2.7 Multiplying Out and Factoring

Distributive laws can be used to multiply out Boolean expressions into a special form called sum-of-products (SOP).

Any literals

\[ A + BC' + D'E + F'G'H' \]

Each product term is a product of only single variables.

Ex: \( (A + C + D)B + BE' \) Not SOP

\( L \) not a single variable

We can convert any expression to sum-of-products form:

Ex:

\[ z = (A \cdot B) + (C + D) \]

\[ = AB + C + D \checkmark \]

Ex:

\[ z = (A+B) \cdot (C \cdot D) \]

\[ = ACD + BCD \checkmark \]

Distributive laws can also be used to factor Boolean expressions into product-of-sums (POS) form.

Any literals

\[ (A + C + D) \cdot B \cdot (E' + F) \] POS

\[ (A + C)B + (E' + F) \] Not POS
2.8 Demorgan's Laws

\[(X + Y)' = X'Y'\]

\[(XY)' = X' + Y'\]

**EX:** \[(C + (A + B'))' = C' \cdot (A + B')'\]

**Duality** To obtain the dual of an expression:

- AND \(\rightarrow\) OR
- OR \(\rightarrow\) AND
- 0 \(\rightarrow\) 1
- 1 \(\rightarrow\) 0

 literals remain unchanged. If the original expression is true, so is its dual.

**Ex:** \[X + 0 = X \quad \rightarrow \quad X \cdot 1 = X\] Dual is also true (theorems with constant 1 or 0).

**Ex:** If \(A = B + DE'\) is true, then \(A = B \cdot (D + E')\) is also true.

**Ex:** Dual of \((X'YZ)' + W'V = (X'YZ)' + (W'V) = (X' + Y + Z)' \cdot (W' + V)\)

**Ex:** Find dual of \((X + Y)(X' + Z) = XY + X'Z\)

**Ex:** Re-forming circuits using \((X')' = X\)

\[= \quad = \quad = \quad = \]

**Ex:**

\[= \quad = \quad = \quad = \quad \text{Demorgan} \]
Ex: Sum-of-Products example: converting to NAND/NOR

\[
\begin{align*}
A & \quad = \quad \text{NAND} \\
B & \quad = \quad \text{NAND} \\
C & \quad = \quad \text{NAND} \\
D & \quad = \quad \text{NAND}
\end{align*}
\]

Ex: Write equation for the following circuit:

\[
K = [(A \cdot A') + B]' \\
Z = [(A \cdot A') + B]' \cdot (B \oplus C)'
\]

\[
= [(A \cdot A') + B]' \cdot (B \oplus C)'
\]

\[
= [B' \cdot (B \oplus C)]' + B + (B \oplus C)'
\]