6. Quine - McCluskey Method

Karnaugh Maps work well for up to 4 (possibly 5) input variables.

Quine - McCluskey Method allows us to work with more input variables. The method forms the basis for other CAD tool logic minimization algorithms.

Recall: To find a minimum sum-of-products expression, we need to

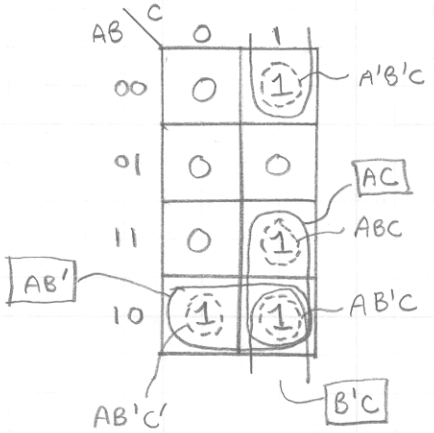
1. Find all prime implicants.
2. Find all essential prime implicants.
3. Minimum solution has all essential prime implicants.

Caveats: None/some/all prime implicants if needed to cover all 1's. It may take some work to find all prime implicants that cover all 1's and make the minimum solution. There may be more than 1 minimum solution.

Quine-McCluskey Method has two steps:
1. Find prime implicants.
2. Use a prime implicant chart to select a minimum set of prime implicants.

1. (a) Start with minterm expansion (list of implicants).
   (b) Combine as much as possible to yield prime implicants.

Ex: Using Karnaugh maps, simplify $Z = \sum m(1,4,5,7) = A'B'C + AB'C' + AB'C + ABC$



$Z = AC + B'C + AB'$

Key concept: groups can only be combined when they differ by one variable.

Therefore, we only need to check groups that differ by one variable to try to group them.

Ex: Simplify using Quine-McCluskey.

1.) Find prime implicants by grouping terms which differ in one variable.

$$Z = \sum m(1,4,5,7) = \underset{001}{A'B'C} + \underset{100}{AB'C'} + \underset{101}{AB'C} + \underset{111}{ABC}$$

Set up chart by grouping terms with same number of 1's and combining:

		Column I	Column II
group 0	1's		
group 1	1	1 001 ✓	−01 (1,5)
		4 100 ✓	10− (4,5)
group 2	1's	5 101 ✓✓	1−1 (5,7)
group 3	1's	7 111 ✓	

} None of these terms can be combined, so these are prime implicants. (unchecked).

2.) Select a minimum set of Implicants using prime implicant chart.

Prime Implicants		Minterms 1	4	5	7
(1,5)	B'C	X		X	
(4,5)	AB		X	X	
(5,7)	AC				X

- Put X at intersection of prime implicant and minterm covered by the prime implicant.
- Select essential prime implicants with columns containing only 1 "X"
- Cross out row
- Cross out columns of minterms in the same row.

$\Rightarrow Z = B'C + AB + AC$ ✓