Review

Minterm/Maxterm Expansions

Each minterm in expression forces output to 1 (specific Sum of Products).

Each maxterm in expression forces output to 0 (specific Product of Sums).

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Minterms: \( Z = \Sigma m(3) = xy \)

Maxterms: \( Z = \Pi M(0,1,2) = (x+y)(x+y')(x'+y) \)

Karnaugh Maps

K-map is an alternate way of representing a truth table.

Rules for combining groups:
1) Groups must contain 1, 2, 4, 8, \( \ldots \), \( 2^n \) minterms

2) Except for Smallest/largest group cases, a group can be split/joined with \( xy + xy' = x \)

<table>
<thead>
<tr>
<th>K-map Size (# of variables)</th>
<th>Group Size/# of literals in term</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1/2, 2/1</td>
</tr>
<tr>
<td>3</td>
<td>1/3, 2/2, 4/1</td>
</tr>
<tr>
<td>4</td>
<td>1/4, 2/3, 4/2, 8/1</td>
</tr>
<tr>
<td>5</td>
<td>1/5, 2/4, 4/3, 8/2, 16/1</td>
</tr>
</tbody>
</table>

Ex:

\[ AB \]

\[ \begin{array}{cc|cc|c}
 A & B & C & & \Rightarrow C \\
 0 & 0 & 0 & 1 & \Rightarrow 0/1, 0/1, 1 \\
 0 & 1 & 0 & 1 & \Rightarrow 1/1, 1/1, 0 \\
 1 & 1 & 1 & 1 & \Rightarrow 0/0, 0/0, 1 \\
 1 & 0 & 0 & 1 & \Rightarrow 1/1, 1/1, 0 \\
\end{array} \]

\( A \overline{B} C \Rightarrow A'B \) is 1 with either \( C=0 \) or \( C=1 \)

\( ABC \Rightarrow AB'C' \); minterm is 1 at this combination of inputs only.
Some theorems with Karnaugh Maps:

1. \( XY + XY' = X \)

2. \( X + XY = X \)

3. \( XY' + Y = X + Y \)

4. Consensus:
   \( XY + YZ + X'Z = XY + X'Z \)

   \( YZ \) consensus term is redundant
4 Variable Karnaugh Maps

Ex: What not to do:

\[
\begin{array}{c|c|c|c}
A & B & CD & F \\
\hline
00 & 01 & 11 & 10 \\
00 & | & | & | \\
01 & | & | & | \\
11 & | & | & | \\
10 & | & | & | \\
\end{array}
\]

No
No (not 2')

Ex:

\[
\begin{array}{c|c|c|c}
W & X & Y & F \\
\hline
00 & 01 & 11 & 10 \\
00 & | & | & | \\
01 & | & | & | \\
11 & | & | & | \\
10 & | & | & | \\
\end{array}
\]

Given a valid group, we can find expressions:

\[
WXY = W'Z \quad \text{Group of 4 1's has 2 literals.}
\]

\[
WXY \quad \text{1/0/1} \quad \Rightarrow \quad WZ
\]

Ex: \(AB\)

\[
\begin{array}{c|c|c|c}
A & B & CD & F \\
\hline
00 & 01 & 11 & 10 \\
00 & | & | & | \\
01 & | & | & | \\
11 & | & | & | \\
10 & | & | & | \\
\end{array}
\]

How many groups of 2 contain the minterm \(m_{12}\)?

\[
m_{12} = ABC'D' \quad \text{is contained by}
\]

1) \(BC'D'\)
2) \(ABD'\)
3) \(AC'D'\)
4) \(ABC'\)

\[\text{Answer} = 4\]

K-Map Hint: certain regions correspond to literals = 1

\[
\begin{array}{c|c|c|c}
A & B & CD & F \\
\hline
00 & 01 & 11 & 10 \\
00 & | & | & | \\
01 & | & | & | \\
11 & | & | & | \\
10 & | & | & | \\
\end{array}
\]

\[\text{A = 1, B = 1, C = 1, D = 1}\]