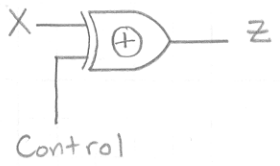
3.2 XOR / XNOR Operations

Basic Properties: $X \oplus 0 = X$ $\quad X \oplus 1 = X'$ $\quad X \oplus Y = XY' + X'Y$

The symbol we use for XNOR (Equivalence) is: $\equiv$

$X \equiv 0 \iff (X \oplus 0)' = X'$ $\quad X \equiv 1 \iff (X \oplus 1)' = X'' = X$ $\quad X \equiv Y \iff XY + X'Y'$

Ex: "Selectable inversion"



Control	Z
0	X
1	X'

Ex: Three (or more) inputs: $A \oplus B \oplus C = (A \oplus B) \oplus C$ Associative Law

$(A \oplus B) \oplus C = (AB' + A'B) \oplus C = AB'C' + A'BC' + A'B'C + ABC$
(100, 010, 001, 111)

$\Rightarrow$ odd # of 1's

Converting expressions from/to Sum-of-Products/ Product-of-Sums

1) POS $\rightarrow$ SOP : Multiply out Ex: $(A+B)(C+D) = AC + AD + BC + BD$

You can always "multiply" out all terms, but the resulting expression can get out of hand.

Ex: $(A+B+C+D)(E+F+G)(H+I+J) = AEH + AEI + AEJ + AFH + \ldots$

$= 4 \times 3 \times 3 = 36$ terms!

To simplify expressions, use theorems and laws to reduce terms as early as possible.

2.) SOP $\rightarrow$ POS : Factor

The following theorem is very helpful: $(X+Y)(X'+Z) = XZ + X'Y$

Ex: $X + YZ = (X+Y)(X+Z)$

$\underbrace{A+B}_{X} + \underbrace{(C+D)}_{Y}\underbrace{E}_{Z} = (A+B+C+D)(A+B+E)$

Clearing up SOP expressions, it's helpful to use $X + XY = X$.
In English, XY is redundant because
if XY is true, X must already be true (Y is also true), so XY is a subset of X.

Venn diagram:

Consensus Theorem $XY + X'Z + YZ = XY + X'Z$
↑ consensus term

Dual form: $(X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)$
↑ consensus term

The consensus term is redundant and can be eliminated.
Look for a literal and its inverse in two product terms. the product of the remaining literals is the consensus term.

Ex: $ABD + D'C + A\cancel{BC} = ABD + D'C$
↑ ↑
Consensus term: ABC

Verify theorem using truth table:

X	Y	Z	XY + X'Z	YZ
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1 ←
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1 ←

Simplifying expressions step by step:

0.) Clearly understand what the problem is asking.

1.) Combine terms using laws and theorems.
Ex: $(A+B)C + C'(A+B) = (A+B)(C+C') = A+B$
$AB + CD + AB = AB + CD$

2.) Eliminate terms, e.g. by using the Consensus Theorem.
Ex: $A'C + A'CD = A'C$ (uses $X + XY = X$)

3.) Eliminate literals (smaller # of terms in expression).
Ex: $A'CD + (A+C'+D')B = \underbrace{A'CD}_{X} + \underbrace{(A'CD)'}_{X'}B$ (by DeMorgan)
$= A'CD + B$ (by $X + X'Y = X + Y$)

4.) Adding redundant terms.

Ex: $AB + A'C + BCD = AB + A'C + BC + BCD = AB + A'C + BC(1+D)$

↑ consensus term

$= AB + A'C + BC$

Practice: Unit 3 Programmed Exercises