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STABILITY UNDER SENSOR OR ACTUATOR FAILURES

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ABSTRACT

We propose compensators that achieve closed-loop stability in the presence of sensor or actuator failures.

1. INTRODUCTION

We consider sensor or actuator failures in linear, time-invariant multiinput-multioutput feedback systems. We propose compensators that achieve stability under arbitrary sensor failures or actuator failures. We find the class of all compensators that achieve stability under sensor failures and diagonalize the map from the external-input to the plant-output.

Notation: Let $\mathcal{U}$ be a subset of the complex-plane $\mathbb{C}$, where $\mathcal{U}$ contains the closed right-half-plane, $\mathcal{U}$ is closed closed and symmetric about the real axis and $\mathbb{C} \setminus \mathcal{U}$ is nonempty. Let $R_{\mathcal{U}}(s)$ denote the ring of proper scalar rational functions in s (with real coefficients) which have no poles in $\mathcal{U}$. Let $\mathcal{J}$ denote the group of units of $R_{\mathcal{U}}(s)$; i.e., $f \in \mathcal{J}$ implies that f is a proper rational function which has neither poles nor zeros in $\mathcal{U} \cup \{\infty\}$. The ring of proper rational functions in s with real coefficients is denoted by $\mathbb{R}_p(s)$. The set of matrices whose entries are in $R_{\mathcal{U}}(s)$ is denoted by $\mathfrak{M}(R_{\mathcal{U}}(s))$. A matrix whose entries are in $R_{\mathcal{U}}(s)$ is called an $R_{\mathcal{U}}$–stable matrix. A square nonsingular $R_{\mathcal{U}}$–stable matrix is called $R_{\mathcal{U}}$–unimodular iff its inverse is $R_{\mathcal{U}}$–stable; equivalently, $A \in \mathfrak{M}(R_{\mathcal{U}}(s))$ is $R_{\mathcal{U}}$–unimodular iff $\det A \in \mathcal{J}$.

2. MAIN RESULTS

Consider the linear, time-invariant feedback systems $S(F_s, P, C)$ and $S(P, F_a, C)$ shown in Figure 1 and Figure 2, respectively. In these systems, P and C represent the plant and the compensator, respectively; F_s and F_a represent the sensor and the actuator connections, respectively, where

$F_s \in \mathcal{F}_s := \{ F_s = diag\,[f_1 \cdots f_{n_o}] \mid f_j = 1 \text{ or } 0 \}$,

$F_a \in \mathcal{F}_a := \{ F_a = diag\,[f_1 \cdots f_{n_i}] \mid f_j = 1 \text{ or } 0 \}$.

The j-th diagonal entry f_j of F_s (F_a) becomes zero if the j-th sensor (actuator) fails; otherwise, $f_j = 1$. The external inputs are denoted by $\bar{u} := \begin{bmatrix} u \\ u' \end{bmatrix}$, the plant and the compensator outputs are denoted by $\bar{y} := \begin{bmatrix} y \\ y' \end{bmatrix}$. The closed-loop input-output (I/O) map of $S(F_s, P, C)$ is denoted by $H_{\bar{y}\bar{u}} : \bar{u} \mapsto \bar{y}$; the closed-loop I/O map of $S(P, F_a, C)$ is denoted by $\tilde{H}_{\bar{y}\bar{u}} : \bar{u} \mapsto \bar{y}$.

2.1. Assumptions: i) The plant $P \in \mathbb{R}_p(s)^{n_o \times n_i}$. ii) The compensator $C \in \mathbb{R}_p(s)^{n_i \times n_o}$. iii) The system $S(F_s, P, C)$ is well-posed; equivalently, the closed-loop I/O map $H_{\bar{y}\bar{u}} : \bar{u} \mapsto \bar{y}$ is in $\mathfrak{M}(\mathbb{R}_p(s))$. iv) The system $S(P, F_a, C)$ is well-posed; equivalently, the closed-loop I/O map $\tilde{H}_{\bar{y}\bar{u}} : \bar{u} \mapsto \bar{y}$ is in $\mathfrak{M}(\mathbb{R}_p(s))$.

2.2. Closed-loop input-output maps: Let Assumptions 2.1 hold; then the I/O maps $H_{\bar{y}\bar{u}}$ and $\tilde{H}_{\bar{y}\bar{u}}$ are in $\mathfrak{M}(\mathbb{R}_p(s))$, where

$$H_{\bar{y}\bar{u}} = \begin{bmatrix} (I_{n_o} - PC(I_{n_o} + F_sPC)^{-1}F_s)P & PC(I_{n_o} + F_sPC)^{-1} \\ -C(I_{n_o} + F_sPC)^{-1}F_sP & C(I_{n_o} + F_sPC)^{-1} \end{bmatrix}$$

and
$$\tilde{H}_{\bar{y}\bar{u}} = \begin{bmatrix} PF_a(I_{n_i} + CPF_a)^{-1} & PF_a(I_{n_i} + CPF_a)^{-1}C \\ -(I_{n_i} + CPF_a)^{-1}CPF_a & (I_{n_i} + CPF_a)^{-1}C \end{bmatrix}.$$

2.3. Definition ($R_{\mathcal{U}}$–stability): a) The system $S(F_s, P, C)$ is said to be $R_{\mathcal{U}}$*–stable* iff $H_{\bar{y}\bar{u}} \in \mathfrak{M}(R_{\mathcal{U}}(s))$. b) The system $S(P, F_a, C)$ is said to be $R_{\mathcal{U}}$*–stable* iff $\tilde{H}_{\bar{y}\bar{u}} \in \mathfrak{M}(R_{\mathcal{U}}(s))$.

2.4. Theorem (conditions for $R_{\mathcal{U}}$–stability under failure)
Let Assumptions 2.1 hold; then
a) the system $S(F_s, P, C)$ is $R_{\mathcal{U}}$–stable for all $F_s \in \mathcal{F}_s$ if and only if i) $P \in \mathfrak{M}(R_{\mathcal{U}}(s))$ and ii) $C \in \mathfrak{M}(R_{\mathcal{U}}(s))$ and iii) $(I_{n_o} + F_sPC)^{-1} \in \mathfrak{M}(R_{\mathcal{U}}(s))$ for all $F_s \in \mathcal{F}_s$;
b) the system $S(P, F_a, C)$ is $R_{\mathcal{U}}$–stable for all $F_a \in \mathcal{F}_a$ if and only if i) $P \in \mathfrak{M}(R_{\mathcal{U}}(s))$ and ii) $C \in \mathfrak{M}(R_{\mathcal{U}}(s))$ and iii) $(I_{n_i} + CPF_a)^{-1} \in \mathfrak{M}(R_{\mathcal{U}}(s))$ for all $F_a \in \mathcal{F}_a$.

2.5. Corollary: Let Assumptions 2.1 hold; let $P \in R_{\mathcal{U}}(s)^{n_o \times n_i}$ and let $C \in R_{\mathcal{U}}(s)^{n_i \times n_o}$; then
a) the system $S(F_s, P, C)$ is $R_{\mathcal{U}}$–stable for all $F_s \in \mathcal{F}_s$ if and only if $(I_{n_o} + F_sPC)$ is $R_{\mathcal{U}}$–unimodular for all $F_s \in \mathcal{F}_s$;
b) the system $S(P, F_a, C)$ is $R_{\mathcal{U}}$–stable for all $F_a \in \mathcal{F}_a$ if and only if $(I_{n_i} + CPF_a)$ is $R_{\mathcal{U}}$–unimodular for all $F_a \in \mathcal{F}_a$.

2.6. Definition (compensators that achieve stabilization under failure): a) The compensator C in the system $S(F_s, P, C)$ is said to *achieve stabilization under sensor failures* iff $C \in R_{\mathcal{U}}(s)^{n_i \times n_o}$ and $S(F_s, P, C)$ is $R_{\mathcal{U}}$–stable for all $F_s \in \mathcal{F}_s$. **b)** The compensator C in the system $S(P, F_a, C)$ is said to *achieve stabilization under actuator failures* iff $C \in R_{\mathcal{U}}(s)^{n_i \times n_o}$ and $S(P, F_a, C)$ is $R_{\mathcal{U}}$–stable for all $F_a \in \mathcal{F}_a$. **c)** In the case that $n_o \le n_i$, the compensator C in the system $S(F_s, P, C)$ is said to *achieve decoupling and stabilization under sensor failures* iff $C \in R_{\mathcal{U}}(s)^{n_i \times n_o}$ achieves stabilization under sensor failures and the closed-loop map $H_{yu'} : u' \mapsto y$ is diagonal and nonsingular for all $F_s \in \mathcal{F}_s$.

2.7. Remark: Let Assumptions 2.1 hold; let $P \in R_{\mathcal{U}}(s)^{n_o \times n_i}$; then by Corollary 2.5, the set of all compensators that achieve stabilization under sensor failures in $S(F_s, P, C)$ is:

$$\{\, C \in R_{\mathcal{U}}(s)^{n_i \times n_o} \mid \det(I_{n_o} + F_sPC) \in \mathcal{J} \text{ for all } F_s \in \mathcal{F}_s \,\};$$

the set of all compensators that achieve stabilization under actuator failures in $S(P, F_a, C)$ is:

$$\{\, C \in R_{\mathcal{U}}(s)^{n_i \times n_o} \mid \det(I_{n_i} + CPF_a) \in \mathcal{J} \text{ for all } F_a \in \mathcal{F}_a \,\}.$$

2.8. Proposition (compensators that achieve stailization under sensor failures): Consider the system $S(F_s, P, C)$. Let the $n_o \times n_i$ plant P be $R_{\mathcal{U}}$–stable.
a) If $n_o \le n_i$, then there exists an $R_{\mathcal{U}}$–unimodular matrix $R_s \in R_{\mathcal{U}}(s)^{n_i \times n_i}$ such that $P R_s = \begin{bmatrix} P_s & 0 \end{bmatrix}$, where $P_s \in R_{\mathcal{U}}(s)^{n_o \times n_o}$ is lower-triangular; for $j = 1, \cdots, n_o$, let $p_{sj}(s) \in R_{\mathcal{U}}(s)$ denote the j-th diagonal entry of the lower-triangular matrix P_s. Let $C_s \in R_{\mathcal{U}}(s)^{n_o \times n_o}$ be any lower-triangular $R_{\mathcal{U}}$–stable matrix whose j-th diagonal entry is denoted by $k_{sj}\, c_{sj}(s)$, where, for $j = 1, \cdots, n_o$, $c_{sj}(s) \in R_{\mathcal{U}}(s)$ is arbitrary and $k_{sj} \in \mathbb{R}$ is chosen so that

$$1 + k_{sj}\, p_{sj}(s)\, c_{sj}(s) \in \mathcal{J}. \qquad (1)$$

Let $C' \in R_{\mathcal{U}}(s)^{(n_i - n_o) \times n_i}$ be arbitrary. Under these conditions,

$$C = R_s \begin{bmatrix} C_s \\ C' \end{bmatrix} \in R_{\mathcal{U}}(s)^{n_i \times n_o} \qquad (2)$$

is a compensator that achieves stabilization under sensor failures in the system $S(F_s, P, C)$.
b) If $n_o > n_i$, then there exists an $R_{\mathcal{U}}$–unimodular matrix

$\tilde{R}_s \in \mathrm{R_U}(s)^{n_i \times n_i}$ such that $P\,\tilde{R}_s = \begin{bmatrix} \tilde{P}_s \\ \tilde{P}' \end{bmatrix}$, where $\tilde{P}_s \in \mathrm{R_U}(s)^{n_i \times n_i}$ is lower-triangular and $\tilde{P}' \in \mathrm{R_U}(s)^{(n_o - n_i) \times n_i}$; for $j = 1, \cdots, n_i$, let $\tilde{p}_{sj}(s) \in \mathrm{R_U}(s)$ denote the j-th diagonal entry of $\tilde{P}_s$. Let $\tilde{C}_s \in \mathrm{R_U}(s)^{n_i \times n_i}$ be any lower-triangular $\mathrm{R_U}$-stable matrix whose j-th diagonal entry is denoted by $\tilde{k}_{sj}\,\tilde{c}_{sj}(s)$, where, for $j = 1, \cdots, n_i$, $\tilde{c}_{sj}(s) \in \mathrm{R_U}(s)$ is arbitrary and $\tilde{k}_{sj} \in \mathbb{R}$ is chosen so that

$$1 + \tilde{k}_{sj}\,\tilde{p}_{sj}(s)\,\tilde{c}_{sj}(s) \in \mathbf{J}\ . \tag{3}$$

Under these conditions,

$$C = \tilde{R}_s \begin{bmatrix} \tilde{C}_s & 0 \end{bmatrix} \in \mathrm{R_U}(s)^{n_i \times n_o} \tag{4}$$

is a compensator that achieves stabilization under sensor failures in the system $\mathbf{S}(F_s, P, C)$.

2.9. Proposition (compensators that achieve stabilization under actuator failures): Consider the system $\mathbf{S}(P, F_a, C)$. Let the $n_o \times n_i$ plant P be $\mathrm{R_U}$-stable.

a) If $n_o \le n_i$, then there exists an $\mathrm{R_U}$-unimodular matrix $L_a \in \mathrm{R_U}(s)^{n_o \times n_o}$ such that $L_a P = \begin{bmatrix} P_a & P' \end{bmatrix}$, where $P_a \in \mathrm{R_U}(s)^{n_o \times n_o}$ is lower-triangular and $P' \in \mathrm{R_U}(s)^{n_o \times (n_i - n_o)}$; for $j = 1, \cdots, n_o$, let $p_{aj}(s) \in \mathrm{R_U}(s)$ denote the j-th diagonal entry of P_a. Let $C_a \in \mathrm{R_U}(s)^{n_o \times n_o}$ be any lower-triangular $\mathrm{R_U}$-stable matrix whose j-th diagonal entry is denoted by $k_{aj}\,c_{aj}(s)$, where, for $j = 1, \cdots, n_o$, $c_{aj}(s) \in \mathrm{R_U}(s)$ is arbitrary and $k_{aj} \in \mathbb{R}$ is chosen so that

$$1 + k_{aj}\,p_{aj}(s)\,c_{aj}(s) \in \mathbf{J}\ . \tag{5}$$

Under these conditions,

$$C = \begin{bmatrix} C_a \\ 0 \end{bmatrix} L_a \in \mathrm{R_U}(s)^{n_i \times n_o} \tag{6}$$

is a compensator that achieves stabilization under actuator failures in the system $\mathbf{S}(P, F_a, C)$.

b) If $n_o > n_i$, then there exists an $\mathrm{R_U}$-unimodular matrix $\tilde{L}_a \in \mathrm{R_U}(s)^{n_o \times n_o}$ such that $\tilde{L}_a P = \begin{bmatrix} \tilde{P}_a \\ 0 \end{bmatrix}$, where $\tilde{P}_a \in \mathrm{R_U}(s)^{n_i \times n_i}$ is lower-triangular; for $j = 1, \cdots, n_i$, let $\tilde{p}_{aj}(s) \in \mathrm{R_U}(s)$ denote the j-th diagonal entry of $\tilde{P}_a$. Let $\tilde{C}_a \in \mathrm{R_U}(s)^{n_i \times n_i}$ be any lower-triangular $\mathrm{R_U}$-stable matrix whose j-th diagonal entry is denoted by $\tilde{k}_{aj}\,\tilde{c}_{aj}(s)$, where, for $j = 1, \cdots, n_i$, $\tilde{c}_{aj}(s) \in \mathrm{R_U}(s)$ is arbitrary and $\tilde{k}_{aj} \in \mathbb{R}$ is chosen so that

$$1 + \tilde{k}_{aj}\,\tilde{p}_{aj}(s)\,\tilde{c}_{aj}(s) \in \mathbf{J}\ . \tag{7}$$

Let $\tilde{C}' \in \mathrm{R_U}(s)^{n_i \times (n_o - n_i)}$ be arbitrary. Under these conditions,

$$C = \begin{bmatrix} \tilde{C}_a & \tilde{C}' \end{bmatrix} \tilde{L}_a \in \mathrm{R_U}(s)^{n_i \times n_o} \tag{8}$$

is a compensator that achieves stabilization under actuator failures in the system $\mathbf{S}(P, F_a, C)$.

2.10. Proposition (compensators that achieve decoupling and stabilization under sensor failures): Consider the system $\mathbf{S}(F_s, P, C)$. Let $P \in \mathrm{R_U}(s)^{n_o \times n_i}$, where $n_o \le n_i$ and P has normal rank n_o. For $k = 1, \cdots, n_o$, let $\Delta_{Lk} \in \mathrm{R_U}(s)$ be a greatest-common-divisor over $\mathrm{R_U}(s)$ of the entries in the k-th row of P. Let $\Delta_L := diag \begin{bmatrix} \Delta_{L1} \cdots \Delta_{Ln_o} \end{bmatrix}$ and $P =: \Delta_L\,\tilde{P}$, where the matrix $\tilde{P}$ has normal rank n_o since P is full row-rank and $\det\Delta_L \not\equiv 0$. Let $\tilde{P}^r$ be such that $P\,\tilde{P}^r = I_{n_o}$; note that $\tilde{P}^r$ is not necessarily $\mathrm{R_U}$-stable or even proper. Write the ij-th entry of $\tilde{P}^r$ as m_{ij} / d_{ij}, where $m_{ij}, d_{ij} \in \mathrm{R_U}(s)$, $d_{ij} \ne 0$ and (m_{ij}, d_{ij}) is a coprime pair over $\mathrm{R_U}(s)$. For $j = 1, \cdots, n_o$, let $\Delta_{Rj} \in \mathrm{R_U}(s)$ be a least-common-multiple of the denominators of the entries in the j-th column of $\tilde{P}^r$. Let $\Delta_R := diag \begin{bmatrix} \Delta_{R1} \cdots \Delta_{Rn_o} \end{bmatrix}$. Under these conditions, C is a compensator that achieves decoupling and stabilization under sensor failures in the system $\mathbf{S}(F_s, P, C)$ if and only if

$$C = \tilde{P}^r\,\Delta_R\,K_d\,Q_d \tag{9}$$

for some $Q_d = diag\,[\,q_1 \cdots q_{n_o}\,] \in \mathfrak{m}(\mathrm{R_U}(s))$ and for some $K_d = diag\,[\,k_1 \cdots k_{n_o}\,] \in \mathfrak{m}(\mathbb{R})$, where, for $j = 1, \cdots, n_o$,

$$1 + k_j\,\Delta_{Lj}\,\Delta_{Rj}\,q_j \in \mathbf{J}\ . \tag{10}$$

2.11. Comments: a) In the system $\mathbf{S}(F_s, P, C)$, suppose that $P \in \mathrm{R_U}(s)^{n_o \times n_i}$, where $n_o \le n_i$. If the compensator C is chosen as in (2), then the closed-loop map $H_{yu'}$ from the external-input u' to the plant-output y becomes lower-triangular, where $H_{yu'} = P\,C\,(I_{n_o} + F_s P\,C\,)^{-1} = P_s C_s (I_{n_o} + F_s P_s C_s\,)^{-1}$. For $j = 1, \cdots, n_o$, the j-th diagonal entry of $H_{yu'}$ is $k_{sj}\,p_{sj}(s)\,c_{sj}(s)\,(\,1 + f_j\,k_{sj}\,p_{sj}(s)\,c_{sj}(s)\,)^{-1}$, where, f_j is 1 or 0.

In the case that $n_o > n_i$, suppose that the compensator C is chosen as in (4); then the last $n_o - n_i$ columns of $H_{yu'}$ become zero and the upper-left $n_i \times n_i$ submatrix of $H_{yu'}$ becomes lower-triangular, where for $j = 1, \cdots, n_i$, the j-th diagonal entry of $H_{yu'}$ is $\tilde{k}_{sj}\,\tilde{p}_{sj}(s)\,\tilde{c}_{sj}(s)\,(\,1 + f_j\,\tilde{k}_{sj}\,\tilde{p}_{sj}(s)\,\tilde{c}_{sj}(s)\,)^{-1}$.

b) Under the conditions of Proposition 2.10, $H_{yu'}$ is an achievable diagonal, nonsingular, external-input to plant-output map for all $F_s \in \mathbf{F_S}$ if and only if $H_{yu'}$ is of the form

$$H_{yu'} = \Delta_L\,\Delta_R\,K_d\,Q_d\,(\,I_{n_o} + F_s \Delta_L\,\Delta_R\,K_d\,Q_d\,)^{-1} \tag{11}$$

for some $Q_d = diag\,[\,q_1 \cdots q_{n_o}\,] \in \mathfrak{m}(\mathrm{R_U}(s))$ and for some $K_d = diag\,[\,k_1 \cdots k_{n_o}\,] \in \mathfrak{m}(\mathbb{R})$, where condition (10) is satisfied for $j = 1, \cdots, n_o$.

c) For any proper stable rational function $p_{sj}(s)\,c_{sj}(s) \in \mathrm{R_U}(s)$, condition (1) can be satisfied by choosing a sufficiently small $k_{sj} \in \mathbb{R}$. After the compensator parameters $c_{sj}(s) \in \mathrm{R_U}(s)$ are chosen arbitrarily, choosing $k_{sj} \in \mathbb{R}$ to satisfy condition (2) is a standard singleinput-singleoutput problem (root-locus, Nyquist, etc.) Similar comments apply to conditions (3), (5), (7) and (10). □

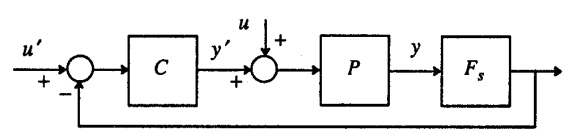

Figure 1. The feedback system $\mathbf{S}(F_s, P, C)$.

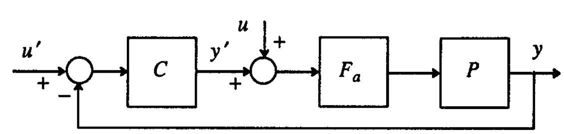

Figure 2. The feedback system $\mathbf{S}(P, F_a, C)$.

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