be *$\mathcal{R_U}$-stable* iff $H_{yu} \in \mathcal{M}(\mathcal{R_U})$. **b)** $S(P, C)$ is said to be *decoupled* iff $S(P, C)$ is $\mathcal{R_U}$-stable and the map $H_{pc} : u_c \mapsto y_p$ is *diagonal* and *nonsingular*. **c)** C is said to be an *$\mathcal{R_U}$-stabilizing compensator* for P (or C $\mathcal{R_U}$-stabilizes P) iff $C \in \mathbb{R}_{\rm p}(s)^{ni\times no}$ and $S(P, C)$ is $\mathcal{R_U}$-stable. **d)** C is said to be a *decoupling compensator* for P (or C decouples P) iff C is an $\mathcal{R_U}$-stabilizing compensator and the map $H_{pc} : u_c \mapsto y_p$ is *diagonal* and *nonsingular.* **e)** The set $\mathcal{S}(P) := \{ C \mid C \in \mathbb{R}_{\rm p}(s)^{ni\times no}$ and $S(P, C)$ is $\mathcal{R_U}$-stable $\}$ is called the *set of all $\mathcal{R_U}$-stabilizing compensators* for P. **f)** The set $\mathcal{A}(P) := \{ H_{pc} : u_c \mapsto y_p \mid C \in \mathcal{S}(P) \}$ is called the *set of all achievable input-ouput maps* for $S(P, C)$ from the input u_c to the output y_p. **g)** The set $\mathcal{S_D}(P) := \{ C \mid C \in \mathcal{S}(P)$ and H_{pc} is diagonal and nonsingular $\}$ is called the *set of all decoupling compensators* for P. **h)** The set $\mathcal{A_D}(P) := \{ H_{pc} \mid C \in \mathcal{S_D}(P) \}$ is called the *set of all achievable decoupled input-ouput maps* H_{pc}. □

2.3. Smith-McMillan form of the Plant: Let $P \in \mathbb{R}_{\rm p}(s)^{ni\times no}$. Let ${\rm rank}P = n_o$. Then there exist $\mathcal{R_U}$-unimodular matrices $L \in \mathcal{R_U}^{no\times no}$ $R \in \mathcal{R_U}^{ni\times ni}$ such that $L^{-1}PR^{-1} = \Lambda\Psi^{-1} = \widetilde{\Psi}^{-1}\Lambda$; equivalently, $P = L\Lambda\Psi^{-1}R = L\widetilde{\Psi}^{-1}\Lambda R$, where $\widetilde{\Lambda} = {\rm diag}[\lambda_1 \ \dots \ \lambda_{no}]$, $\widetilde{\Psi} = {\rm diag}[\psi_1 \ \dots \ \psi_{no}]$, $\Lambda = \begin{bmatrix} \widetilde{\Lambda} & \vdots & 0_{no\times(ni-no)} \end{bmatrix}$, $\Psi = {\rm diag}[\widetilde{\Psi} \ \ I_{(ni-no)}]$. Here λ_j and $\psi_j \in \mathcal{R_U}$ are the invariant-factors of the numerator and denominator matrices, where, for $j = 1, \dots, n_o$, $\lambda_j, \psi_j \in \mathcal{R_U}$, the pair (λ_j, ψ_j) is coprime (equivalently, there exist $u_j \in \mathcal{R_U}$, $v_j \in \mathcal{R_U}$ such that $u_j \lambda_j + v_j \psi_j = 1$); λ_j divides λ_{j+1} and ψ_{j+1} divides ψ_j. The invariant-factors $\psi_j \in \mathcal{I}$ if and only if $P \in \mathcal{M}(\mathbb{R}_{\rm p}(s))$; rank $P = n_o$ implies that $\lambda_{no} \neq 0$. An **rcfr** of P is given by $(N_P, D_P) = (L\Lambda, R^{-1}\Psi)$ and **lcfr** of P is given by $(\widetilde{D}_P, \widetilde{N}_P) = (\widetilde{\Psi}L^{-1}, \Lambda R)$. Let $\widetilde{U} := {\rm diag}[u_1 \ \dots \ u_{no}]$, $\widetilde{V} := {\rm diag}[v_1 \ \dots \ v_{no}]$, $U := \begin{bmatrix} \widetilde{U} \\ 0_{(ni-no)\times no} \end{bmatrix}$, $V := {\rm diag}\begin{bmatrix} \widetilde{V} & I_{(ni-no)} \end{bmatrix}$.

2.4. All $\mathcal{R_U}$-stabilizing Compensators: The set $\mathcal{S}(P)$ of all $\mathcal{R_U}$-stabilizing compensators is given by $\mathcal{S}(P) =$ $\{R^{-1}(V - Q\Lambda)^{-1}(U + Q\widetilde{\Psi})L^{-1} \mid Q \in \mathcal{R_U}^{ni\times no}, \det(\widetilde{V} - \Lambda Q) \in \mathcal{I}\}$. Using $C \in \mathcal{S}(P)$ in the map $H_{pc} = PC(I_{no} + PC)^{-1}$, the set $\mathcal{A}(P)$ of all achievable maps is obtained as $\mathcal{A}(P) = \{L\Lambda(U + Q\widetilde{\Psi})L^{-1} = I_{no} - L(\widetilde{V} - \Lambda Q)\widetilde{\Psi}L^{-1} \mid Q \in \mathcal{R_U}^{ni\times no}, \det(\widetilde{V} - \Lambda Q) \in \mathcal{I}\}$. If P is strictly proper, then $\det(V - Q\Lambda) \in \mathcal{I}$ (equivalently, $\det(\widetilde{V} - \Lambda Q) \in \mathcal{I}$) for *all* $Q \in \mathcal{M}(\mathcal{R_U})$.

3. DECOUPLING

Let (N_P, D_P) be an **rcfr** and $(\widetilde{D}_P, \widetilde{N}_P)$ be an **lcfr** of $P \in \mathbb{R}_{\rm p}(s)^{no\times ni}$. Let ${\rm rank}P$ denote the normal rank of P. Note that rank P = rank N_P = rank $\widetilde{N}_P$.

3.1. Lemma: Let $P \in \mathbb{R}_{\rm p}(s)^{ni\times no}$. If the system $S(P, C)$ is decoupled, then rank $P = n_o \leq n_i$. □.

Now $p_o \in \mathcal{U}$ is a $\mathcal{U}$-pole of P if and only if $\psi_1(p_o) = 0$; $z_o \in \mathcal{U}$ is a $\mathcal{U}$-zero of P if and only if $\lambda_{no}(z_o) = 0$. The plant P has no $\mathcal{U}$-poles coinciding with $\mathcal{U}$-zeros if and only if (λ_{no}, ψ_1) is a coprime pair; equivalently, there exist $\hat{\alpha}$, $\hat{\beta} \in \mathcal{R_U}$ such that, for all $q \in \mathcal{R_U}$, $\alpha\lambda_{no} + \beta\psi_1 := (\hat{\alpha} + q\psi_1)\lambda_{no} + (\hat{\beta} - q\lambda_{no})\psi_1 = 1$. If $\lambda_{no} \in \mathbb{R}_{\rm sp}(s)$, then $\beta := (\hat{\beta} - q\lambda_{no}) \in \mathcal{I}$ for *all* $q \in \mathcal{R_U}$. If $\lambda_{no} \notin \mathbb{R}_{\rm sp}(s)$, then $\beta = (\hat{\beta} - q\lambda_{no}) \in \mathcal{I}$ for all $q \in \mathcal{R_U}$ such that $q(\infty) \neq \hat{\beta}(\infty)/\lambda_{no}(\infty)$.

Let $\widetilde{U}^\bullet := {\rm diag}[\alpha\lambda_{no}/\lambda_1 \ \ \alpha\lambda_{no}/\lambda_2 \dots \alpha\lambda_{no}/\lambda_{no-1} \ \ \alpha]$, $U^\bullet := \begin{bmatrix} \widetilde{U}^\bullet \\ 0_{(ni-no)\times no} \end{bmatrix}$, $\widetilde{V}^\bullet := {\rm diag}[\beta \ \ \beta\psi_1/\psi_2 \dots \beta\psi_1/\psi_{no-1} \ \ \beta\psi_1/\psi_{no}]$, $V^\bullet := {\rm diag}\begin{bmatrix} \widetilde{V}^\bullet & I_{(ni-no)} \end{bmatrix}$.

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Since for $j = 1, \ldots, n_o - 1$, λ_j divides λ_{j+1} and ψ_{j+1} divides ψ_j, and since $P \in \mathcal{M}(\mathbb{R}_p(s))$ implies that $\psi_j \in \mathcal{I}$, it is clear that $\lambda_{no}/\lambda_j \in \mathcal{R_U}$ and $\psi_1/\psi_j \in \mathcal{I}$. The matrices $\tilde{U}^*, U^*, \tilde{V}^*, V^* \in \mathcal{M}(\mathcal{R_U})$. If P has no $\mathcal{U}$-poles coinciding with $\mathcal{U}$-zeros, then $V^*\Psi + U^*\Lambda = I_{ni}$ and $\tilde{\Psi}\tilde{V}^* + \Lambda U^* = \tilde{\Psi}\tilde{V}^* + \tilde{\Lambda}\tilde{U}^* = I_{no}$.

3.2. Lemma: Let $P \in \mathbb{R}_p(s)^{ni \times no}$. Let rank$P = n_o$. Then there exists a decoupling compensator C for P if P has no $\mathcal{U}$-poles coinciding with $\mathcal{U}$-zeros. □

3.3. Parametrization of Decoupling Compensators: Let $P \in \mathbb{R}_p(s)^{ni \times no}$. Let rank$P = n_o$. Let P have no $\mathcal{U}$-poles coinciding with $\mathcal{U}$-zeros. Under these assumptions, it is possible to parametrize the class of *all* decoupling compensators for P and the class of all achievable decoupled maps H_{pc}. From the Smith-McMillan form, $N_P = L\Lambda = \left[\, L\tilde{\Lambda} \,\vdots\, 0_{no \times (ni-no)} \right]$, where $L\tilde{\Lambda} \in \mathcal{R_U}^{no \times no}$ is nonsingular. Let $\delta_j \in \mathcal{R_U}$ be a greatest-common-divisor (**gcd**) of the entries in the j-th row of $L\tilde{\Lambda}$. Let $\Delta := \mathrm{diag}[\, \delta_1 \ \ldots \ \delta_{no} \,]$. Since $\delta_j \neq 0$, the square matrix $\Delta \in \mathcal{R_U}^{no \times no}$ is nonsingular. Define $\hat{N}$ as $L\tilde{\Lambda} = \Delta \hat{N}$, where $\hat{N} := \Delta^{-1} L\tilde{\Lambda} \in \mathcal{R_U}^{no \times no}$ is nonsingular since $L\tilde{\Lambda}$ and Δ are both nonsingular; therefore $\hat{N} \in \mathcal{R_U}^{no \times no}$ has an inverse, $\hat{N}^{-1}$. Let n_{ij}/d_{ij} denote the ij-th entry of $\hat{N}^{-1}$; then $\hat{N}^{-1} \in \mathbb{R}(s)^{no \times no}$, where the pair (n_{ij}, d_{ij}) is coprime, $n_{ij} \in \mathcal{R_U}$, $d_{ij} \in \mathcal{R_U}$, $d_{ij} \neq 0$ (d_{ij} need not be in $\mathcal{I}$).

Let $\tilde{\delta}_j \in \mathcal{R_U}$ be a least-common-multiple (**lcm**) of $(d_{1j}, d_{2j}, \ldots, d_{noj})$; equivalently, $\tilde{\delta}_j$ is an **lcm** of all denominators in the j-th column of $\hat{N}^{-1}$; for each j, $\tilde{\delta}_j \neq 0$ since $d_{ij} \neq 0$. Let $\widetilde{\Delta} := \mathrm{diag}[\, \tilde{\delta}_1 \ \ldots \ \tilde{\delta}_{no}]$. Since $\tilde{\delta}_j \neq 0$, the square matrix $\widetilde{\Delta} \in \mathcal{R_U}^{no \times no}$ is nonsingular. Note that $\hat{N}^{-1}\widetilde{\Delta} \in \mathcal{R_U}^{no \times no}$.

Let $\theta_j \in \mathcal{R_U}$ be a **gcd** of the entries in the j-th column of $\widetilde{D}_P = \tilde{\Psi}L^{-1}$. Let $\Theta := \mathrm{diag}[\, \theta_1 \ \ldots \ \theta_{no} \,]$. Since $\theta_j \neq 0$, the square matrix $\Theta \in \mathcal{R_U}^{no \times no}$ is nonsingular. Let $\widetilde{D}_P = \hat{D}\,\Theta$, where $\hat{D} := \widetilde{D}_P\,\Theta^{-1} \in \mathcal{R_U}^{no \times no}$. The matrix $\hat{D}$ is nonsingular since $\widetilde{D}_P$ and Θ are both nonsingular; in fact, det $\hat{D} \in \mathcal{I}$ since $P \in \mathcal{M}(\mathbb{R}_p(s))$ by assumption. Consequently, $\hat{D} \in \mathcal{R_U}^{no \times no}$ has an inverse, $\hat{D}^{-1}$. Let x_{ij}/y_{ij} denote the ij-th entry of $\hat{D}^{-1}$; then $\hat{D}^{-1} \in \mathbb{R}_p(s)^{no \times no}$, where the pair (x_{ij}, y_{ij}) is coprime, $x_{ij} \in \mathcal{R_U}$, $y_{ij} \in \mathcal{R_U}$ ($y_{ij} \in \mathcal{I}$ since y_{ij} is a factor of det $\hat{D} \in \mathcal{I}$).

Let $\tilde{\theta}_i \in \mathcal{R_U}$ be a **lcm** of $(y_{i1}, \ldots, y_{ino})$; equivalently, $\tilde{\theta}_i$ is an **lcm** of all denominators in the i-th row of $\hat{D}^{-1}$, where $\tilde{\theta}_i \in \mathcal{I}$ since $y_{ij} \in \mathcal{I}$. Let $\tilde{\Theta} := \mathrm{diag}[\, \tilde{\theta}_1 \ \ldots \ \tilde{\theta}_{no} \,]$. Since $\tilde{\theta}_i \in \mathcal{I}$, the square matrix $\tilde{\Theta} \in \mathcal{R_U}^{no \times no}$ is nonsingular. Note that $\tilde{\Theta}\,\hat{D}^{-1} \in \mathcal{R_U}^{no \times no}$.

3.4. Theorem: Let $P \in \mathbb{R}_p(s)^{ni \times no}$. Let rank$P = n_o$. Let P have no $\mathcal{U}$-poles coinciding with $\mathcal{U}$-zeros. Then

i) the set $\mathcal{A_D}(P)$ of all decoupled input-output maps H_{pc} is:

$$\mathcal{A_D}(P) = \{\, \alpha\lambda_{no} I_{no} + \Delta\,\widetilde{\Delta}\,Q_D\,\tilde{\Theta}\,\Theta = (1-\beta\psi_1) I_{no} + \Delta\,\widetilde{\Delta}\,Q_D\,\tilde{\Theta}\,\Theta$$

$$\mid\ Q_D = \mathrm{diag}[\, q_1 \ \ldots \ q_{no} \,],\ \text{for}\, j = 1, \ldots, n_o,$$

$$q_j \in \mathcal{R_U}\ ,\ q_j \neq \frac{\beta\psi_1 - 1}{\delta_j\,\tilde{\delta}_j\,\tilde{\theta}_j\,\theta_j}\ ,\ q_j(\infty) \neq \frac{\beta\psi_1}{\delta_j\,\tilde{\delta}_j\,\tilde{\theta}_j\,\theta_j}(\infty)\ \};$$

ii) the set $\mathcal{S_D}(P)$ of all decoupling compensators is: $\mathcal{S_D}(P) =$

$$\{ R^{-1} \begin{bmatrix} \tilde{U}^* + \tilde{\Psi}\,\hat{N}^{-1}\widetilde{\Delta}\,Q_D\,\tilde{\Theta}\,\hat{D}^{-1} \\ Q_A \end{bmatrix} (L\tilde{V}^* - \Delta\,\widetilde{\Delta}\,Q_D\,\tilde{\Theta}\,\hat{D}^{-1})^{-1}$$

$$\mid\ Q_A \in \mathcal{R_U}^{(ni-no) \times no},\ Q_D = \mathrm{diag}[\, q_1 \ \ldots \ q_{no} \,],\ \text{for}\, j = 1, \ldots, n_o,$$

$$q_j \in \mathcal{R_U}\ ,\ q_j \neq \frac{\beta\psi_1 - 1}{\delta_j\,\tilde{\delta}_j\,\tilde{\theta}_j\,\theta_j}\ ,\ q_j(\infty) \neq \frac{\beta\psi_1}{\delta_j\,\tilde{\delta}_j\,\tilde{\theta}_j\,\theta_j}(\infty)\ \}.$$

3.5. Comment: **i)** For $j = 1, \ldots, n_o$, the condition $q_j \neq (\beta\psi_1 - 1)/\delta_j\,\tilde{\delta}_j\,\tilde{\theta}_j\,\theta_j$ on $q_j \in \mathcal{R_U}$ guarantees that the achieved decoupled input-output maps H_{pc} are nonsingular, where $H_{pc} = \alpha\lambda_{no} I_{no} + \Delta\,\widetilde{\Delta}\,Q_D\,\tilde{\Theta}\,\Theta = (1-\beta\psi_1) I_{no} + \Delta\,\widetilde{\Delta}\,Q_D\,\tilde{\Theta}\,\Theta$. If $(\theta_j\,\tilde{\theta}_j)$ is coprime with ψ_1, then this condition is satisfied for *any* $q_j \in \mathcal{R_U}$. **ii)** For $j = 1, \ldots, n_o$, the condition $q_j(\infty) \neq \beta\psi_1(\infty)/\delta_j\,\tilde{\delta}_j\,\tilde{\theta}_j\,\theta_j(\infty)$ on $q_j \in \mathcal{R_U}$ guarantees that the decoupling compensators are proper. If the plant is strictly proper, then this condition is satisfied for *any* $q_j \in \mathcal{R_U}$. **iii)** If $P \in \mathcal{R_U}^{no \times ni}$ then without loss of generality, $\tilde{\Psi} = I_{no}$ and $\Psi = I_{ni}$. Since $\psi_1 = 1$, one choice for $\hat{\alpha}$ is 0, $\hat{\beta}$ is 1 and $\tilde{U}^* = 0$ and $\tilde{V}^* = I_{no}$. In this case, $\Theta = I_{no} = \tilde{\Theta}$ and $\hat{D} = L^{-1}$. Therefore when $P \in \mathcal{M}(\mathcal{R_U})$, the parametrizations $\mathcal{S_D}(P)$ and $\mathcal{A_D}(P)$ become: $\mathcal{A_D}(P) = \{\, \Delta\,\widetilde{\Delta}\,Q_D \mid Q_D = \mathrm{diag}[\, q_1 \ \ldots \ q_{no} \,]$, for $j = 1, \ldots, n_o$, $q_j \in \mathcal{R_U} \setminus 0$, $q_j(\infty) \neq 1/\delta_j\,\tilde{\delta}_j(\infty)$ $\}$; $\mathcal{S_D}(P) = \{ R^{-1} \begin{bmatrix} \hat{N}^{-1}\widetilde{\Delta}\,Q_D L \\ Q_A \end{bmatrix} (I_{no} - L^{-1}\Delta\,\widetilde{\Delta}\,Q_D L)^{-1} L^{-1}$ $\mid Q_A \in \mathcal{R_U}^{(ni-no) \times no}$, $Q_D = \mathrm{diag}[\, q_1 \ \ldots \ q_{no} \,]$, for $j = 1, \ldots, n_o$, $q_j \in \mathcal{R_U} \setminus 0$, $q_j(\infty) \neq 1/\delta_j\,\tilde{\delta}_j(\infty)$ $\}$.

4. CONCLUSIONS

For LTI, MIMO plants which have no undesirable hidden-modes, full row-rank transfer-function matrices and no undesirable poles coinciding with zeros, we parametrized the class of all compensators such that the unity-feedback system is (internally) stable and the closed-loop transfer-function from the command-input to the plant-output is diagonal and nonsingular. If the plant has undesirable poles coinciding with zeros, then this class of compensators cannot be used; however, any full row-rank plant which has no undesirable hidden-modes can be decoupled using two-parameter compensation [2, 3].

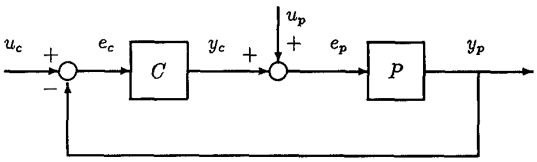

Figure 1: The system $S(P, C)$

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