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SIMPLE LOW-ORDER AND INTEGRAL-ACTION CONTROLLER SYNTHESIS FOR MIMO SYSTEMS WITH TIME DELAYS

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ABSTRACT. A simple finite-dimensional controller synthesis method is developed for some classes of linear, time-invariant, multi-input multi-output systems that are subject to time delays. The proposed synthesis procedure gives low-order controllers that achieve closed-loop stability, with simple modifications, also achieve integral-action.

Keywords: Stability, Delay Systems, Integral-Action, Controller Design.

AMS Subject Classification: 93D15.

1. Introduction

A wide range of dynamical phenomena cannot be modeled sufficiently accurately as finite-dimensional linear time-invariant (LTI) systems due to being subject to time delays. The effects of these delays often cannot be ignored and have to be included in the model [3].

This paper presents a stabilizing controller synthesis method for some classes of linear, time-invariant (LTI), multi-input multi-output (MIMO) systems that are subject to time delays. The proposed controllers are simple finite-dimensional LTI controllers that achieve closed-loop stability of the delayed plants. The controller design can be modified easily so that these controllers also provide integral-action in order to achieve asymptotic tracking of step-input references with zero steady-state error. For the plant classes considered here, the finite-dimensional part of the plants may have any (finite) number of poles; while the poles in the stable region are unrestricted, those in the unstable region (closed right-half complex plane) have to be 'small', i.e., close to the origin. There are no restrictions on the number or location of the transmission-zeros of the finite-dimensional parts of the delayed plants. The controllers designed for these plants are low-order, and in the SISO case they are either stable (if integral-action is not required in the design) or have all poles in the open left-half complex plane except a pole at the origin for integral-action. The simplicity of the designed controllers explains why the plant classes have restrictions on the poles in the region of instability: these plants are in fact strongly stabilizable.

Stability of delay systems of retarded type and of neutral type was studied extensively and many delay-independent and delay-dependent stability results are available (see [3], [6]). Most tuning and internal model control techniques used in process control systems apply to delay systems [8]. The results on robust control of infinite-dimensional systems apply to the subclass of systems with delays [1]. The problem of existence of proportional-derivative (PD) and proportional-integral-derivative (PID) controllers for time delay systems was also considered and computational PID-stabilization methods have been extended to cover scalar, single-delay systems (see e.g., [5, 7]). For MIMO stable plants and for unstable plants whose finite-dimensional parts have no more than two poles in the unstable region, PD and PID controller synthesis methods were developed for I/O delays, i.e., delays that affect the plant's inputs and outputs (see [4]). The synthesis method developed in this paper allows arbitrary delay terms to affect

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different entries of the plant's transfer-matrix. The proposed controller design applies under certain assumptions on the poles of the finite-dimensional parts of the unstable delayed plants:

We use the following standard notation:

Notation: Let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}_+$ denote complex, real, and positive real numbers. The extended closed right-half complex plane is $\mathcal{U} = \{s \in \mathbb{C} \mid \mathcal{R}e(s) \geq 0\} \cup \{\infty\}$; $\mathbf{R_p}$ denotes real proper rational functions (of s); $\mathbf{S} \subset \mathbf{R_p}$ is the stable subset with no poles in $\mathcal{U}$; $\mathcal{M}(\mathbf{S})$ is the set of matrices with entries in $\mathbf{S}$. The space $\mathcal{H}_\infty$ is the set of all bounded analytic functions in $\mathbb{C}_+$. For $h \in \mathcal{H}_\infty$, the norm is defined as $\|h\|_\infty = ess\ sup_{s\in\mathbb{C}_+}|h(s)|$, where $ess\ sup$ denotes the essential supremum. A matrix-valued function H is in $\mathcal{M}(\mathcal{H}_\infty)$ if all its entries are in $\mathcal{H}_\infty$; in this case $\|H\|_\infty = ess\ sup_{s\in\mathbb{C}_+}\bar{\sigma}(H(s))$, where $\bar{\sigma}$ denotes the maximum singular value. From the induced L^2 gain point of view, a system whose transfer-matrix is H is stable iff $H \in \mathcal{M}(\mathcal{H}_\infty)$. A square transfer-matrix $H \in \mathcal{M}(\mathcal{H}_\infty)$ is unimodular iff $H^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$. We drop (s) in transfer-matrices such as $G(s)$ where this causes no confusion. Since all norms of interest here are $\mathcal{H}_\infty$ norms, we drop the norm subscript, i.e., $\|\cdot\|_\infty \equiv \|\cdot\|$. We use coprime factorizations over $\mathbf{S}$; i.e., for $G \in \mathbf{R_p}^{m\times m}$, $G = D^{-1}N$ denotes a left-coprime-factorization (LCF), where $N, D \in \mathbf{S}^{m\times m}$, $\det D(\infty) \neq 0$.

2. Problem description

Consider the feedback system $Sys(G^\Lambda, C)$ in Fig. 1, where $C \in \mathbf{R_p}^{m\times m}$ is the transfer-function of the controller and G^Λ is the transfer-function of the plant with time delays. It is assumed that the feedback system is well-posed and that the delay-free part of the plant (i.e, the plant without the time delay terms) and the controller have no unstable hidden-modes. The finite-dimensional part of the plant is denoted by $G \in \mathbf{R_p}^{m\times m}$.

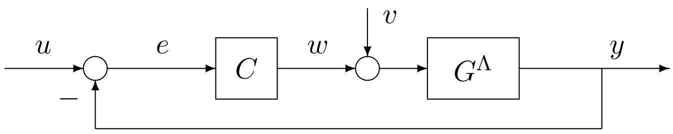

Figure 1. The feedback system $Sys(G^\Lambda, C)$.

Let $G = D^{-1}N$ be an LCF of G. Suppose that each ij-th entry N_{ij}^Λ of N^Λ contains any arbitrary delay terms and that the delays are known. We assume that G^Λ can be written as

$$G^\Lambda = D^{-1}N^\Lambda \quad , \quad where \quad N_{ij}^\Lambda = e^{-h_{ij}s}N_{ij} \ , \quad i,j = 1,\ldots,m. \tag{1}$$

If the finite-dimensional part G of the delayed plant G^Λ is stable, then (1) implies that the entries of G^Λ may contain all different arbitrary known delay terms. If the finite-dimensional part G of the delayed plant G^Λ is not stable, then we assume that the delayed plant transfer-function G^Λ has restrictions on its poles in the unstable region.

In the system $Sys(G^\Lambda, C)$, let u, v, w, y denote the input and output vectors. The closed-loop transfer-matrix $\widehat{H}$ from (u,v) to (w,y) is

$$\widehat{H} = \begin{bmatrix} C(I+G^\Lambda C)^{-1} & -C(I+G^\Lambda C)^{-1}G^\Lambda \\ G^\Lambda C(I+G^\Lambda C)^{-1} & (I+G^\Lambda C)^{-1}G^\Lambda \end{bmatrix}. \tag{2}$$

Let the (input-error) transfer-function from u to e be denoted by H_{eu} and let the (input-output) transfer-function from u to y be denoted by H_{yu}; then

$$H_{eu} = (I+G^\Lambda C)^{-1} = I - G^\Lambda C(I+G^\Lambda C)^{-1} = I - H_{yu} \ . \tag{3}$$

Definition 1. a) The feedback system $Sys(G^\Lambda, C)$ shown in Fig. 1 is stable if the closed-loop map $\widehat{H}$ is in $\mathcal{M}(\mathcal{H}_\infty)$. **b)** The controller C stabilizes G^Λ if C is proper and $Sys(G^\Lambda, C)$ is stable. **c)** The system $Sys(G^\Lambda, C)$ is stable and has integral-action if the closed-loop transfer-function from (u,v) to (w,y) is stable, and the (input-error) transfer-function H_{eu} has blocking-zeros at

$s = 0$. **d)** The controller C is said to be an integral-action controller if C stabilizes G^Λ and $Y(0) = 0$ for any RCF $C = XY^{-1}$.

Let $G = D^{-1}N$ be an LCF of the finite-dimensional part of G^Λ, and let $G^\Lambda = D^{-1}N^\Lambda$; let $C = XY^{-1}$ be an RCF, where $D, N, Y, X \in \mathbf{S}^{m\times m}$, $\det D(\infty) \neq 0$, $\det Y(\infty) \neq 0$. Then C stabilizes G^Λ if and only if $M^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, where

$$M := DY + N^\Lambda X \ . \tag{4}$$

Suppose that the system $Sys(G^\Lambda, C)$ is stable and that step input references are applied at $u(t)$. Then the steady-state error $e(t)$ due to step inputs at $u(t)$ goes to zero as $t \to \infty$ if and only if $H_{eu}(0) = 0$. Therefore, by Definition 1-(c), the stable system $Sys(G^\Lambda, C)$ achieves asymptotic tracking of constant reference inputs with zero steady-state error if and only if it has integral-action. By (4), write $H_{eu} = (I + G^\Lambda C)^{-1} = YM^{-1}D$. Then by Definition 2-(d), $Sys(G^\Lambda, C)$ has integral-action if $C = XY^{-1}$ is an integral-action controller since $Y(0) = 0$ implies $H_{eu}(0) = (YM^{-1}D)(0) = 0$. Note that the system $Sys(G^\Lambda, C)$ would also have integral-action if every entry of the MIMO plant has poles at $s = 0$ since $D(0) = 0$ implies $H_{eu}(0) = 0$ even if the controller's $Y(0) \neq 0$. However, for robust designs, integral-action is achieved with poles duplicating the dynamic structure of the exogenous signals that the regulator has to process; these integral-action controllers obey the well-known internal model principle [2].

We assume throughout that the finite-dimensional part G has no transmission-zeros at $s = 0$. In fact, this condition is a necessary condition for existence of integral-action controllers: Let $G \in \mathbf{R_p}^{m\times m}$, with (normal) rank$G(s) = m$. If G^Λ admits an integral-action controller, then G has no transmission-zeros at $s = 0$.

3. Controller synthesis

In this section, we propose stabilizing controllers for a class of (MIMO as well as SISO) plants with time delays. Theorem 1 presents controller synthesis for closed-loop stability. Corollary 1 includes integral-action in the stabilizing controller synthesis.

Let $G = D^{-1}N \in \mathbf{R_p}^{m\times m}$ have full (normal) rank m. Let G have no transmission zeros at $s = 0$, equivalently, rank$N(0) = m$. Let $p_j \in \mathcal{U}$, $j = 1, \ldots, r$, denote the poles of G with non-negative real parts (counting multiplicities). Define

$$\varphi := \prod_{j=1}^{r} (s - p_j) \ . \tag{5}$$

Since the set $\{ p_j \}_{j=1}^r$ includes all $\mathcal{U}$-poles of G, for any $\alpha \in \mathbb{R}_+$, $\frac{\varphi}{(s+\alpha)^r}$ is a largest invariant-factor of the denominator D. Therefore, $\frac{\varphi}{(s+\alpha)^r} G \in \mathcal{M}(\mathbf{S})$, equivalently, $\frac{\varphi}{(s+\alpha)^r} D^{-1} \in \mathcal{M}(\mathbf{S})$. Hence, $\frac{\varphi}{(s+\alpha)^r} G^\Lambda \in \mathcal{M}(\mathcal{H}_\infty)$.

Theorem 1. (Stabilizing controller synthesis): Let $G^\Lambda = D^{-1}N^\Lambda$ be as in (1), with φ as in (5). Define

$$\Phi := \frac{1}{s} [\, N^\Lambda(s)\, N(0)^{-1} - I \,] \ . \tag{6}$$

Suppose that

$$\sum_{j=1}^{r} p_j \ < \ \|\Phi\|^{-1} \ . \tag{7}$$

Choose $\alpha \in \mathbb{R}_+$ such that

$$\alpha \geq |p_j| \ \text{ for } \ j = 1, \ldots, r \ , \tag{8}$$

and

$$\alpha \ < \ \frac{1}{r} \ (\ \|\Phi\|^{-1} - \sum_{j=1}^{r} p_j \ ) \ . \tag{9}$$

Then a controller that stabilizes G^{Λ} is given by

$$C = \frac{[\,(s+\alpha)^r - \varphi\,]}{\varphi} \, N(0)^{-1} D(s) \ . \tag{10}$$

Proof of Theorem 1. Since the order of the polynomial term $d := [\,(s+\alpha)^r - \varphi\,]$ is $r-1$, the term $X := \frac{[\,(s+\alpha)^r-\varphi\,]}{(s+\alpha)^r} N(0)^{-1} \in \mathcal{M}(\mathbf{S})$ (it is in fact strictly-proper and stable). The term $Y := \frac{\varphi}{(s+\alpha)^r} D^{-1} \in \mathcal{M}(\mathbf{S})$ since φ contains all $\mathcal{U}$-poles of G. Therefore, $C = XY^{-1}$ is an RCF of the proposed controller in (10). By (4), C stabilizes G^{Λ} if and only if $M^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, where

$$M = D\,Y + N^{\Lambda} X = \frac{\varphi}{(s+\alpha)^r} D D^{-1} + \frac{[\,(s+\alpha)^r - \varphi\,]}{(s+\alpha)^r} N^{\Lambda} N(0)^{-1}$$

$$= \frac{\varphi}{(s+\alpha)^r} I + \frac{[\,(s+\alpha)^r - \varphi\,]}{(s+\alpha)^r} N^{\Lambda} N(0)^{-1} = I + \frac{[\,(s+\alpha)^r - \varphi\,]}{(s+\alpha)^r} \, [N^{\Lambda} N(0)^{-1} - I\,]$$

$$= I + \frac{s\,[\,(s+\alpha)^r - \varphi\,]}{(s+\alpha)^r} \, \frac{[N^{\Lambda} N(0)^{-1} - I\,]}{s} = I + \frac{s\,[\,(s+\alpha)^r - \varphi\,]}{(s+\alpha)^r} \, \Phi = I + \frac{s\,d}{(s+\alpha)^r} \, \Phi \, .$$

A sufficient condition for M to be unimodular is that $\| \frac{s\,d}{(s+\alpha)^r} \Phi \| < 1$. Under the condition $\alpha > |p_j|$, the norm $\| \frac{s\,d}{(s+\alpha)^r} \|$ is:

$$\| \frac{s\,d}{(s+\alpha)^r} \| = \| \frac{s\,[\,(s+\alpha)^r - \prod_{j=1}^{r}(s-p_j)\,]}{(s+\alpha)^r} \| = r\alpha + \sum_{j=1}^{r} p_j \ . \tag{11}$$

Since α satisfies (9), $\| \frac{s\,d}{(s+\alpha)^r} \Phi \| \le \| \frac{s\,d}{(s+\alpha)^r} \| \|\Phi\| = (r\alpha + \sum_{j=1}^{r} p_j)\|\Phi\| < 1$. Therefore, M is unimodular and hence, C stabilizes G^{Λ}.

We now prove (11): For all $r \ge 1$, $\left(\frac{s\,[(s+\alpha)^r - \varphi]}{(s+\alpha)^r}\right)\Big|_{s=\infty} = r\alpha + \sum_{j=1}^{r} p_j$ implies the norm in (11) is greater than or equal to the right-hand side. We prove the norm in (11) is less than or equal to the right-hand side by iteration: For $r = 1$, $p_1 \in \mathbb{R}_+$ and (11) holds since $\|\frac{(\alpha+p_1)s}{s+\alpha}\| = \alpha + p_1$. For $r = 2$,

$$\|\frac{s\,[(2\alpha+p_1+p_2)s + \alpha^2 - p_1p_2]}{(s+\alpha)^2}\| = (2\alpha + p_1 + p_2)$$

since $\alpha^2 \ge p_1p_2$ and hence, (11) holds. For $r = 3$, let $p_3 \in \mathbb{R}_+$ since at least one of the three $\mathcal{U}$-poles has to be real. Then $\|\frac{s}{s+\alpha}\| = 1$ and $\|\frac{\prod_{j=1}^{2}(s-p_j)]}{(s+\alpha)^2}\| = 1$ for $\alpha^2 \ge p_1p_2$ imply that

$$\|\frac{s\,[(s+\alpha)^3 - \prod_{j=1}^{3}(s-p_j)]}{(s+\alpha)^3}\|$$

$$\le \|\frac{s}{s+\alpha}\| \, [\,\|\frac{s\,[(s+\alpha)^2 - \prod_{j=1}^{2}(s-p_j)]}{(s+\alpha)^2} + \alpha + p_3 \|\frac{\prod_{j=1}^{2}(s-p_j)]}{(s+\alpha)^2}\|\,]$$

$$= (2\alpha + p_1 + p_2 + \alpha + p_3)$$

and hence, (11) holds. Continuing similarly, suppose that (11) holds for r and show that it holds for $(r+1)$: Case (i): if at least one of the $\mathcal{U}$-poles of G is real, let $p_{(r+1)} \in \mathbb{R}_+$. Then

$\|\frac{\prod_{j=1}^{r}(s-p_j)]}{(s+\alpha)^r}\| = 1$ for $\alpha \geq |p_j|$ implies that

$$\|\frac{s\,[(s+\alpha)^{r+1} - \prod_{j=1}^{r+1}(s-p_j)]}{(s+\alpha)^{r+1}}\|$$

$$\leq \|\frac{s}{s+\alpha}\|\,[\,\|\frac{s\,[(s+\alpha)^{r} - \prod_{j=1}^{r}(s-p_j)]}{(s+\alpha)^{r}} + \alpha + p_{(r+1)}\|\frac{\prod_{j=1}^{r}(s-p_j)]}{(s+\alpha)^{r}}\|\,]$$

$$= (r\alpha + \sum_{j=1}^{r} p_j + \alpha + p_{(r+1)})$$

and hence, (11) holds. Case (ii): if none of the $\mathcal{U}$-poles of G is real, let $p_{(r+1)} = \bar{p}_r$ be the complex-conjugate pair of the pole $p_r \in \mathcal{U}$. Then $\|\frac{\prod_{j=1}^{r-1}(s-p_j)]}{(s+\alpha)^{r-1}}\| = 1$ and $\|\frac{\alpha(2s+\alpha)(s+\alpha)^r - p_r\bar{p}_r\prod_{j=1}^{r-1}(s-p_j)}{(s+\alpha)^r}\| = 2\alpha$ for $\alpha \geq |p_j|$ imply that

$$\|\frac{s\,[(s+\alpha)^{r+1} - \prod_{j=1}^{r+1}(s-p_j)]}{(s+\alpha)^{r+1}}\|$$

$$\leq \|\frac{s}{s+\alpha}\|\,[\,\|\frac{s}{(s+\alpha)}\frac{s[(s+\alpha)^{(r-1)} - \prod_{j=1}^{r-1}(s-p_j)]}{(s+\alpha)^{r-1}}\|$$

$$+\|\frac{\alpha(2s+\alpha)(s+\alpha)^r - p_r\bar{p}_r\prod_{j=1}^{r-1}(s-p_j)}{(s+\alpha)^r}\| + (p_r+\bar{p}_r)\|\frac{s}{s+\alpha}\|\|\frac{\prod_{j=1}^{r-1}(s-p_j)]}{(s+\alpha)^{r-1}}\|\,]$$

$$= (\,[r-1]\alpha + \sum_{j=1}^{r-1} p_j + 2\alpha + p_r + p_{(r+1)})$$

and hence, (11) holds.

Corollary 1. (Integral-action controller synthesis): Under the assumptions of Theorem 1, choose $\alpha \in \mathbb{R}_+$ satisfying (8) and

$$\alpha \;<\; \frac{1}{r+1}\;(\;\|\Phi\|^{-1} - \sum_{j=1}^{r} p_j\;)\;. \tag{12}$$

Then an integral-action controller that stabilizes G^{Λ} is given by

$$C_i = \frac{[\,(s+\alpha)^{r+1} - s\,\varphi\,]}{s\,\varphi}\,N(0)^{-1}D(s)\;. \tag{13}$$

Proof of Corollary 1. With $X := \frac{[\,(s+\alpha)^{r+1} - s\,\varphi\,]}{(s+\alpha)^{r+1}}\,N(0)^{-1} \in \mathcal{M}(\mathbf{S})$, and $Y := \frac{s\,\varphi}{(s+\alpha)^{r+1}}D^{-1} \in \mathcal{M}(\mathbf{S})$, $C_i = XY^{-1}$ is an RCF of the proposed controller in (13). By (4), C_i stabilizes G^{Λ} if and only if $M^{-1} \in \mathcal{M}(\mathcal{H}_\infty)$, where

$$M = D\,Y + N^{\Lambda}\,X = \frac{s\,\varphi}{(s+\alpha)^{r+1}}DD^{-1} + \frac{[\,(s+\alpha)^{r+1} - s\,\varphi\,]}{(s+\alpha)^{r+1}}N^{\Lambda}\,N(0)^{-1}$$

$$= I + \frac{s\,[\,(s+\alpha)^{r+1} - s\,\varphi\,]}{(s+\alpha)^{r+1}}\,\frac{[N^{\Lambda}\,N(0)^{-1} - I\,]}{s} = I + \frac{s\,[\,(s+\alpha)^{r+1} - s\,\varphi\,]}{(s+\alpha)^{r+1}}\,\Phi = I + \frac{s\,\tilde{d}}{(s+\alpha)^{r}}\,\Phi\;,$$

where $\tilde{d} := [\,(s+\alpha)^{r+1} - s\,\varphi\,]$. Under the condition $\alpha > |p_j|$, the norm $\|\, \frac{s\,\tilde{d}}{(s+\alpha)^{r+1}} \,\|$ is:

$$\|\, \frac{s\,\tilde{d}}{(s+\alpha)^{r+1}} \,\| = \|\, \frac{s\,[\,(s+\alpha)^{r+1} - s\,\prod\limits_{j=1}^{r}(s-p_j)\,]}{(s+\alpha)^{r+1}} \,\|\,(r+1)\alpha + \sum_{j=1}^{r} p_j \ . \tag{14}$$

Since α satisfies (12), $\|\, \frac{s\,\tilde{d}}{(s+\alpha)^{r+1}}\,\Phi \,\| \leq= [(r+1)\alpha + \sum\limits_{j=1}^{r} p_j\,]\|\Phi\,\| < 1$. Therefore, M is unimodular and hence, C_i stabilizes G^{Λ}.

Remark 1. a) The controller (10) proposed in Theorem 1 is strictly-proper since the polynomial term $d := [(s+\alpha)^r - \varphi]$ is of order $r-1$. Similarly, the integral-action controller (13) proposed in Corollary 1 is strictly-proper.

b) In the case that G^{Λ} is SISO, for any r-th order strictly-Hurwitz polynomial $\theta(s)$, the finite-dimensional part of G^{Λ} can be written as

$$G = \frac{\eta}{\gamma\varphi} = \left(\frac{\varphi}{\theta}\right)^{-1}\left(\frac{\eta}{\gamma\,\theta}\right) = D^{-1}N \ , \tag{15}$$

where γ is a strictly-Hurwitz polynomial of degree $\delta(\gamma)$, whose roots are the (finitely many) poles of G in the stable region $\mathbb{C}\setminus\mathcal{U}$. The order of G is $[r+\delta(\gamma)]$, and the numerator polynomial η is of order $\leq [r+\delta(\gamma)]$. Therefore, for the SISO case, the controller in (10) becomes

$$C = \frac{[\,(s+\alpha)^r - \varphi\,]}{\varphi}\,N(0)^{-1}D(s) = \gamma(0)\theta(0)\eta(0)^{-1}\,\frac{[\,(s+\alpha)^r - \varphi\,]}{\theta(s)} \ , \tag{16}$$

which is a stable controller of order r, the same as the number of $\mathcal{U}$-poles of G. Therefore, C in (16) strongly stabilizes SISO plant G^{Λ}.

For the case of SISO plants with time delays, the integral-action controller in (13) becomes

$$C_i = \gamma(0)\theta(0)\eta(0)^{-1}\,\frac{[\,(s+\alpha)^{r+1} - s\,\varphi\,]}{s\,\theta(s)} \ . \tag{17}$$

The controller in (17) is of order $r+1$, and has poles in the stable region $\mathbb{C}\setminus\mathcal{U}$ except for the pole at $s=0$ providing integral-action.

c) As in the SISO case, whenever the finite-dimensional part G of the MIMO plant G^{Λ} is such that an LCF $G = D^{-1}N$ is given by

$$D = \frac{\varphi}{\theta}I \ , \quad N = \frac{\varphi}{\theta}G \ ,$$

i.e., rank $\left(\frac{\varphi}{\theta}G\right)\big|_{s=p_j} = m$ for each $\mathcal{U}$-pole p_j of G, then the controller in (10) becomes

$$C = \frac{[\,(s+\alpha)^r - \varphi\,]}{\varphi}\,N(0)^{-1}D(s) = \frac{[\,(s+\alpha)^r - \varphi\,]}{\theta(s)}\,N(0)^{-1} \ , \tag{18}$$

which is a stable controller, whose poles are the r roots of the arbitrarily chosen strictly-Hurwitz polynomial $\theta(s)$. Delayed plants G^{Λ} in this class are therefore strongly stabilized by the controller in (18).

d) Any number of the $\mathcal{U}$-poles of G may be at the origin $s=0$. In fact, if $p_j = 0$ for $j = 1,\ldots,r$, then condition (7) is obviously satisfied since $0 < \|\Phi\|^{-1}$. In this case, choose $\alpha \in \mathbb{R}_+$ in (9) such that $\alpha < \frac{1}{r}\|\Phi\|^{-1}$.

e) In the MIMO case, the finite-dimensional part G of G^{Λ} may have coinciding poles and zeros (in the unstable region $\mathcal{U}$ as well as the stable region $\mathbb{C}\setminus\mathcal{U}$ of the complex plane). The only assumption is that G has no zeros at $s=0$.

f) If the finite-dimensional part G of G^{Λ} has only one $\mathcal{U}$-pole (although it may obviously have any number of poles in the open left-half complex plane), then the controller (10) and the integral-action controller in (13) become

$$C = \frac{[\,(s+\alpha)-(s-p_1)\,]}{s-p_1}\,N(0)^{-1}D(s) = \frac{(\,\alpha+p_1\,)}{s-p_1}\,N(0)^{-1}D(s)\ , \tag{19}$$

$$C_i = \frac{[\,(2\alpha+p_1)s+\alpha^2\,]}{s\,(s-p_1)}\,N(0)^{-1}D(s)\ . \tag{20}$$

Following Remark-(b), if G can be factorized as in (15), i.e., $D = \frac{(s-p_1)}{\theta}I$, where θ is a first-order strictly-Hurwitz polynomial, then the controller (19) and (20) become PD and PID controllers, respectively:

$$C = \frac{(\,\alpha+p_1\,)}{\theta}\,N(0)^{-1}\ , \tag{21}$$

$$C_i = \frac{[\,(2\alpha+p_1)s+\alpha^2\,]}{s\,\theta}\,N(0)^{-1}\ . \tag{22}$$

4. Conclusions

The main result (Theorem 1) presented a simple LTI controller synthesis method to achieve closed-loop stability for some classes of LTI, MIMO plants that are subject to time delays. The procedure is modified in Corollary 1 to include integral-action in the controller design so that asymptotic tracking of step-input references with zero steady-state error is also achieved in addition to closed-loop stability. The results apply to plants whose finite-dimensional part has restrictions on the poles in the unstable region of the complex plane. Performance implications for choices within the controller parameters can also be explored for specific applications of the synthesis methods presented here.

References

[1] Foias, C.C., Özbay, H., Tannenbaum, A. Robust control of infinite dimensional systems, LNCIS 209, Springer-Verlag, London, 1996.

[2] Francis, B.A., Wonham, W.A. The internal model principle for linear multivariable regulators, Applied Mathematics & Optimization, V.2, N.2, 1975, pp.170-195.

[3] Gu, K., Kharitonov, V.L., Chen, J. Stability of time-delay systems, Birkhäuser, Boston, 2003.

[4] Gündeş, A.N., Özbay, H., Özgüler, A.B. PID controller synthesis for a class of unstable MIMO plants with I/O delays, Automatica, V.43, N.1, 2007, pp.135-142.

[5] Larin, V.B. On stabilization of systems with delay, International Applied Mechanics, V.44, N.10, 2008, pp.1148-1160.

[6] Niculescu, S.-I. Delay effects on stability: a robust control approach, Springer-Verlag: Heidelberg, LNCIS, V.269, 2001.

[7] Silva, G.J., Datta, A., Bhattacharyya, S.P. PID controllers for time-delay systems, Birkhäuser, Boston, 2005.

[8] Skogestad, S. Simple analytic rules for model reduction and PID controller tuning, Journal of Process Control, V.13, 2003, pp.291-309.

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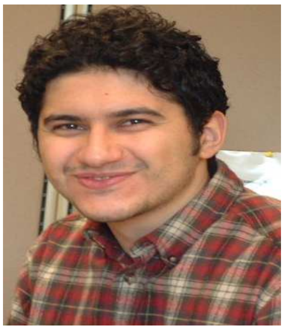

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