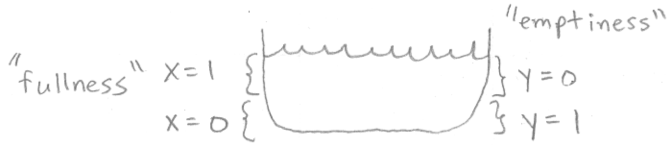
2.1 Boolean Algebra

Boolean algebra (for our purposes) operates on variables that can take on 1 of 2 values.

<u>Ex</u>: X = ON or OFF
= TRUE or FALSE
= full or empty
= 1 or 0
= +5 V or 0 V

It's arbitrary how we assign values as long as we're consistent.

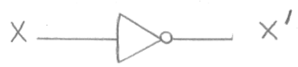


2.2 Basic Operations

<u>Invert/NOT/Complement Operator</u>

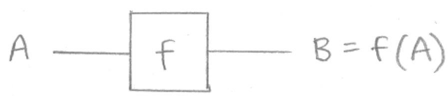
Denote inversion/complementation by $\overline{X}$ or X' or $!X$ or $\sim X$
If X = ON, then $\overline{X}$ = OFF
If Y = 0, then $Y' = 1$

Symbol:

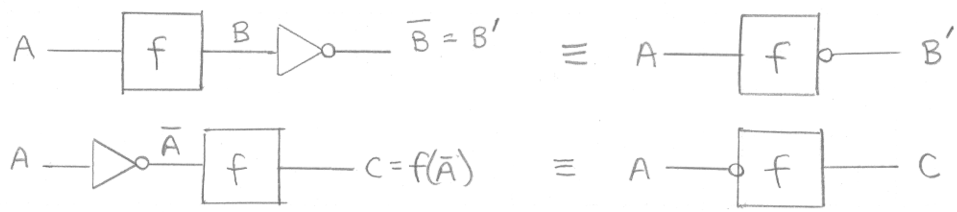


Inverter

Sometimes denoted as just a "bubble"



If we want to invert B or A, add an inverter or bubble to the output or input, respectively.



Boolean variables have a (very!) limited number of possible values so we can enumerate all possibilities in a <u>truth table</u>.

X	X'
0	1
1	0

Note: N input Boolean variables have 2^N possible input combinations, so we can build truth tables only for a small number of inputs.

AND Operator

AND must have two or more inputs:
X AND Y AND Z or X·Y·Z or XYZ or "Product of X, Y, Z"

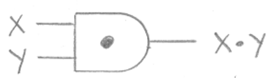
By which we mean logical product, not multiplication.

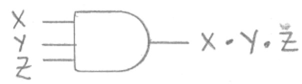
Def: Output is 1 or TRUE only when all inputs ar 1 or TRUE.

X	Y	X·Y
0	0	0
0	1	0
1	0	0
1	1	1

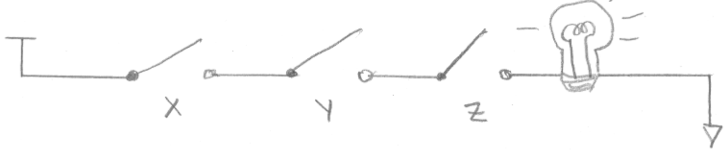
X	Y	Z	X·Y·Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Symbol:





Ex: Switches in series



OR Operator

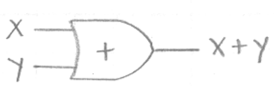
Also has two or more inputs: X OR Y OR Z or X+Y+Z or "Sum of X, Y, Z"
By which we mean logical sum, not addition.

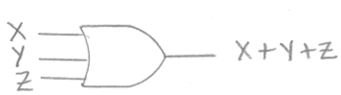
Def: Output is 1 or TRUE if any input is 1 or TRUE.

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

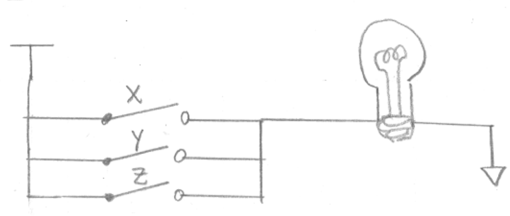
X	Y	Z	X+Y+Z
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Symbol:

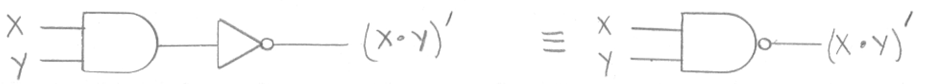




Ex: Switches in parallel



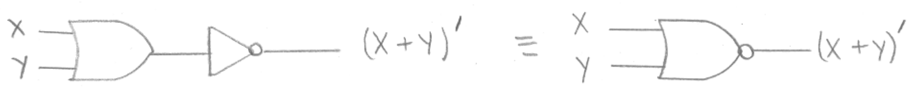
NAND (NOT AND) Operator



X	Y	$\overline{X \cdot Y}$
0	0	1
0	1	1
1	0	1
1	1	0

NAND is easier to build (much more common) in hardware.

NOR (NOT OR) Operator



X	Y	$\overline{X+Y}$
0	0	1
0	1	0
1	0	0
1	1	0

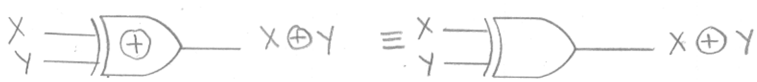
XOR (eXclusive OR) Operator

XOR has two or more inputs : X XOR Y or $X \oplus Y$

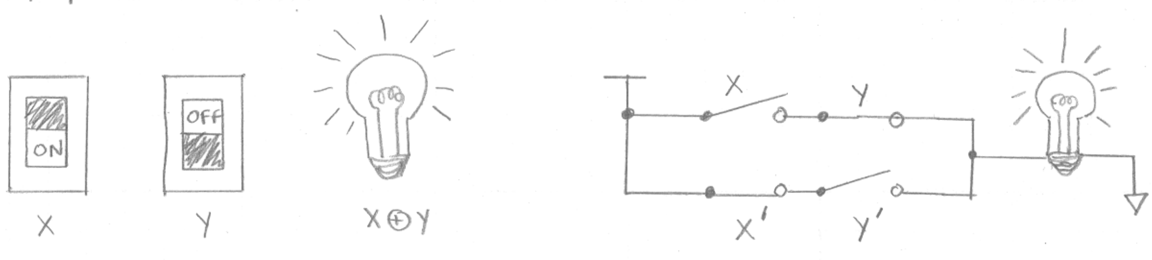
Def: Output is 1 or TRUE if one or the other inputs (but not both) are 1 or TRUE, i.e. X=1 or Y=1, exclusively.

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

Symbol:



Ex:

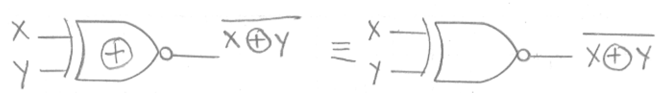


Useful in addition : Sum = X PLUS Y ≡ Sum = $X \oplus Y$

XNOR Operator

Complement of XOR

Symbol:

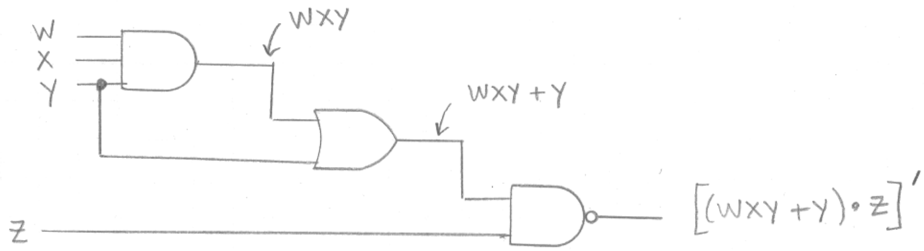


Def: Output is 1 or TRUE if both inputs are the same.

X	Y	$\overline{X \oplus Y}$
0	0	1
0	1	0
1	0	0
1	1	1

XNOR is sometimes called the "equivalence" operator.

More complex expressions are built out of these fundamental blocks:



Literal Def: A variable or its complement in an expression.

Ex: $[(WXY+Y)\cdot Z]'$ has 5 literals (W, X, Y twice, and Z)

2.4 Basic Theorems

Operations with 0 or 1

$X + 0 = X$
$X + 1 = 1$ (set)
$X \cdot 0 = 0$ (clear)
$X \cdot 1 = X$

We frequently need to set or clear part of a larger binary word (number).

Ex: Set bit 1

```
   XXXX
OR 0001
   ----
   XXX1
```

Ex: Clear bits 2 and 3

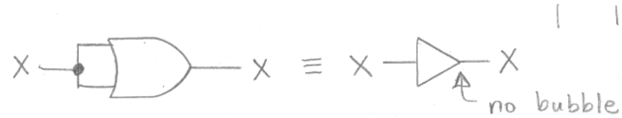
```
    YYYY
AND 0011
    ----
    00YY
```

Idempotent Laws

OR: $X + X = X$ Verify using truth table:

X	Y	X+Y	
0	0	0	← Y=X
0	1	1	
1	0	1	
1	1	1	← Y=X

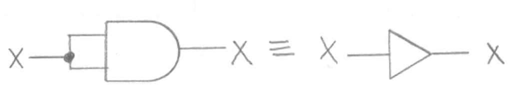
Symbolically,



AND: $X \cdot X = X$ Verify using truth table:

X	Y	X•Y	
0	0	0	← Y=X
0	1	0	
1	0	0	
1	1	1	← Y=X

Symbolically,



Involution Law "Double flip" $(X')' = \overline{\overline{X}} = X$

Laws of Complementarity $X + X' = 1$ (Either X or X' is always 1)

$X \cdot X' = 0$ (Either X or X' is always 0.)

Truth table:

X	X'	$X+X'$	$X \cdot X'$
0	1	1	0
1	0	1	0

Ex: $(X+Y) \cdot (X+Y) = X+Y$ $\quad (X' + Y \cdot Z') \cdot (X' + Y \cdot Z')' = 0$

Commutative Law AND: $ABC = CBA = BAC$

OR: $A+B+C = C+B+A = C+A+B$

Symbolically,

Associative Law AND: $(AB)C = A(BC) = ABC$

OR: $(A+B)+C = A+(B+C) = A+B+C$

Symbolically,

Distributive Law

a) $A(B+C) = AB + AC$

b) $A + BC = (A+B)(A+C)$ dual

Looks like multiplication and addition, but meaning is very different!

Helpful Theorems (Roth, p.41)

Ex: $X + XY = X$ Help simplify complicated expressions:

$$DC(KL' + M)(G+H) + C = C$$