Suppression of Cochannel Interference in GSM by Pre-demodulation Signal Processing

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Abstract

It is shown that FRESH (frequency-shift) filtering prior to Viterbi demodulation in GSM basestations and/or handsets can multiply the number of cochannel signals that can be separated with M antennas by (2M-1)/(M-1): suppression of 1 interferer with one antenna, 3 interferers with a diversity antenna pair, and 2M-1 interferers with M antennas. The network-level simulations reported in this paper clearly demonstrate that FRESH filtering prior to demodulation in GSM basestations and/or handsets provides a cost-effective means of substantially improving system performance regardless of the antenna subsystem used. In all cases considered, single antenna, pair of diversity antennas, fully adaptive array of four omnidirectional antennas per 120° sector, and switched-beam array of four 30° antennas per 120° sector, FRESH filtering substantially reduces BER and FER (thereby improving quality) and/or reduces required SNR (thereby increasing range) and/or enables reduced frequency reuse factor (thereby increasing capacity).

1. Introduction

The primary alternatives for accomplishing cochannel interference (CCI) mitigation in FDMA/TDMA systems, such as GSM, are well known in the industry and consist of cell sectorization, dynamic channel assignment, frequency hopping, and smart antennas. FRESH filtering is a new alternative that is compatible with each and every one of the conventional alternatives, and that can enhance the interference mitigation capability in all cases for GSM. Moreover, when FRESH filtering is used in both the basestation (uplink) and handset (downlink) for GSM, then system capacity can be as much as doubled. Furthermore, capacity can be increased incrementally as the number of handsets and basestation channels are upgraded with FRESH filters. This is illustrated in Figure 1, which is the composite result of extensive network-level simulations, some of which are reported in this paper. Unlike the better-known alternatives for CCI mitigation, the introduction of FRESH filtering into a system requires only a DSP modification to the demodulator.

![Graph](image_url)

Figure 1. Incremental increase in capacity with gradual deployment of FRESH filters in GSM.

The new technique for cochannel interference suppression in GSM presented in this paper is based on FRESH (frequency-shift) filtering and the concept of spectral redundancy [1]-[4]. Because of the high correlation between a complex-valued GMSK signal and frequency-shifted and conjugated versions of that signal, three such versions can be individually convolved with appropriate complex-valued FIR filter
weights and then added together such that one of multiple interfering GMSK signals in these three versions adds constructively, and the remaining interfering GMSK signals in these three versions add destructively. The particular signal selected is determined by the training signal used in the training-assisted property-restoral algorithm for adaptive adjustment of the FIR weights. By adding the outputs of M of these three-path structures, one per antenna for an M-antenna receiving system, M times as many interfering signals can be separated by joint adaptation [5],[7]. Thus, for a two-antenna diversity receiver, the technique can ideally remove three interfering cochannel GMSK signals perfectly, and can potentially suppress more than three. The spectral redundancy property that is exploited in this manner is a characteristic of the particular conjugate cyclostationarity property of GMSK signals. The particular implementation of this filtering structure, called an LCL filter, that is reported on here has low computational cost and can be fully re-adapted in every GSM time slot in order to track nonstationarities. No auxiliary algorithms (other than those present in commercial GSM receivers) are required to operate the adaptive LCL filter.

2. Applications

The “CCI-hardening” in the basestation for uplink channels, and in the handset for downlink channels resulting from FRESH filtering, provides the network designer with greater flexibility in setting frequency reuse distances and transmitted power levels that can enable improvement in capacity, coverage, and quality of service, at minimal cost.

The capacity of GSM networks can be substantially increased by making only one modification to the physical layer: incorporation of LCL filtering in the baseband DSP in basestations and handsets. By using LCL filtering to harden the network to CCI, smaller frequency reuse distances can be used, thereby greatly increasing spectral efficiency. The capacity of a network can be increased gradually, as demand requires, by providing CCI protection with LCL filtering, enabling frequency reuse improvement on a channel-by-channel basis. The simulations to be reported in this paper show that in CCI-limited environments, LCL filtering decreases frame-error rates by as much as a factor of ten with dual-antenna reception compared with conventional diversity combining. As shown in Figure 1, increases in the number of subscribers per cell that can be served are essentially linear in the number of baseband channels and corresponding number of handsets that are CCI hardened (and used with a frequency reuse factor of 4 instead of 7), with an ultimate doubling of capacity when all channels are CCI hardened.

LCL filtering can substantially improve grade of service also. When cochannel interference would cause reception to be marginal or unacceptable, resulting in calls being dropped or blocked, LCL filtering, through frame-error-rate reduction, will increase voice fidelity by decreasing lost speech frames; and decrease probability of dropped calls; and decrease probability of blocked calls. CCI hardening also can enable the use of softer cell boundaries, and this has two beneficial effects: handoffs become less urgent and less frequent; and requirements on spot beams for satellite service can be relaxed, thereby reducing cost and power consumption of phased arrays.

The FRESH filter is quite versatile. It can provide performance improvements for any cellular or PCS wireless communication system using MSK (minimum-shift keying) modulation. This presently includes GSM900, DSC1800, PCS1900, IS661, DECT, and GPRS systems, all of which use GSM signaling; and Mobitex, and some ETS 300 220 and ETS 300 113 systems, which use GMSK modulation. Moreover, FRESH filtering can be used with existing antennas and RF hardware to convert 2-element switched diversity, by DSP replacement only, into a system that performs as well as if it were a 4-element smart-antenna receiving system. FRESH filtering also can be used with smart antenna systems to substantially increase system capacity. It can be used with smart-antenna systems including fully-adaptive totally overlapping beams at one extreme and essentially non-overlapping fixed beams used in switched-beam systems at the other extreme, and an arbitrary number of antenna elements.

3. FRESH Filter Structures

We shall denote the discrete-time complex envelope of data received at an M-sensor array by the $M \times 1$ vector $\mathbf{x}(t)$. It is assumed that the
received data is a linear superposition of uncorrelated GMSK signals, each convolved with the impulse response of a corresponding linear time-invariant (LTI) spatio-temporal channel, and each containing additive stationary noise. Since the filters considered in this paper are re-adapted in every GSM time slot, time-invariance and stationarity are required over only the length of a time slot, ½ ms. The GMSK signals share a common bit rate \( f_b = 1/T \), where \( T \geq 1 \) samples per bit are collected, and have zero nominal carrier offset.

We first consider filters that operate on over-sampled data \( T > 1 \) to produce over-sampled estimates of the GMSK signals. We then consider filters that operate on over-sampled data to produce bit-rate-sampled estimates, and we finally consider filters that operate on bit-rate-sampled data \( T = 1 \) to produce bit-rate-sampled estimates.

In the following, the filters are described for the case in which only one of the multiple GMSK signals is to be estimated. That is, the filter output \( \hat{s}(t) \) is a scalar-valued signal. It is understood that the extension to multiple desired signals is trivial, consisting of simply increasing the number of columns, one per desired signal, in the matrix \( w \) of filter coefficients that is defined for each filter and denoting the vector-valued filter output by \( \hat{\mathbf{s}}(t) \).

Also, the various LTI filters that comprise each FRESH filter or modification thereof are finite impulse response (FIR) filters. To facilitate the descriptions of these filters, we denote the output of a tapped delay line (TDL) having memory length \( L - 1 \) samples (i.e., there are \( L \) outputs) and being driven by input \( x(t) \) by the \( LM \times 1 \) vector

\[
\text{TDL}_L \{ x(t) \} = \begin{bmatrix} x(t) \\ x(t-1) \\ \vdots \\ x(t-L+1) \end{bmatrix} \tag{1}
\]

Also, we denote the collection of \( M \times 1 \) vectors of weights in an \( L \)-sample FIR filter \( h(0), h(1), \ldots, h(L-1) \), by the \( LM \times 1 \) vector \( h(0:L-1) \).

The filters presented here are eventually expressed in terms of a time-invariant linear combiner. That is, the filter output \( \hat{s}(t) \) or \( \hat{s}(nT) \) is expressed as

\[
\hat{s}(t) = \mathbf{w}^H y(t), \quad \text{or} \\
\hat{s}(nT) = \mathbf{w}^H y(nT), \quad \text{or} \\
\hat{s}(nT) = \mathbf{w}^H y(n).
\]

where \( y \) is an appropriate function of the received data \( x(t) \). That is, the definition of \( y \) defines the filter structure.

### 3-Path LCL-FRESH Filter

As explained in Section 1, the GMSK signals encountered in GSM exhibit substantial spectral redundancy as a result of their conjugate cyclostationarity [2], [3]. In particular, the signal, when conjugated and frequency-shifted down by half the bit rate, is nearly perfectly correlated with the original signal throughout the lower half of the original signal band. Similarly, the signal, when conjugated and frequency-shifted up by half the bit rate, is nearly perfectly correlated with the original signal throughout the upper half of the band. It follows from this that the bit-rate sampled signal also exhibits substantial spectral redundancy. That is, \( x(n) \bar{x}^* \) and its conjugate are nearly perfectly correlated throughout the entire band of width equal to the bit rate. In other words, the magnitude of the cross coherence function [2] for these two signals is nearly unity, as shown in Figure 2.

![Figure 2](image-url)
As a result of this spectral redundancy, the signal can be linearly combined with its frequency-shifted conjugated versions at each frequency and, depending on the values of the complex-valued frequency-dependent weights used, the signal can be either amplified by two, or cancelled. By this means, two GSM signals added together can be separated by canceling one but not the other (assuming the signals have either distinct carrier phases or bit phases [5]-[7]). This observation leads to the three-path frequency-shift linear-conjugate-linear (LCL) filter described as follows. The filter output, which is the desired-signal estimate, is given by

\[
\hat{s}(t) = \sum_{l=0}^{L-1} h(l)^H x(t-l) \\
+ \sum_{l=0}^{L-1} g_+(l)^H x^*(t-l) \exp[j\pi f_b (t-l)] \\
+ \sum_{l=0}^{L-1} g_-(l)^H x^*(t-l) \exp[-j\pi f_b (t-l)] \\
= w^H y(t)
\]  

(3)

with the \(3LM \times 1\) vectors defined by

\[
y(t) = \begin{bmatrix}
\text{TDL}_L \{x(t)\} \\
\text{TDL}_L \{x(t)^* \exp(j\pi f_b t)\} \\
\text{TDL}_L \{x(t)^* \exp(-j\pi f_b t)\}
\end{bmatrix}
\]

(4a)

\[
w = \begin{bmatrix}
h(0:L-1) \\
g_+(0:L-1) \\
g_-(0:L-1)
\end{bmatrix}
\]

(4b)

This 3-path LCL-FRESH filter is shown in Fig. 3.

2-path LCL-FRESH-FSE

By observing \(\exp(-j\pi f_b (nT-l)) = \exp(j\pi f_b (nT-l)) \exp(j2\pi f_b l)\), we see that (5) can be re-expressed as

\[
\hat{s}(nT) = \sum_{l=0}^{L-1} h(l)^H x(nT-l) \\
+ \sum_{l=0}^{L-1} g_+(l)^H x^*(nT-l) \exp[j\pi f_b (nT-l)] \\
+ \sum_{l=0}^{L-1} g_-(l)^H x^*(nT-l) \exp[-j\pi f_b (nT-l)] \\
= w^H y(nT)
\]

(6)

where \(g(l) = g_+(l) + g_-(l) \exp(-j2\pi f_b l)\) and called the 3-path LCL-FRESH fractionally-spaced equalizer (FSE). That is, the output \(\hat{s}(nT)\) of this filter can be expressed as

\[
\hat{s}(nT) = \sum_{l=0}^{L-1} h(l)^H x(nT-l) \\
+ \sum_{l=0}^{L-1} g_+(l)^H x^*(nT-l) \exp[j\pi f_b (nT-l)] \\
+ \sum_{l=0}^{L-1} g_-(l)^H x^*(nT-l) \exp[-j\pi f_b (nT-l)] \\
= w^H y(nT)
\]
This implementation is easily observed to be more computationally efficient than the 3-path LCL-FRESH-FSE because the 2-path LCL-FRESH-FSE (6) uses only $2L$ complex weights instead of $3L$, with no loss in capability.

In the typical case for which $T = 2$, $y(t)$ can be expressed without use of complex sinusoids,

$$y(t) = \begin{bmatrix} \text{TDL}_L \{ x(t) \} \\ \text{TDL}_L \{ x^*(t) \} \end{bmatrix}$$

(7a)

$$w = \begin{bmatrix} h(0: L - 1) \\ g(0: L - 1) \end{bmatrix}$$

(7b)

This leads to

$$\hat{s}(n) = \sum_{l=0}^{L-1} h(l)^H x(n - l) + \sum_{l=0}^{L-1} g(l)^H [x^*(n - l)(-1)^{n-l}] = w^H y(n)$$

(9)

where $g(l) = g_+(l) + g_-(l)(-1)^l$ and

$$\begin{bmatrix} TDL_L \{ x(n) \} \\ \text{TDL}_L \{ x^*(n)(-1)^n \} \end{bmatrix}$$

(10a)

$$w = \begin{bmatrix} h(0: L - 1) \\ g(0: L - 1) \end{bmatrix}$$

(10b)

This 2-path LCL-FRESH filter is shown in Figure 5. This is the particular FRESH filter that was simulated to obtain the results reported in Section 5.

**Figure 3.** Block diagram of 3-path LCL-FRESH filter.
4. Adaptation of FRESH Filter

There are two properties of GSM signals that can be exploited for adaptive adjustment of the $2L$ FIR weights in the LCL FRESH filter shown in Figure 5: (i) the 26 known midamble bits present in each 157.25-bit normal burst that fills one GSM time slot lasting 577 microsecs, and (ii) the constant modulus property of the GMSK signal, which is a digital FM signal with no amplitude modulation. One could ignore (i) and exploit (ii) using, for example, an iterative block least square constant modulus algorithm for blind adaptation. Or, one could ignore (ii) and exploit (i) using block least squares training over the midamble. However, the complementary advantages of these two conventional algorithms can both be realized by exploiting both these properties simultaneously using a training-assisted iterative-block-least-squares constant-modulus (TA-IBLS-CM) algorithm. (A similar approach for another application is investigated in [6].) In particular, the TA-IBLS-CM algorithm minimizes the following linear combination of sums of squared errors with respect to the vector $w_k$ of $2L$ FIR weights at iteration $k$:

$$
\gamma \left( \left\| \begin{bmatrix} w_k^H & y(t) - z_{k-1}(t) \end{bmatrix} \right\|^2_{\text{re} T_i} \right) + (1 - \gamma) \left( \left\| \begin{bmatrix} w_k^H & y(t) - s_c(t) \end{bmatrix} \right\|^2_{\text{re} T_o} \right)
$$

(11)
where \( z_{k-1}(t) = \hat{s}_{k-1}(t)/|\hat{s}_{k-1}(t)| \) is the modulus-normalized version of the signal estimate \( \hat{s}_{k-1} = w_{k-1}^H y(t) \). \( T_0 \) is the time interval occupied by the embedded 26-bit training signal \( \hat{s}_0(t) \) with unity modulus, and \( T_1 \) is the time period over which the departure from modulus constancy of the filter output from the previous iteration is measured. The interval \( T_1 \) can be either the entire time slot, or the time slot excluding the training signal period \( T_0 \). Either way, the closed-form solution is given by
\[
 w_k = \left[ \gamma_k R_y^{(T_0)} + (1 - \gamma_k) R_{yy}^{(T_0)} \right]^{-1} \left[ \gamma_k R_{ys}^{(T_0)} + (1 - \gamma_k) R_{yy}^{(T_0)} \right] \tag{12}
\]
where the matrix inverse need be computed only once (prior to or as part of the first iteration). The linear combiner weight \( \gamma_k \) takes on values in \([0,1]\), and controls the relative influence of the two cost functions and thereby controls the emphasis given to the two distinct types of knowledge used in them. Values of \( \gamma_k \) closer to unity give more influence to the exploitation of the CM property, whereas values closer to zero give more influence to the exploitation of the known training signal.

In (12), we have used the notation
\[
 R_{ab}^{(T)} = \left\langle a(t) b^H(t) \right\rangle_T
\]
for correlation matrices, where \( T \) denotes the interval over which the time-averaging operation is carried out.

Once the TA-IBLS-CM algorithm has been iterated enough for the FRESH filter to suppress CCI well enough that the BER at the output of the Viterbi demodulator is low enough, then adaptation of the FRESH filter can be switched to decision-direction using the bits decided by the Viterbi algorithm to obtain convergence to the lowest attainable BER.

A special case of the TA-IBLS-CM algorithm arises when the initial condition is zero (i.e., \( w_0 = 0 \), \( \gamma_1 = 0 \), and \( \gamma_k = 1 \) for \( k > 1 \)). In this case, we have an algorithm in which the known training signal is used to initialize the conventional CM algorithm at iteration \( k = 2 \) with
\[
 w_1 = \left[ R_y^{(T_0)} \right]^{-1} R_{y_0}^{(T_0)}.
\]

5. Simulations

One and two antennas.

Network-level simulations were conducted to evaluate performance improvement provided by FRESH filtering in terms of signal-to-interference-and-noise ratio (SINR) just prior to bit decision, and frame error rate (FER) following bit decision, de-interleaving, and convolutional decoding. These evaluations were performed for FRESH filtering in handsets and in basestation receivers with and without diversity antennas. The number of weights used in each of the two FIR filters in the FRESH filter shown in Figure 5 was \( L = 5 \), which was found to be optimum for the environments simulated and for the TA-IBLS-CM adaptation algorithm used. The number of iterations of the adaptation algorithm was 10, although as few as 5 can result in convergence to the best attainable performance and even only one iteration can provide good performance. The following simulated signal environments replicate conditions found in networks operating with reuse distances of 3, 4, and 7.

- Uniform hexagonal cells
- Cell-centered base stations with two 120-degree antennas per sector (Celwave PD10188: 3dB-BW = 120º; 10dB-BW = 200º, F/B ratio = 20dB)
- Diversity antennas at base stations separated by 10 wavelengths
- Each handset uses a single omnidirectional antenna
- Traffic loads in cochannel cells range from lightly to fully loaded
- Random slot-timing offset among cells (asynchronous cells): both adjacent slots of CCI are active (worst case)
- Propagation power loss: (range)3.8
- Log-normal shadowing: \( \sigma = 8\)dB for CCI, \( \sigma = 2\)dB for SOI
- Locations of mobile units are drawn at random (uniform distribution over each cell)
- Temporal channel: ETSI/GSM TU50 (12-path profile)
- Average input SNR: \( E_b/N_0 = 15\)dB
• Spatial channel: 20° angular spread for cell of interest, 10° for first-tier cells

The simulations focus on traffic channels. Beacon channels/BCCH (C0,T0) are assumed to be satisfactorily received at the mobile units with the same frequency reuse as used for traffic channels (if not, a larger reuse factor can be used for C0). Mobiles’ call initiation channels/RACH are assumed to be satisfactorily received at the base station with the same frequency reuse as used for traffic channels (if not, a larger reuse factor can be used for RACH). RACH uses a shorter burst and a longer training sequence, so a larger reuse factor is not likely to be needed.

The Erlang-B formula is used to determine the average traffic offered ($A$, Erlangs) for a given number $n$ of time/frequency channels in a cell such that the blocking probability $E(n,A) \leq 1\%$:

$$E(n, A) = \frac{A^n}{n!} \sum_{m=0}^{n} \frac{A^m}{m!} \leq 1\%$$

The probability $p = A/n$ of any particular time/frequency channel being active is then used to determine the probability that $k$ out of 6 co-channel cells are active in this time/frequency channel.

$$P_k = \text{prob}\{k \text{ CCI cells active}\} = \binom{6}{k} p^k (1 - p)^{6-k}$$

for $k = 0, ..., 6$.

The results of Monte Carlo trials for each $k$ are then combined; e.g.,

$$\text{mean}\{\text{BER}\} = \sum_{k=0}^{6} P_k \text{ mean}\{\text{BER}\} | k \text{ CCI cells active}\}$$

The results of the simulations are shown in the FER vs SINR graphs in Figures 6 & 7. It can be seen that when voice quality and reliability would be marginally acceptable (input SINR < 9dB and FER > 7%), the FRESH filter in handsets increases SINR by 2.5dB to 3dB. In similar environments, FRESH filtering in base stations increases SINR by 2.5dB to 3.5dB relative to switched diversity processing.
In especially harsh environments (FER > 15%), FRESH filtering in base stations and handsets decreases FER by a factor of 3 and, with a pair of diversity antennas, decreases FER by a factor of 10, relative to switched diversity.

As a result of the reduction of FER due to CCI suppression, the increases in the number of subscribers per cell that can be served are essentially linear in the number of baseband channels and corresponding number of handsets that are equipped with the FRESH filter (and used with a frequency reuse factor of 4 instead of 7), with an ultimate doubling of capacity. This is illustrated in Figure 1 with the performance results obtained from realistic network-level simulations for the case in which two antennas are used at the basestations and the spectrum allocation is 15MHz. Not only is the capacity doubled, but also the FER is reduced from 3.6% to 2.2%.

More than two antennas.

To further illustrate the advantages of FRESH filtering, we consider their use for enhancing the capabilities of antenna arrays, including both fully-adaptive omnidirectional antennas and switched-beam antennas. The environments simulated are described as follows:

- Four 120° antennas (Celwave PD10188) per sector separated vertically by two wavelengths (1 foot at 1900MHz) for “fully adaptive” case
- Four 30° antennas (Celwave PD10272) with 3-dB BW = 30°; 10dB-BW = 52°; F/B ratio > 40dB) per sector spanning 120°, vertically separated by 2 wavelengths, for the “switched beam” case
- No Log-normal fading
- Network loading is adjusted for a blocking probability of 1%
- $E_b/N_0$ = 8dB or 20dB

For the fully-adaptive array, we use a four-input FRESH filter. For the switched-beam array, we use either a single-input FRESH filter on the best one of four beams or a two-input FRESH filter on the best two of four beams. We compare the performances of these FRESH-filter-enhanced systems with their conventional counterparts. We define signal-to-interference ratio (SIR) reliability to be the probability that SIR ≥ 9dB, and we define BER reliability to be the probability that BER ≤ 3%. We also select 95% to be the desired minimum acceptable value of reliability.

The results are shown in the graphs of reliability vs frequency reuse depicted in Figures 8-14, wherein the dashed horizontal line represents 95% reliability. Reliability curves are shown for ideal adaptation using MMSE linear combiner weights and for practical adaptation using the 26-bit midamble in the TA-IBLS-CM algorithm. The processors evaluated include the multi-input FRESH filter, a linear time-invariant (LTI) spatio-temporal filter (STF), and a memoryless spatial filter.

It can be seen from Figures 8-10 that FRESH filtering applied to a four-element fully-adaptive array can allow a frequency reuse factor of $K = 4$ to be used instead of $K ≥ 7$; and that further improvement, to $K = 3$, might be possible through algorithm refinement and tuning. This reduction in the frequency reuse factor from $K = 7$ to $K = 4$ (or lower) can double (or more than double) the number of subscribers through the increased number of physical channels made available and the improvements in trunking efficiency arising from the increased numbers of channels. In other words, FRESH filtering with a fully-adaptive array significantly improves the bottom-line benefit derived from a given number of antennas.

From Figures 11-14, it can be seen that FRESH filtering applied to the best 1 or 2 out of 4 beams in a switched-beam array can allow a frequency reuse factor of $K = 4$ to be used instead of $K = 7$, and that further improvement, to $K = 3$, might be possible through algorithm refinement and tuning. This reduction in the frequency reuse factor from $K = 7$ to $K = 4$ (or lower) can double (or more than double) the number of subscribers through the increased number of physical channels made available, and the improvements in trunking efficiency arising from the increased numbers of channels. In other words, FRESH filtering with a switched-beam array significantly improves the bottom-line benefit derived from a given number of antennas.
6. Implementation

To illustrate the modesty of the system requirements imposed by the adaptive FRESH filter, we present the following results on computational cost and memory requirement. For one antenna, the approximate minimum number of clock cycles (on an Analog Devices 210xx or similar DSP chip) needed to process a single time slot can be as low as 22,000. Clock speeds and memory required are as follows:

- Base station (real-time processing of all 8 time slots): requires 38 MHz clock speed, 1 k-word of data memory, and 1 to 2 k-words of program memory.
- Handset (processing delay of 4 time slots): requires 10 MHz clock speed, and same memory requirements as for base station. If more cycles are available (e.g., up to 93,000/slot), then FER can be further reduced by 1 to 1.5%, as shown in the performance graphs in Section 5.

7. Conclusions

In conclusion the network-level simulations reported in this paper clearly demonstrate that FRESH filtering prior to demodulation in GSM basestations and/or handsets provides a cost-effective means of substantially improving system performance regardless of the antenna subsystem used. In all cases considered, single antenna, pair of diversity antennas, fully adaptive array of four omnidirectional antennas per 120° sector, and switched-beam array of four 30° antennas per 120° sector, FRESH filtering substantially reduces BER and FER (thereby improving quality) and/or reduces required SNR (thereby increasing range) and/or enables reduced frequency reuse factor (thereby increasing capacity).

8. Acknowledgment

The author acknowledges the contributions of S.V. Schell to 1) the software development and experimental design for the simulations and 2) the verification of all analytical results (by independent derivation).

9. References


Figure 8. SIR performance for $E_b/N_0 = 8$ dB with four omnidirectional antennas.
**Figure 9.** BER performance for $E_b/N_0 = 8$ dB with four omnidirectional antennas.

**Figure 10.** BER performance for $E_b/N_0 = 20$ dB with four omnidirectional antennas.

**Figure 11.** SIR performance for $E_b/N_0 = 8$ dB with four switched-beam antennas.

**Figure 12.** SIR performance for $E_b/N_0 = 20$ dB with four switched-beam antennas.

**Figure 13.** BER performance for $E_b/N_0 = 8$ dB with four switched-beam antennas.
Figure 14. BER performance for $E_b/N_0 = 20$ dB with four switched-beam antennas.