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Controller synthesis for single-area and multi-area power systems with communication delays

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***Abstract*— Finite-dimensional controller synthesis methods are developed for load-frequency control of power systems subject to communication time delays. The controllers proposed for single service area systems are simple to implement. The systematic controller synthesis methods are extended to multi-area systems under a decentralized control structure. The proposed synthesis procedures give low-order stabilizing controllers that also achieve integral-action so that constant reference inputs are tracked asymptotically with zero steady-state error. The freedom in the controller parameters can be used to improve system performance.**

I. Introduction

The objective of power system control is to maintain stability, performance, and system integrity after the occurrence of failures or system disturbances, such as short circuits and loss of generation or load. An important function of automatic generation control (AGC) systems is the control of frequency and power generation, which is called load-frequency control (LFC). The stability of a power system (for a given initial operating condition) is defined as its ability to regain a state of operating equilibrium after a physical disturbance occurs, with most system variables bounded so that practically the entire system remains intact [12]. The presence of time delays arising during transmission becomes a very important consideration for power systems as in any large-scale system. Load-frequency control is a topic that has been studied extensively, and recent advances can be found in e.g., [1], [13], [17], [18], [19], [10] and the references therein. Although the problem is well-known, very few detailed studies that propose systematic methods of finite-dimensional controller synthesis for single-area or decentralized multi-area systems subject to delays exist, and almost none were developed in a transfer-function setting.

A power system is a highly nonlinear and large-scale multi-input multi-output (MIMO) dynamical system. However, for the purpose of frequency control synthesis and analysis in the presence of load disturbances, a simple low-order linearized model can be used. In a modern large-scale power system, the generation, transmission and distribution of electric energy requires rigorous robust and optimal control methodologies, infrastructure communication and information technology services in designing control units and supervisory control and data acquisition system (SCADA) centers [1]. Due to their practical advantages, decentralized structures are used for large-scale multi-area frequency control synthesis since it is difficult to implement centralized control design in a large-scale power system environment. Low-order controller designs that provide integral-action for steady-state accuracy are preferred for simple implementation. In open communication systems, time delays can arise during transmission from the control center to the individual units and also from telemetry delays. In this work, finite-dimensional stabilizing controller synthesis methods are developed for single-area and multi-area power systems that are subject to time delays. The proposed controllers provide stability and integral-action so that step-input references are tracked asymptotically with zero steady-state error. Stability of delay systems has been studied using various approaches and many delay-independent and delay-dependent stability results are available [3], [7], [14], [10]. Proportional-derivative (PD) and proportional-integral-derivative (PID) controllers were proposed for several linear, time-invariant plant classes subject to input-output delays [16], [8]. Arbitrary delay terms in addition to input-output delays were considered in decentralized controller structures [9]. Infinite-dimensional integral-action controllers were designed in [15] to maximize the allowable controller gain using the robust control techniques that are available for infinite-dimensional systems [4].

The contribution of this work is the development of synthesis methods applicable to power systems subject to time delays using novel approaches based on transfer-function descriptions of single-area systems as well as complex multi-area interconnections. An integral-action controller design method is proposed in Section II for a single-area power system subject to time delays using different types of turbine models in the plant description (non-reheated, reheated, and hydraulic turbines). The freedom in the controller parameters may be used to improve system performance. A decentralized controller using the single-area design in each channel is used for a a multi-area control system in Section III. The individual service areas are connected by a tie-line network. Numerical examples are given to illustrate the controller synthesis method using typical model parameters for load, generator, and turbine models as given in e.g., [11], [17], [19] and the references therein.

Notation: Let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}_+$ denote complex, real, and positive real numbers. The closed right-half complex plane is $\mathbb{C}_+ = \{s \in \mathbb{C} \mid \mathcal{R}e(s) \geq 0\}$ and the extended closed right-half complex plane is $\mathcal{U} = \mathbb{C}_+ \cup \{\infty\}$; $\mathbf{R_p}$ denotes real proper rational functions (of s); $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$; I_r is the $r \times r$ identity matrix. The space $\mathcal{H}_\infty$ is the set of all bounded analytic functions in $\mathbb{C}_+$. For $h \in \mathcal{H}_\infty$,

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the norm is defined as $\|h\|_\infty = \text{ess}\sup_{s\in\mathbb{C}_+}|h(s)|$, where ess sup denotes the essential supremum. A matrix-valued function H is in $\mathcal{M}(\mathcal{H}_\infty)$ if all its entries are in $\mathcal{H}_\infty$; in this case $\|H\|_\infty = \text{ess}\sup_{s\in\mathbb{C}_+}\bar\sigma(H(s))$, where $\bar\sigma$ denotes the maximum singular value. All norms of interest here are $\mathcal{H}_\infty$ norms, $\|\cdot\|_\infty \equiv \|\cdot\|$. A system with transfer-matrix H is stable if $H \in \mathcal{M}(\mathcal{H}_\infty)$. A square $H \in \mathcal{M}(\mathcal{H}_\infty)$ is unimodular if $H^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. We drop (s) in transfer-matrices such as $G(s)$ when this is clear. The $(k\times k)$ diagonal matrix, whose diagonal entries are $a_1,\ldots,a_m$, is denoted by diag $[a_1\,,\,a_2\,,\,\cdots\,,\,a_k]$. We use coprime factorizations over $\mathbf{S}$; i.e., $V = D_v^{-1}N_v \in \mathbf{R_p}^{k\times k}$ denotes a left-coprime-factorization, where $N_v\,,\,D_v \in \mathbf{S}^{k\times k}$, $\det D_v(\infty) \neq 0$.

II. Single-area LFC design with time delay

Consider the load frequency control problem for a single generator supplying power to a single service area. A linearized low-order model of the plant for purposes of system frequency analysis and control synthesis consists of three main parts. In the block-diagram shown in Fig. 1, $G_p(s)$, $G_g(s)$, $G_t(s)$ represent the transfer-functions of the load and machine, the speed governor, and the turbine, respectively. The speed regulation due to governor action is represented by the constant R, called the speed droop characteristic. With T_p, T_g, T_t as the load, governor, turbine time-constants, and K_p a constant inversely proportional to the generator damping coefficient, the transfer-functions are given by

$$G_p(s) = \frac{K_p}{T_p s+1}\;, \qquad G_g(s) = \frac{1}{T_g s+1}\;. \tag{1}$$

For a non-reheated turbine, $G_t(s)$ is

$$G_t(s) = \frac{1}{T_t s+1}\;. \tag{2}$$

For a reheated turbine, with T_r and c_r as constants, the turbine transfer-function $G_t(s)$ becomes second order:

$$G_t(s) = \frac{c_r T_r s+1}{(T_r s+1)(T_t s+1)}\;. \tag{3}$$

In the case of hydraulic turbines, $G_t(s)$ has a zero in $\mathbb{C}_+$ and the governors of hydraulic units include transient droop compensation for stable speed control performance [11]:

$$G_t(s) = \frac{1-T_w s}{1+0.5T_w s}\;, \tag{4}$$

$$G_g(s) = \frac{1}{(T_g s+1)}\cdot\frac{(1+T_c s)}{(1+(R_t/R)T_c s)}\;. \tag{5}$$

In Fig. 1, let the transfer-function from w to f be called $P(s)$. With $G(s) := G_t(s)G_g(s)$, define $X(s)$ and $Y(s)$ as

$$X(s) := G_p(s)G_t(s)G_g(s) = G_p(s)G(s)\;, \tag{6}$$

$$Y(s) := 1+\frac{1}{R}G_p(s)G_t(s)G_g(s) = 1+\frac{1}{R}X(s). \tag{7}$$

Then the plant $P(s)$ is given by

$$P(s) = \frac{G_p(s)G_t(s)G_g(s)}{1+\frac{1}{R}G_p(s)G_t(s)G_g(s)} = Y(s)^{-1}X(s)\;. \tag{8}$$

For the three types of turbines used in generation as in (2), (3), (4), the plant transfer-function in (8) is stable (assuming that governor transfer-functions for hydraulic units include transient droop compensation as needed). We design simple controllers for the plant $P(s)$ such that the closed-loop system is stable under communication delays. For single-input single-output systems, the representation of input delays and output delays are equivalent. The block diagram of the unity-feedback system is shown in Fig. 2, where e^{-hs} represents a time delay of h-seconds, and $G(s) = G_t(s)G_g(s)$. In Fig. 2, with $L := (1+e^{-hs}PC)$, the closed-loop map H from the inputs (u,v) to the outputs (w,f) is

$$H = \begin{bmatrix} e^{-hs}CL^{-1} & e^{-hs}CL^{-1}Y^{-1}G_p \\ e^{-hs}PCL^{-1} & -L^{-1}Y^{-1}G_p\end{bmatrix}. \tag{9}$$

Let the (input-output) map from u to f be denoted by $H_{fu} = e^{-hs}PCL^{-1}$. Let $e := u-f$ be the tracking error. Then the (input-error) map H_{eu} from u to e is given by

$$H_{eu} = 1-H_{fu} = (1+e^{-hs}PC)^{-1} = L^{-1}. \tag{10}$$

Definition 1: **a)** The feedback system, shown in Fig. 2, is stable if the closed-loop map H in (9) is in $\mathcal{M}(\mathcal{H}_\infty)$. **b)** The feedback system is stable and has integral-action if the closed-loop map from (u,v) to (w,f) is stable, and the (input-error) map H_{eu} has blocking-zeros at $s=0$. **c)** The controller C is called a stabilizing controller if C is proper and $H \in \mathcal{M}(\mathcal{H}_\infty)$. **d)** The controller C is called an integral-action controller if C is a stabilizing controller, and $D(0) = 0$ for any coprime factorization $C = ND^{-1}$. □

Let $C(s) = ND^{-1}$ be a coprime factorization of the controller, where $N,\,D \in \mathbf{S}$, $D(\infty) \neq 0$. Then C is a stabilizing controller if and only if $M^{-1} \in \mathcal{H}_\infty$, where

$$M := (1+\frac{1}{R}G_pG)\,D + e^{-hs}G_pG\,N = YD + e^{-hs}XN\;. \tag{11}$$

If the feedback system in Fig. 2 is stable and step input references are applied at $u(t)$, then the steady-state error $e(t)$ goes to zero as $t\to\infty$ if and only if $H_{eu}(0) = 0$. Therefore, by Definition 1-(b), the stable system achieves asymptotic tracking of constant reference inputs with zero

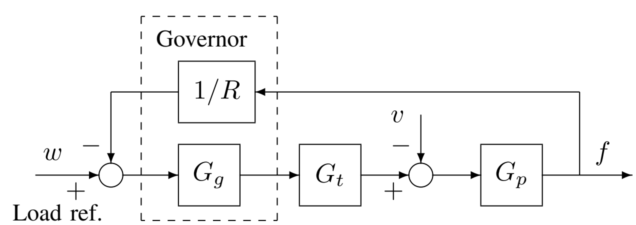

Fig. 1. Plant with speed droop characteristic.

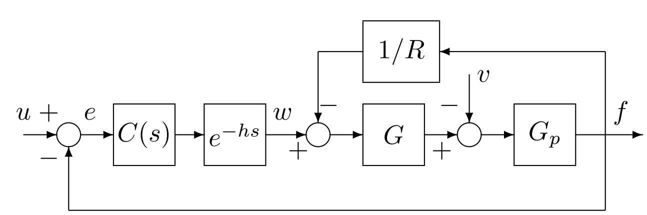

Fig. 2. Feedback system with time delay in the loop.

steady-state error if and only if it has integral-action. By (11), write $H_{eu} = (I + \widehat{G}C)^{-1} = DM^{-1}Y$. Then by Definition 1-(d), the system has integral-action if $C = ND^{-1}$ is an integral-action controller since $D(0) = 0$ implies $H_{eu}(0) = (DM^{-1}Y)(0) = 0$. The system would also have integral-action if the plant had poles at $s = 0$ since $Y(0) = 0$ implies $H_{eu}(0) = 0$ even if the controller's $D(0) \neq 0$. However, the plant $P(s)$ in (8) is stable, and hence, has no poles at $s = 0$. Therefore, the system has integral-action if and *only if* C is an integral-action controller. In the general case, if $P(s)$ has poles at $s = 0$ and $Y(0) = 0$, then this already ensures integral-action for the system. However, integral-action is still designed into the controller for the sake of robustness, following the well-known internal model principle, with poles duplicating the dynamic structure of the exogenous signals that the regulator has to process [5].

Theorem 1 presents a finite-dimensional controller synthesis method for closed-loop stability with integral-action for a single-area LFC control system.

Theorem 1: (Single-area stabilizing controller synthesis): Let $P(s)$ be as in (8). Choose any proper, stable transfer-function $\tilde{N}(s) \in \mathbf{S}$ such that

$$\tilde{N}(0) = P(0)^{-1} \ .$$

Choose any $\alpha \in \mathbb{R}_+$. Let $\beta \in \mathbb{R}_+$ satisfy

$$\beta < \| \frac{e^{-hs}\,(\alpha s + 1)\,P(s)\tilde{N}(s) - 1}{s} \|^{-1} \ . \qquad (12)$$

Then the integral-action controller $C(s)$ given by

$$C(s) = \beta \frac{(\alpha s + 1)}{s} \tilde{N}(s) \qquad (13)$$

is a stabilizing controller. □

Remarks: 1) In Theorem 1, $C(s)$ in (13) can be chosen as a *proper* PID-controller in the following realizable form [6]:

$$C_{pid}(s) = K_P + \frac{K_I}{s} + \frac{K_D\ s}{\tau\, s + 1} \ , \qquad (14)$$

where $K_P,\ K_I,\ K_D \in \mathbb{R}$ are the proportional, integral, and derivative constants, respectively, and $\tau \in \mathbb{R}_+$ (typically very small), where $C_{pid}(s)$ has integral-action when $K_I \neq 0$. To put the controller design proposed in Theorem 1 into a PID-controller form, choose $\tilde{K}_P \in \mathbb{R}$ and $\tilde{K}_D \in \mathbb{R}$ completely arbitrarily. Then let $\tilde{N}(s)$ be

$$\tilde{N}(s) = \frac{1}{(\alpha s + 1)} [\tilde{K}_P s + \frac{\tilde{K}_D\ s^2}{(\tau s + 1)} + P(0)^{-1}] \ ;$$

since $\alpha,\ \tau > 0$, $\tilde{N}(s) \in \mathbf{S}$. With $K_P = \beta \tilde{K}_P$, $K_D = \beta \tilde{K}_D$, $K_I = \beta P(0)^{-1}$, the controller $C(s)$ in (13) becomes a PID-controller as in (14):

$$C(s) = \beta \frac{(\alpha s + 1)}{s} \tilde{N}(s) = \beta \tilde{K}_P + \frac{\beta P(0)^{-1}}{s} + \frac{\beta \tilde{K}_D s}{\tau\, s + 1} \ .$$

The constants $\tilde{K}_D$ or $\tilde{K}_P$ can be chosen as zero to achieve PI or pure integral controllers. 2) The freedom in the design parameters of $C(s)$ in (13) comes from the choice of the stable transfer-function $\tilde{N}(s)$. This freedom is important in order to achieve better performance. Therefore, $\tilde{N}(s)$ may be chosen higher order than two as restricted in PID-controllers to satisfy the performance requirements. The simplest choice of $\tilde{N}(s)$ for integral-action is $\tilde{N}(s) = (\alpha s + 1)^{-1} P(0)^{-1}$, resulting in a pure integral controller $C(s) = \beta P(0)^{-1} \frac{1}{s}$; or $\tilde{N}(s) = P(0)^{-1}$, resulting in a PI-controller $C(s) = \beta P(0)^{-1} \frac{(\alpha s + 1)}{s}$. The integral-action controller $C(s)$ in (13) is strictly-proper whenever $\tilde{N}(s)$ is strictly-proper; otherwise, $C(s)$ is bi-proper. 3) By standard robustness arguments, the stabilizing controller $C(s)$ in (13) of Theorem 1 achieves robust stability under 'sufficiently small' plant uncertainty. The controller C in (13) robustly stabilizes the additively perturbed plant $P + \Delta_p$ for all $\Delta_p \in \mathbf{S}$ such that $\| \Delta_p \| < \| e^{-hs} C (1 + e^{-hs} PC)^{-1} \|^{-1}$. For multiplicative perturbations, C robustly simultaneously stabilizes the plants $P\,(1 + \Delta_p)$ under all multiplicative perturbations $\Delta_p \in \mathbf{S}$ such that $\| \Delta_p \| < \| e^{-hs} PC (1 + e^{-hs} PC)^{-1} \|^{-1}$. Some of the free controller parameter choices in the synthesis may be used to maximize the allowable perturbations. □

Proof of Theorem 1: The plant $P(s) = Y^{-1} X$ given in (8) is stable; therefore, $Y^{-1} \in \mathbf{S}$. Write the controller $C(s)$ in (13) as $C(s) = ND^{-1}$, where $N = \beta \tilde{N}(s)$, $D = \frac{s}{\alpha s + 1}$. By (11), $C(s)$ is a stabilizing controller if and only if $M^{-1} = (YD + e^{-hs} XN)^{-1} \in \mathcal{H}_\infty$, where $M = Y \frac{s}{\alpha s + 1} + e^{-hs} X \beta \tilde{N}(s) = \frac{(s+\beta)}{(\alpha s + 1)} Y\, [1 + \frac{\beta\, s}{(s+\beta)} \Theta(s)]$, where $\Theta(s) := \frac{e^{-hs} (\alpha s + 1) P(s) \tilde{N}(s) - 1}{s}$. Since $\tilde{N}(0) = P(0)^{-1}$ by design, at $s = 0$, the numerator of $\Theta(s)$ is $(\, e^{-hs} (\alpha s + 1) P(s) \tilde{N}(s) - 1\,)(0) = 0$ and hence, $\Theta(s) \in \mathcal{H}_\infty$. Now if $\beta > 0$ satisfies (12), then $\| \frac{\beta\, s}{(s+\beta)} \Theta(s) \| \leq \beta\ \| \Theta(s) \| < 1$. It follows from the small gain theorem that this is a sufficient condition to have $[1 + \frac{\beta\, s}{(s+\beta)} \Theta(s)]^{-1} \in \mathcal{H}_\infty$. Since $\frac{(\alpha s + 1)}{(s+\beta)} Y^{-1} \in \mathbf{S}$, we have $M^{-1} \in \mathcal{H}_\infty$ and hence, the proposed $C(s)$ in (13) is a stabilizing controller. □

In the following examples, we apply the controller design in Theorem 1 to plants with non-reheated, reheated, and hydraulic turbines as in (2), (3), (4), respectively. The plant parameters are typical values as in e.g., [11], [17], [18], [19].

Example 1: **a)** Consider the plant $P(s)$ in (8), with a non-reheated turbine, where the load, governor, turbine, droop model parameters are given as

$$K_p = 1 \ ,\ T_p = 10 \ ,\ T_g = 0.2 \ ,\ T_t = 7 \ ,\ R = 0.05 \ . \quad (15)$$

Then from (6)-(7) and (8), $X(s) = \frac{1}{(10s+1)(7s+1)(0.2s+1)}$, $Y(s) = \frac{14s^3 + 73.4s^2 + 17.2s + 21}{(10s+1)(7s+1)(0.2s+1)}$. The plant $P(s) = Y^{-1} X$ is stable, with poles at $\{-5.0586, -0.0921 \pm j0.5367\}$. For a very simple second order controller design, let $\alpha = 1$ and

$$\tilde{N}(s) = \frac{(10s + P(0)^{-1})}{(\tau s + 1)} = \frac{(10s + 21)}{(0.04s + 1)} \ . \qquad (16)$$

Suppose that there is no time delay, i.e., $h = 0$; then β satisfies (12) if $\beta < 0.1824$. The controller in (13) becomes

$$C(s) = \frac{\beta\,(s + 1)(10s + 21)}{s\,(0.04s + 1)} \ . \qquad (17)$$

b) For the case of a reheated turbine, in addition to the parameters in (15), $T_r = 0.3$, $c_r = 0.3$. From (6), (7), (8), $X(s) = \frac{(0.09s+1)}{(10s+1)(0.3s+1)(7s+1)(0.2s+1)}$, $Y(s) = \frac{4.2s^4+36.02s^3+78.56s^2+19.3s+21}{(10s+1)(0.3s+1)(7s+1)(0.2s+1)}$. The plant $P(s) = Y^{-1}X$ is stable, with poles at $\{-4.9281, -3.5236, -0.0622 \pm j0.533\}$. Let $\alpha = 1$ and choose $\tilde{N}$ in (16). With no time delay, $h = 0$, β satisfies (12) if $\beta < 0.1183$.

c) For the case of a hydraulic turbine, in addition to (15), let $T_w = 1$, $T_c = 5$, $R_t = 0.38$. From (6), (7), (8), $X(s) = \frac{(5s+1)(1-s)}{(10s+1)(0.5s+1)(38s+1)(0.2s+1)}$, $Y(s) = \frac{38s^4+270.8s^3+313.7s^2+128.7s+21}{(10s+1)(0.5s+1)(38s+1)(0.2s+1)}$. The plant $P(s) = Y^{-1}X$ is stable, with poles at $\{-5.8011, -0.7223, -0.3015 \pm j0.2025\}$. Let $\alpha = 1$ and choose $\tilde{N}$ in (16). With no time delay, $h = 0$, $\beta < 0.3538$ satisfies (12). When an h-second time delay occurs, β satisfies (12) if $\beta < \| s^{-1}[e^{-hs}(s+1)P(s)\frac{(10s+21)}{(0.04s+1)} - 1] \|^{-1}$. The maximum β that satisfies (12) for h-seconds of time delay, where $0 \leq h \leq 2.5$, is shown in Fig. 3. For all three turbine examples, we can choose $\beta = 0.1$ or $\beta = 0.05$ for a delay of $h < 2.5$ sec.

With no time delay ($h = 0$), the closed-loop step-responses of the output $f(t)$ due to a constant input at $u(t)$ are shown for $\beta = 0.1$ and $\beta = 0.05$ satisfying (12) for non-reheated, reheated, and hydraulic turbine examples in Figures 4, 6, 8, respectively. The same controller given in (17) is used for all three different turbine types. Due to the integral-action in the controller, the steady-state error is zero as expected. Smaller β values correspond to more sluggish response. The performance can be improved exploring the freedom in $\tilde{N}$, α, and by changing β. The step-responses for a delay of $h = 0.5$ are shown for the same β choices for the three types of turbine examples in Figures 5, 7, and 9.

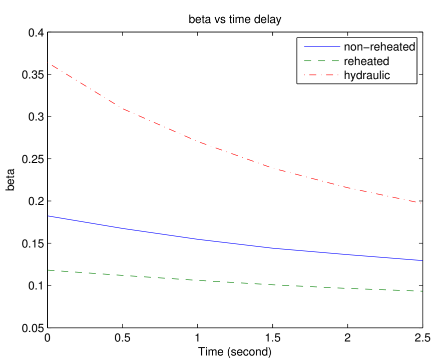

Fig. 3. Maximum β v.s. time delay h for for three types of turbines.

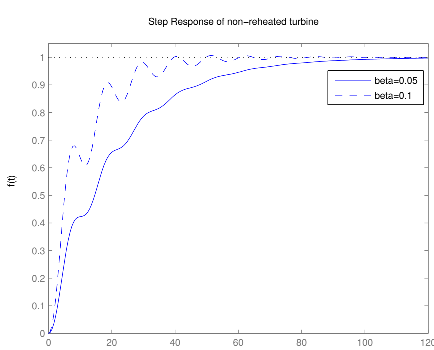

Fig. 4. Step response of non-reheated turbine with no time delay.

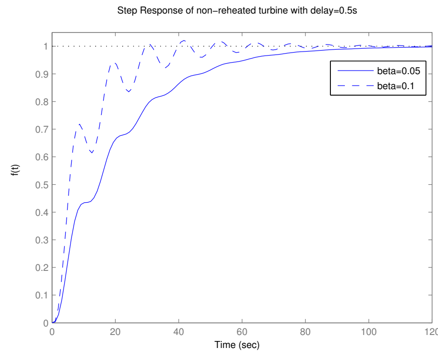

Fig. 5. Step response of non-reheated turbine with time delay $h = 0.5$.

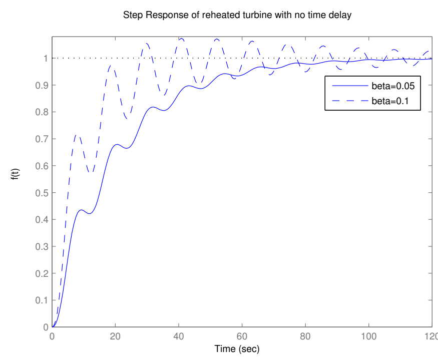

Fig. 6. Step response of reheated turbine with no time delay.

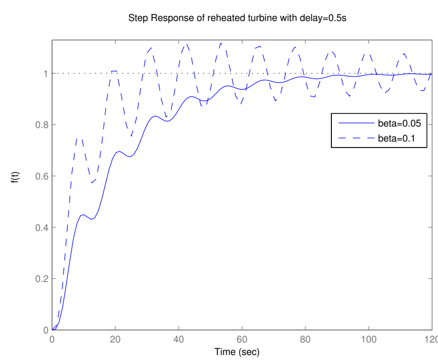

Fig. 7. Step response of reheated turbine with time delay $h = 0.5$.

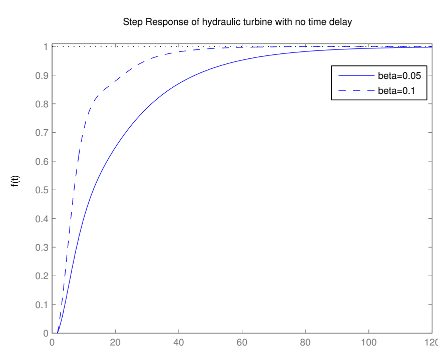

Fig. 8. Step response of hydraulic turbine with no time delay.

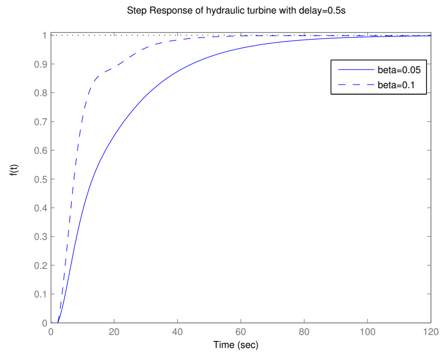

Fig. 9. Step response of hydraulic turbine with time delay $h = 0.5$.

III. Multi-area decentralized LFC design

Consider the multi-area load frequency control problem, where each area has the same structure as shown in Fig. 10. The tie-line power flows among the k areas is represented by the constant matrix $V_t \in \mathbb{R}^{k\times k}$, which may or may not be non-singular. Let $s^{-1}V_t = D_v^{-1}(s)N_v(s)$ be a left-coprime-factorization, where N_v, $D_v \in \mathbf{S}^{k\times k}$, $\det D_v(\infty) \neq 0$. If $\mathrm{rank}V_t = r < k$, then there exist non-singular matrices F_L, $F_R \in \mathbb{R}^{k\times k}$ such that (18) holds, and for any $a \in \mathbb{R}_+$, D_v, N_v are as in (19):

$$V_t = F_L \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} F_R \ , \tag{18}$$

$$D_v = \begin{bmatrix} \dfrac{s}{s+a} I_r & 0 \\ 0 & I_{k-r} \end{bmatrix} F_L^{-1} \ , \ N_v = \begin{bmatrix} \dfrac{1}{s+a} I_r & 0 \\ 0 & 0 \end{bmatrix} F_R \ . \tag{19}$$

When V_t is non-singular, $r = k$; then

$$D_v = \frac{s}{s+a} I_k \ , \ N_v = \frac{1}{s+a} V_t \ ; \ F_L = I_k \ , \ F_R = V_t \ . \tag{20}$$

In the decentralized system of Fig. 10, Φ represents a constant matrix to be designed so that the closed-loop system is stable. Let the i-th area plant $P_i(s) = Y_i^{-1}(s)X_i(s)$ be as in (8). Let $G_p = \mathrm{diag}\ [\ G_{p1}\,, \ G_{p2}\,, \cdots, G_{pk}\ ]$, $G = \mathrm{diag}\ [\ G_1\,, G_2\,, \cdots, G_k\ ]$, $R = \mathrm{diag}\ [\ R_1\,, R_2\,, \cdots, R_k\ ]$. With $X_i(s) = G_{pi}(s)G_i(s)$, $Y_i(s) = 1 + \frac{1}{R_i}X_i(s)$ as in (6)-(7), let

$$X = \mathrm{diag}\ [\ X_1\ ,\ X_2\ ,\ \cdots,\ X_k\ ] = G_pG\ , \tag{21}$$

$$Y = \mathrm{diag}\ [\ Y_1\ ,\ Y_2\ ,\ \cdots,\ Y_k\ ] = I_k + R^{-1}X\ . \tag{22}$$

Then the plant is

$$P(s) = \mathrm{diag}\ [\ P_1(s)\ ,\ P_2(s)\ ,\ \cdots,\ P_k(s)\ ]\ . \tag{23}$$

The constants B_i for areas $i \in \{1,\ldots,k\}$ are called frequency bias factors, and

$$B = \mathrm{diag}\ [\ B_1\ ,\ B_2\ ,\ \cdots,\ B_k\ ]\ . \tag{24}$$

Each area has its own stabilizing integral-action controller $C_i(s) = N_i(s)D_i^{-1}(s)$ and hence, the decentralized controller has a diagonal transfer-matrix given by

$$C(s) = \mathrm{diag}\ [C_1(s)\,, C_2(s)\,, \cdots, C_k(s)] = ND^{-1}, \tag{25}$$

$N = \mathrm{diag}\ [\ N_1\ ,\ N_2\ ,\ \cdots,\ N_k\ ]$, $D = \mathrm{diag}\ [\ D_1\ ,\ D_2\ ,\ \cdots,\ D_k\ ]$. The time delay h_i for each area may be different. Let

$$E = \mathrm{diag}\ \left[\ e^{-h_1 s}\ ,\ e^{-h_2 s}\ ,\ \cdots,\ e^{-h_k s}\ \right]\ . \tag{26}$$

Let each of the controllers $C_i = N_iD_i^{-1} = ND^{-1}$ for the individual areas be designed as in (13), and let

$$M_i = Y_iD_i + e^{-hs}X_iN_iB_i\ , \tag{27}$$

$$M = \mathrm{diag}\ [\ M_1\ ,\ M_2\ ,\ \cdots,\ M_k\ ] = YD + EXNB\ .$$

In the multi-area system of Fig. 10, let the input and output vectors be $u = [u_1\ \ldots\ u_k]^T$, $v = [v_1\ \ldots\ v_k]^T$, $w = [w_1\ \ldots\ w_k]^T$, $f = [f_1\ \ldots\ f_k]^T$. Let $\xi := w - R^{-1}f$. Define $\Psi \in \mathcal{M}(\mathcal{H}_\infty)$ as

$$\Psi = \begin{bmatrix} YD + EXNB & [EN(I - BG_p) - DR^{-1}G_p]\Phi \\ -N_vX & D_v + N_vG_p\Phi \end{bmatrix}. \tag{28}$$

The system in Fig. 10 is described as

$$\Psi \begin{bmatrix} \xi \\ x \end{bmatrix} = \begin{bmatrix} EN & (ENB + DR^{-1})G_p \\ 0 & -N_vG_p \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}, \tag{29}$$

$$\begin{bmatrix} w \\ f \end{bmatrix} = \begin{bmatrix} Y & -R^{-1}G_p \\ X & -G_p \end{bmatrix} \begin{bmatrix} \xi \\ x \end{bmatrix} + \begin{bmatrix} 0 & -R^{-1}G_p \\ 0 & -G_p \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}. \tag{30}$$

Then the closed-loop system with k areas as shown in Fig. 10 is stable if and only if $\Psi^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, where Ψ is given by (28). Define $\Gamma := EN(I - BG_p) - DR^{-1}G_p$, and re-write

$$\Psi = \begin{bmatrix} M & \Gamma\Phi \\ -N_vX & D_v + N_vG_p\Phi \end{bmatrix}$$

$$= \begin{bmatrix} M & 0 \\ -N_vX & I \end{bmatrix} \begin{bmatrix} I & M^{-1}\Gamma\Phi \\ 0 & D_v + N_vG_p\Phi + N_vXM^{-1}\Gamma\Phi \end{bmatrix}. \tag{31}$$

By (31), Ψ in (28) is unimodular if and only if

$$\begin{aligned} \tilde{\Psi} &= D_v + N_vG_p\Phi + N_vXM^{-1}\Gamma\Phi \\ &= D_v + N_vG_p(D + GEN)M^{-1}\Phi \end{aligned} \tag{32}$$

is unimodular.

Theorem 2: (Multi-area stabilizing controller synthesis): Let $C_i(s)$ be designed as in (13) to stabilize each individual subsystem. as in (8). Define $W_r \in \mathcal{M}(\mathcal{H}_\infty)$ as

$$W_r = [I_r\ \ 0]\, F_RG_p(D + GEN)M^{-1} \begin{bmatrix} I_r \\ 0 \end{bmatrix}\ . \tag{33}$$

Let $\varphi \in \mathbb{R}_+$ satisfy

$$\varphi < \| \frac{1}{s} \left(W_r(s)W_r(0)^{-1} - I_r \right) \|^{-1}\ . \tag{34}$$

Let $\Phi_r = \varphi\, W_r(0)^{-1}$ and let Φ in (31) be chosen as

$$\Phi := \begin{bmatrix} \Phi_r & 0 \\ 0 & I_{k-r} \end{bmatrix} F_L^{-1}\ . \tag{35}$$

Then the decentralized system in Fig. 10 is stable. □

Proof of Theorem 2: Since C_i stabilizes P_i, each $M_i^{-1} \in \mathcal{H}_\infty$ implies that $M^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. The multi-area system is stable if and only if Ψ is unimodular, equivalently, $\tilde{\Psi}$ in (32) unimodular. By (19) and (35), write $\tilde{\Psi}$ as

$$\tilde{\Psi} = \begin{bmatrix} \frac{s}{s+a} I_r & 0 \\ 0 & I_{k-r} \end{bmatrix} F_L^{-1}$$

$$+ \begin{bmatrix} \frac{1}{s+a} I_r & 0 \\ 0 & 0 \end{bmatrix} F_R G_p (D + GEN) M^{-1} \begin{bmatrix} \Phi_r & 0 \\ 0 & I_{k-r} \end{bmatrix} F_L^{-1}$$

$$= \begin{bmatrix} \frac{s}{s+a} + \frac{1}{s+a} W_r \Phi_r & \frac{1}{s+a} W_2 \\ 0 & I_{k-r} \end{bmatrix} F_L^{-1} \; ,$$

where $W_2 = [I_r \;\; 0]\, F_R G_p (D + GEN) M^{-1} \begin{bmatrix} 0 \\ I_{k-r} \end{bmatrix}$. Since F_L is unimodular, $\tilde{\Psi}$ is unimodular if and only if

$$\frac{s}{s+a} I_r + \frac{1}{s+a} W_r \Phi_r = \frac{s}{s+a} I_r + \frac{\varphi}{s+a} W_r W_r(0)^{-1}$$

$$= \frac{(s+\varphi)}{(s+a)} \left[I_r + \frac{\varphi\, s}{(s+\varphi)} \frac{1}{s} (W_r W_r(0)^{-1} - I_r) \right] \qquad (36)$$

is unimodular. Therefore, (36) is unimodular since φ satisfies (34), and hence, the closed-loop system is stable. $\square$

Example 2: Consider a two-area power system, where the plant model for each individual area contains any of the three turbines as in Example 1. Let the tie-line network matrix be $V_t = \gamma \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Note that V_t is singular, and $r = 1 < k$. Then $s^{-1} V_t = D_v^{-1} N_v$, where, for any $a \in \mathbb{R}_+$, $D_v = \begin{bmatrix} \frac{s}{s+a} & 0 \\ 1 & 1 \end{bmatrix}$, $N_v = \begin{bmatrix} \frac{\gamma}{s+a} & \frac{-\gamma}{s+a} \\ 0 & 0 \end{bmatrix}$, $F_L = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $F_R = \begin{bmatrix} \gamma & -\gamma \\ 0 & 1 \end{bmatrix}$. Suppose that $\gamma = 0.5$ and the frequency bias factors are $B_1 = B_2 = 0.4$. Let each of the controllers $C_i = N_i D_i^{-1}$ for the individual areas be designed as in (17) and let $\beta = 0.05$. For a time-delay of $h_1 = h_2 = 0.5$, $M_1^{-1} = (Y_1 D_1 + e^{-h_1 s} X_1 N_1 B_1)^{-1} \in \mathcal{H}_\infty$, $M_2^{-1} = (Y_2 D_2 + e^{-h_2 s} X_2 N_2 B_2)^{-1} \in \mathcal{H}_\infty$. The multi-area system is stable if and only if $\Psi^{-1} \in \mathcal{H}_\infty{}^{2\times 2}$, where Ψ is given by (28). By (33), $W_r = \gamma G_{p1}(D_1 + e^{-h_1 s} N_1 G_1) M_1^{-1}$. Since the controllers C_i are designed as in (13) have integral-action, $D_1(0) = D_2(0) = 0$. Therefore, $M_i(0) = X_i(0) N_i(0) B_i(0)$ implies $W_r(0) = \gamma B_1^{-1} = 1.25$. By Theorem 2, the system is stable by choosing $\Phi_r = 0.8\varphi$ for any $\varphi > 0$ satisfying (34). $\square$

IV. Conclusions

A finite-dimensional controller synthesis method is proposed for load-frequency control of single-area power systems with time delays in communication channels. The controllers are simple, and they provide integral-action. The synthesis method allows freedom in the choice of controller parameters, which can be used to achieve better performance. Numerical examples are provided for low-order linearized plant models containing non-reheated, reheated, and hydraulic turbines. The design for each area is extended in a decentralized setting to a multi-area system interconnected with a tie-line network.

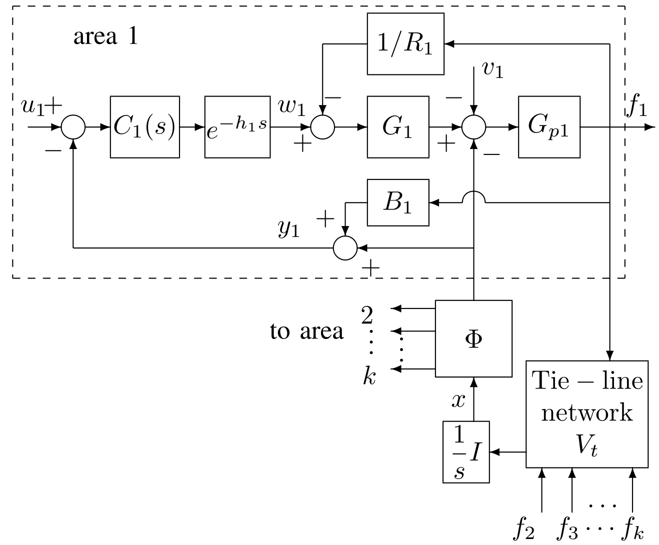

Fig. 10. Multi-area system, with area 1 shown.

References

[1] H. Bevrani, *Robust Power System Frequency Control*, New York: Springer, 2009.

[2] R.F. Curtain, K. Glover, "Robust stabilization of infinite-dimensional systems by finite-dimensional controllers," *Systems Control Letters*, 7 (1), pp. 4147, 1986.

[3] R.F. Curtain, H. Zwart, *An Introduction to Infinite-dimensional Linear Systems Theory*, Texts in Applied Mathematics, Vol. 21, Springer, New York, 1995.

[4] C. Foias, H. Özbay, A. Tannenbaum, *Robust Control of Infinite Dimensional Systems*, LNCIS 209, Springer-Verlag, London, 1996.

[5] B. A. Francis, W. A. Wonham, "The internal model principle for linear multivariable regulators," *Applied Mathematics & Optimization*, 2:2, pp. 170-195, 1975.

[6] G. C. Goodwin, S. F. Graebe, M. E. Salgado, *Control System Design*, Prentice Hall, 2001.

[7] K. Gu, V. L. Kharitonov, J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Boston, 2003.

[8] A. N. Gündeş, H. Özbay, A. B. Özgüler, "PID controller synthesis for a class of unstable MIMO plants with I/O Delays," *Automatica*, vol. 43, no. 1, pp. 135-142, 2007.

[9] A. N. Gündeş, H. Özbay, "Reliable decentralized control of delayed MIMO plants," *Int. Jour. Control*, vol. 83, no. 3, pp. 516-526, 2010.

[10] L. Jiang, W. Yao, Q. H. Wu, J. Y. Wen, S. J. Cheng, "Delay-dependent stability for load frequency control with constant and time-varying delays," *IEEE Trans. Power Systems*, vol. 27, no. 2, pp. 932-941, 2012.

[11] P. Kundur, *Power System Stability and Control*, New York: McGraw-Hill, 1994.

[12] P. Kundur, J. Paserba, V. Ajjarapu, et al., "Definition and classification of power system stability," *IEEE Trans. Power Syst.,*, 19(2), pp. 13871401, 2004.

[13] J. Liu, B. H. Krogh, M. D. Ilić, "Robust control design for frequency regulation in power systems with high wind penetration," *Proc. American Control Conf.*, pp. 4349-4354, 2010.

[14] S. I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*, LNCIS, vol. 269, Heidelberg: Springer-Verlag, 2001.

[15] H. Özbay, A. N. Gündeş, "Integral action controllers for systems with time delays," in *Topics in Time Delay Systems. Analysis, Algorithms and Control*, LNCIS 388, J. J. Loiseau, W. Michiels, S-I. Niculescu, R. Sipahi (Eds.), pp. 197-208, Springer-Verlag, London, 2009.

[16] G. J. Silva, A. Datta, S. P. Bhattacharyya, "New results on the synthesis of PID controllers," *IEEE Trans. Automatic Control*, vol. 47: 2, pp. 241-252, 2002.

[17] W. Tan, "Unified tuning of PID load frequency controller for power systems via IMC," *IEEE Trans. Power Systems*, vol. 25, no. 1, pp. 341-350, 2010.

[18] W. Tan,"Tuning of PID load frequency controller for power systems," *Energy Convers. Manage.*, vol. 50, no. 6, pp. 14651472, 2009.

[19] X. F. Yu, K. Tomsovic, "Application of linear matrix inequalities for load frequency control with communication delays," *IEEE Trans. Power Systems*, vol. 19, no. 3, pp. 15081519, 2004.