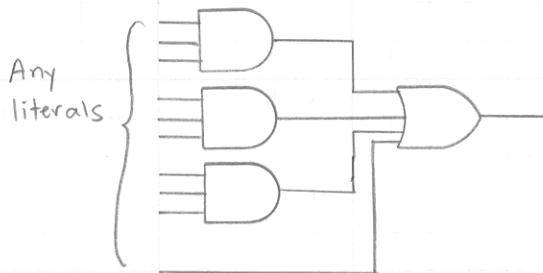


2.7 Multiplying Out and Factoring

Distributive laws can be used to multiply out Boolean expressions into a special form called Sum-of-products (SOP).



$$\text{Ex: } A + BC' + D'E + F'G'H'$$

Each product term is a product of only single variables.

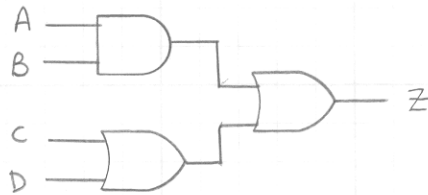
$$\text{Ex: } \underbrace{(A+C+D)}_{\text{not a single variable}} B + BE' \quad \text{Not SOP}$$

↑ not a single variable

↑ AND or nothing
↑ OR (or nothing)

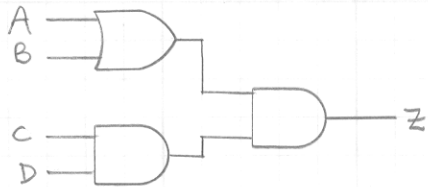
We can convert any expression to sum-of-products form:

Ex:



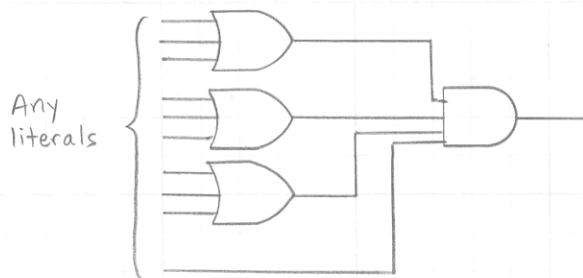
$$\begin{aligned} Z &= (A \cdot B) + (C + D) \\ &= AB + C + D \quad \checkmark \end{aligned}$$

Ex:



$$\begin{aligned} Z &= (A + B) \cdot (C \cdot D) \\ &= ACD + BCD \quad \checkmark \end{aligned}$$

Distributive laws can also be used to factor Boolean expressions into product-of-sums (POS) form.



$$\text{Ex: } (A + C + D) \cdot B \cdot (E' + F) \quad \text{POS}$$

$$(A + C)B + (E' + F) \quad \text{Not POS}$$

↑ OR or nothing
↑ AND (or nothing)

2.8 DeMorgan's Laws

$(X + Y)' = X'Y'$



$(XY)' = X' + Y'$



EX: $(C + (A + B'))' = C' \cdot (A + B')$

Duality To obtain the dual of an expression,

- AND \rightarrow OR
- OR \rightarrow AND
- 0 \rightarrow 1
- 1 \rightarrow 0

Literals remain unchanged. If the original expression is true, so is its dual.

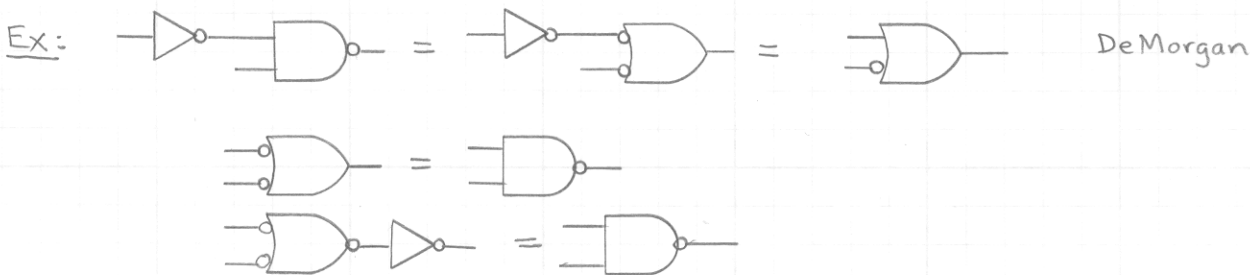
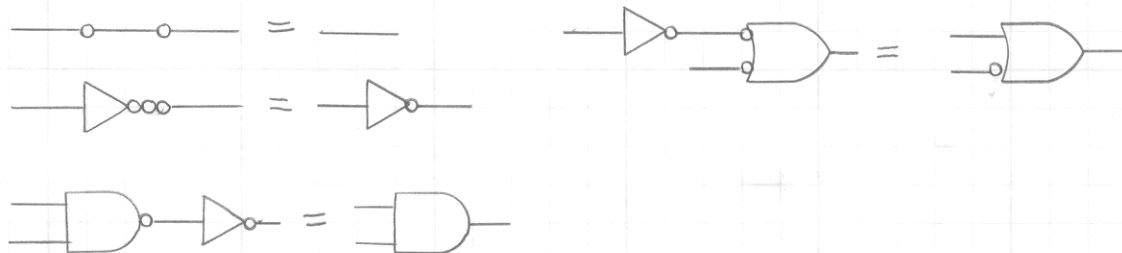
EX: $X + 0 = X \rightarrow$ dual $X \cdot 1 = X$ Dual is also true (theorems with constant 1 or 0).

EX: If $A = B + DE'$ is true, then $A = B \cdot (D + E')$ is also true.

EX: Dual of $(X'YZ)' + W'V = (X'YZ)' + (W'V) = (X' + Y + Z)' \cdot (W' + V)$

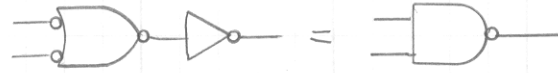
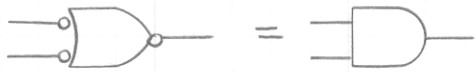
EX: Find dual of $(X + Y)(X' + Z) = XY + X'Z$

EX: Re-forming circuits using $(X')' = X$

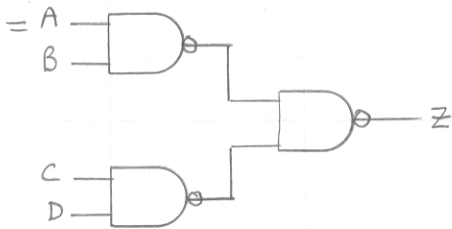
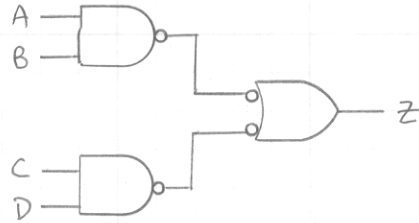
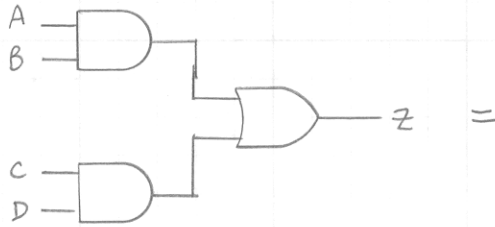


500 SHEETS FILLER 5 SQUARE
 100 SHEETS WEAR 5 SQUARE
 100 SHEETS WEAR 5 SQUARE
 200 SHEETS WEAR 5 SQUARE
 42-382 100 RECYCLED WHITE 5 SQUARE
 42-389 200 RECYCLED WHITE 5 SQUARE
 Made in U.S.A.

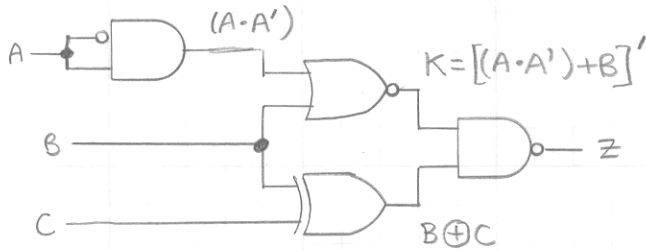




EX: Sum-of-Products example: converting to NAND/NOR



EX: Write equation for the following circuit:



$$\begin{aligned}
 Z &= [[(A \cdot A') + B]' \cdot (B \oplus C)]' \\
 &= [[0 + B]' \cdot (B \oplus C)]' \\
 &= [B' \cdot (B \oplus C)]' = B + (B \oplus C)'
 \end{aligned}$$

13-782 500 SHEETS, FILLER, 5 SQUARE
 42-381 50 SHEETS, FILLER, 5 SQUARE
 42-382 100 SHEETS, FILLER, 5 SQUARE
 42-383 200 SHEETS, FILLER, 5 SQUARE
 42-392 100 SHEETS, FILLER, 5 SQUARE
 42-393 200 SHEETS, FILLER, 5 SQUARE
 42-399 100 RECYCLED WHITE, 5 SQUARE
 42-399 200 RECYCLED WHITE, 5 SQUARE
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