

Oct. 12

A Karnaugh Map (K-Map) is a graphical representation of a Boolean expression (really a T.T.) which is convenient for simplifying expressions. It is systematic.

Thm 9:-

$$XY + XY' = X$$

Ex:

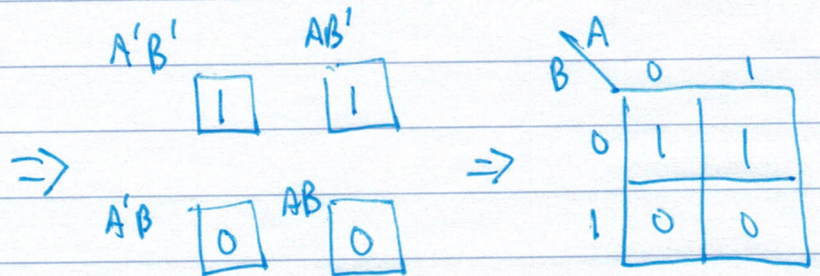
A	B	C	X	Y	Z
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	1	0
0	1	1	0	0	0
1	0	0	0	0	1
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	0	0

$$X = A'B'C' + A'B'C = A'B'$$

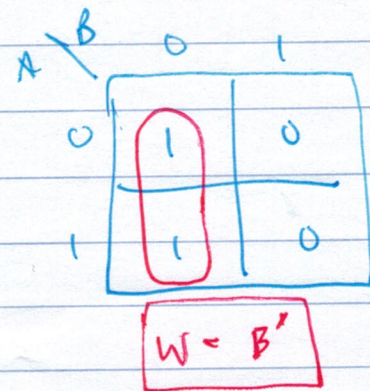
$$Y = A'B'C' + A'BC' = A'C'$$

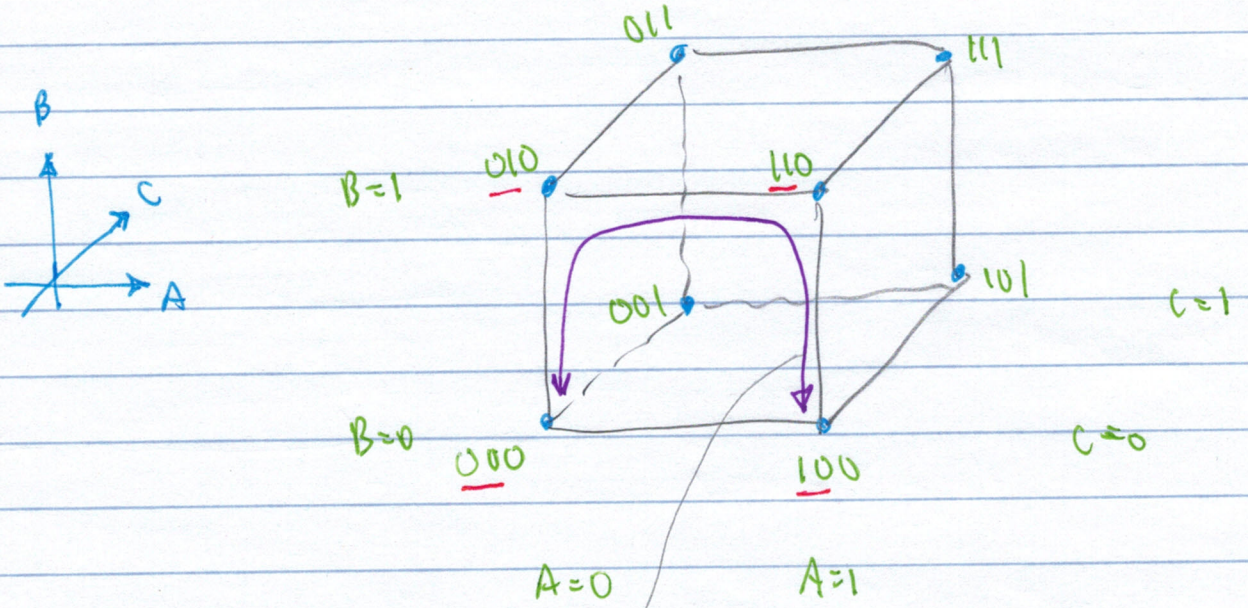
$$Z = A'B'C' + AB'C' = B'C'$$

A	B	W
0	0	1
0	1	0
1	0	1
1	1	0



Alternatively,





AB \ C	0	1
00	000 m_0	001 m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

Labels are not in binary order
 It is a type of gray code

"adjacent"

Each box corresponds to a possible minterm

Ex: $Z(A, B, C) = \sum m(0, 2, 3, 7)$

		C	0	1
AB	00	1		
	01	1	1	
	11		1	
	10			

Group 1: $A'B'C' + A'BC' = A'C'$

Group 2: $B \cdot C$

$Z = A'C' + B \cdot C$

Ex:

		C	0	1
AB	00	1		
	01		1	
	11	1	1	
	10	1	1	

Labels: $A'B'C'$, $B \cdot C$, A , $B' \cdot C'$

$out = A + BC + A'B'C'$

$out = A + BC + B'C'$ (ABC = 100)

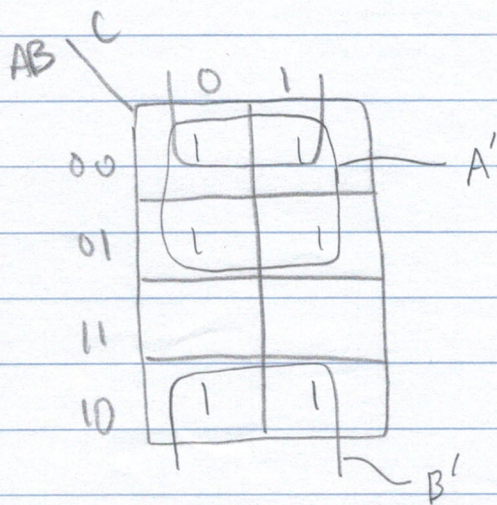
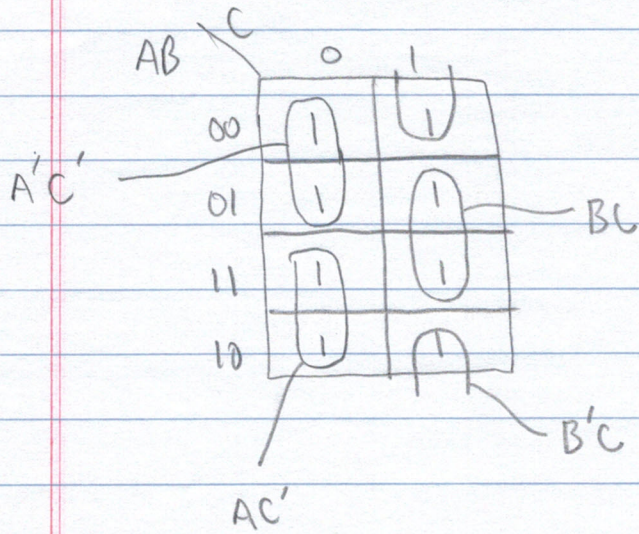
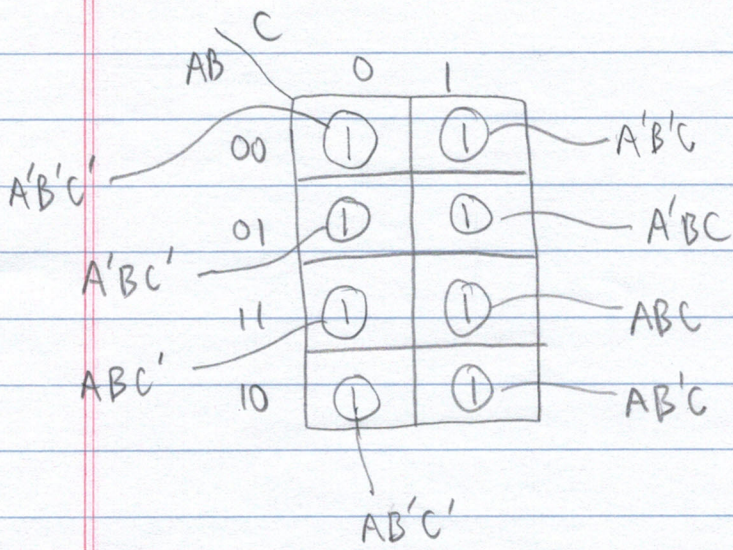
		C	0	1
AB	00	1	1	
	01			
	11			
	10	1	1	

$out = B'$

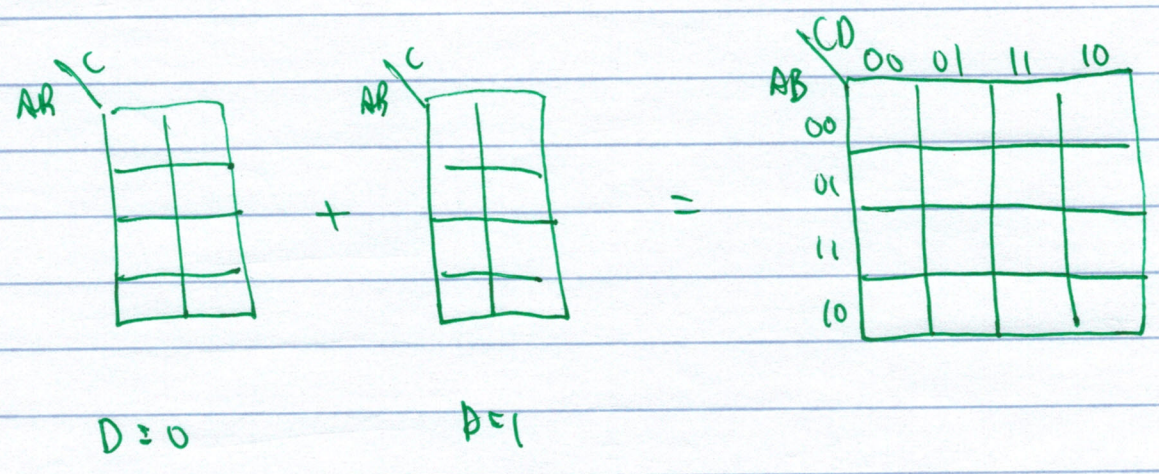
Goal: larger groups

→ simpler logic

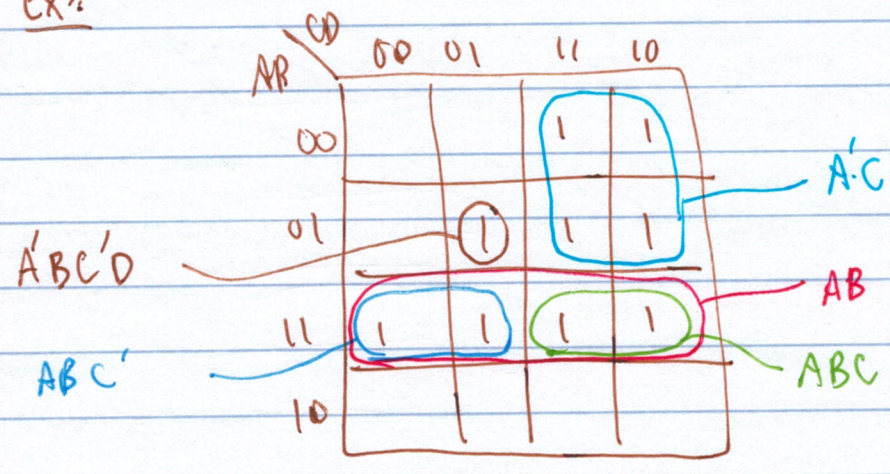
→ fewer terms



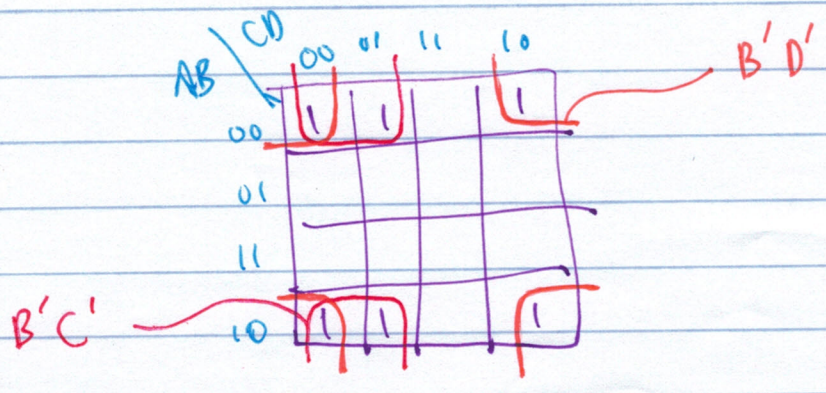
Four-variable K-Map:



Ex:



$$\text{out} = A'BC'D + \underbrace{ABC + ABC'}_{AB} + A'C$$



Ex: Convert don't cares to 1s to make larger groups
 " " " " " 0's if not helpful

		CD			
		00	01	11	10
AB	00				0
	01	1	1		
	11	X	1		
	10				

$out = B \cdot C' + A'B'C'D'$

Ex:

		A	
		0	1
BC	00		
	01	1	1
	11	1	1
	10		

1) Minterms:

$Z = A'B'C + AB'C + A'BC + ABC$

2) Groups of 2

$Z = B'C + BC$ (horiz.)

$Z = A'C + AC$ (vert.)

3) Group of 4

$Z = C$

All identical logically.



Maxterms correspond to input combinations that force the output to 0.

Ex: Given SOP: $Z = B' + A'C$
Find POS.

- Find minterms for Z'
- Convert using DeMorgan's

$$Z' = BC' + AB$$

$$Z = (Z')' = (BC' + AB)'$$

$$= (BC')' \cdot (AB)'$$

$$Z = (B' + C) \cdot (A' + B')$$

Z	CD	00	01	11	10
AB					
00	1	1	1	1	
01	0	0	1	1	
11	0	0	0	0	
10	1	1	1	1	

5-var. K-map

