3.2 XOR/XNOR Operations

Basic Properties:

\[ X \oplus 0 = X \]
\[ X \oplus 1 = X' \]

\[ X \oplus Y = X'Y + XY' \]

The symbol we use for XNOR (Equivalence) is: \( \equiv \)

\[ X \equiv 0 \iff (X \oplus 0)' = X' \]
\[ X \equiv 1 \iff (X \oplus 1)' = X'' = X \]

Ex: "Selectable inversion"

\[ \begin{array}{cc}
X \rightarrow & Z \\
\text{Control} & 0 \\
& X \\
& 1 \\
& X'
\end{array} \]

Ex: Three (or more) inputs:

\[ A \oplus B \oplus C = -(A \oplus B) \oplus C \quad \text{Associative Law} \]

\[ (A \oplus B) \oplus C = (AB' + A'B) \oplus C = AB'C' + A'BC' + A'B'C + ABC \]

\Rightarrow \text{odd # of 1's}

Converting expressions from/to Sum-of-Products/Product-of-Sums

1) POS \( \rightarrow \) SOP: Multiply out

Ex: \((A+B)(C+D) = AC + AD + BC + BD\)

You can always "multiply" out all terms, but the resulting expression can get out of hand.

Ex: \((A+B+C+D)(E+F+G)(H+I+J) = AEH + AEI + AEJ + AFH + \ldots\)

\[ = 4 \times 3 \times 3 = 36 \text{ terms!} \]

To simplify expressions, use theorems and laws to reduce terms as early as possible.

2) SOP \( \rightarrow \) POS: Factor

The following theorem is very helpful: \((X+Y)(X'+Z) = XZ + X'Y\)

Ex: \(X + YZ = (X+Y)(X+Z)\)

\[ A + B + (C+D)E = (A + B + C + D)(A + B + E) \]

\[ \frac{X}{X} \quad \frac{Y}{Y} \quad \frac{Z}{Z} \]
Clearing up SOP expressions, it's helpful to use \( X + XY = X \).

In English, \( XY \) is redundant because if \( XY \) is true, \( X \) must already be true (\( Y \) is also true), so \( XY \) is a subset of \( X \).

Venn diagram:

```
  X
  |
  |
  |
  |
  |
  |
  |
  |
  |
```

**Consensus Theorem**

\[
XY + X'Z + YZ = XY + X'Z
\]

\[\uparrow\text{ consensus term}\]

**Dual form:**

\[
(x+y)(x'+z)(y+z) = (x+y)(x'+z)
\]

\[\uparrow\text{ consensus term}\]

The consensus term is redundant and can be eliminated.

Look for a literal and its inverse in two product terms. The product of the remaining literals is the consensus term.

**Ex:** \( ABD + \overline{D'C} + A\overline{BC} = ABD + \overline{D'C} \)

\[\uparrow\text{ consensus term: } ABC\]

Verify theorem using truth table:

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y )</th>
<th>( Z )</th>
<th>( XY + X'Z )</th>
<th>( YZ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Simplifying expressions step by step:

1. Clearly understand what the problem is asking.

2. Combine terms using laws and theorems.

   **Ex:** \((A+B)C + C'(A+B) = (A+B)(C+C') = A+B\)

   \[AB + CD + AB = AB + CD\]

3. Eliminate terms, e.g. by using the Consensus Theorem.

   **Ex:** \(A'C + A'CD = A'C\) (uses \( X + XY = X \))

4. Eliminate literals (smaller # of terms in expression).

   **Ex:** \(A'CD + (A + C' + D')B = A'CD + (A'CD)'B \)

   \[
   = A'CD + B
   \]

   (by DeMorgan)

   (by \( X + X'Y = X + Y \))
4) Adding redundant terms.
   Ex: \( AB + A'C + BCD = AB + A'C + BC + BCD = AB + A'C + BC(1+D) \)
   \[ \uparrow \text{ consensus term} \]
   \[ = AB + A'C + BC \]

Practice: Unit 3 Programmed Exercises