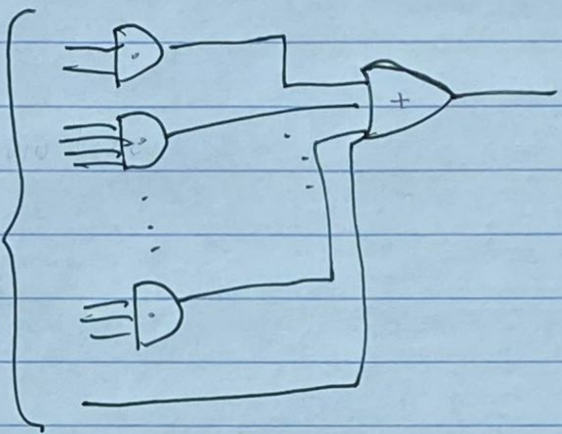
# EEC 18

Oct. 5

"Sum" of "Products" (SOP)
(OR of ANDs)

any literal



AND (or nothing)    OR (or nothing)

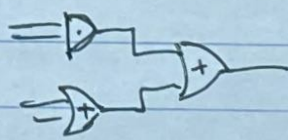
Ex: $A + B \cdot C' + D' \cdot E + F \cdot G \cdot H'$ ✓ SOP

Ex: $(A+C+D) \cdot B + B \cdot E'$    not SOP

Distributive law often helpful

Ex: $Z = (A \cdot B) + (C + D)$

$= A \cdot B + C + D$ ✓ SOP

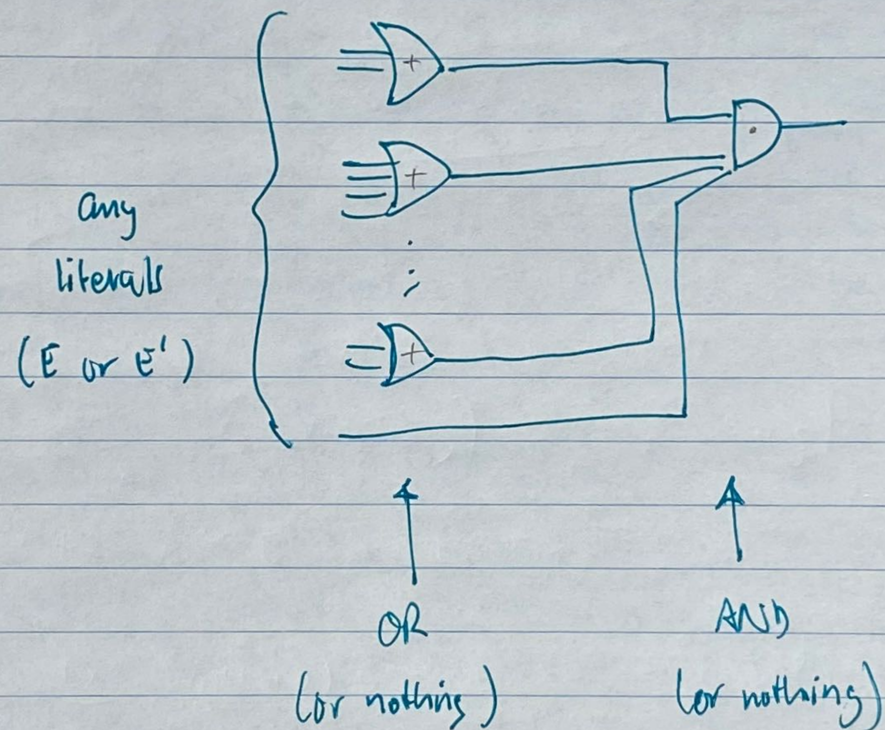


Ex: $Z = (A+B)\cdot(C\cdot D)$

$= A\cdot C\cdot D + B\cdot C\cdot D$ ✓ SOP

"Product" of "Sums" (POS)

(AND of OR)

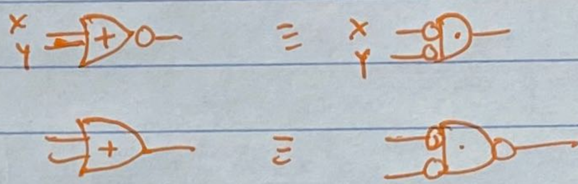



Ex: $\underbrace{(A+C+D)}\cdot\underbrace{B}\cdot\underbrace{(E'+F)}$ ✓ POS

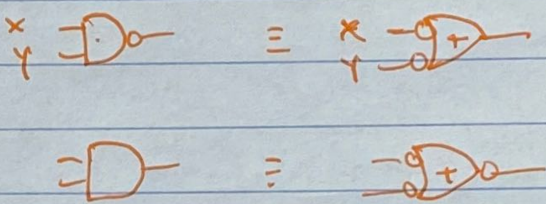
Ex: $(A+C)\cdot B + (E'+F)$ not POS

## De Morgan's Law

$(X+Y)' = X' \cdot Y'$




$(X \cdot Y)' = X' + Y'$




Ex: $(\underline{C} + \underline{(A+B')})'$

$= C' \cdot (A+B')'$

$= C' \cdot (A' \cdot B)$

$= C'A'B$

## Duality

To obtain the dual:

AND → OR

OR → AND

0 → 1

1 → 0

Literals unchanged

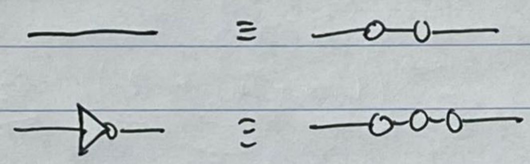
- If an expression is true, its dual is true
- In general, an expression is NOT equal to its Dual.

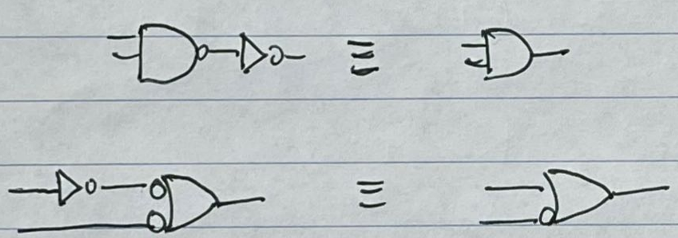
**Ex:** $X + 0 = X$ ✓ $\xrightarrow[\text{Dual}]{}$ $X \cdot 1 = X$ ✓

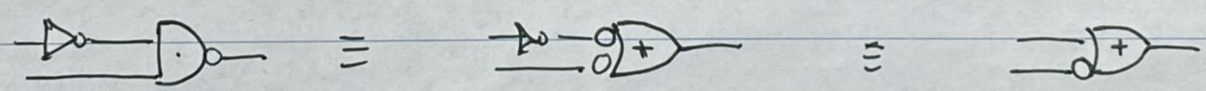
**Ex:** $(X' \cdot Y \cdot Z)' + (W'V) =$

Add parentheses first!

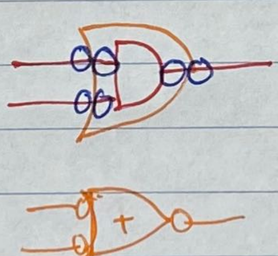
$\xrightarrow{\text{Dual}}$ $(X' + Y + Z)' \cdot (W' + V)$



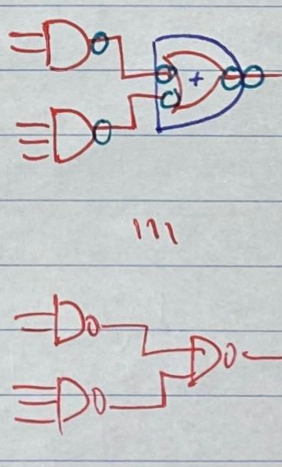




**Ex:**



**Ex:**



Convert to NANDs

## XOR / XNOR

$X \oplus \overset{Y}{0} = X$

$X \oplus 1 = X'$

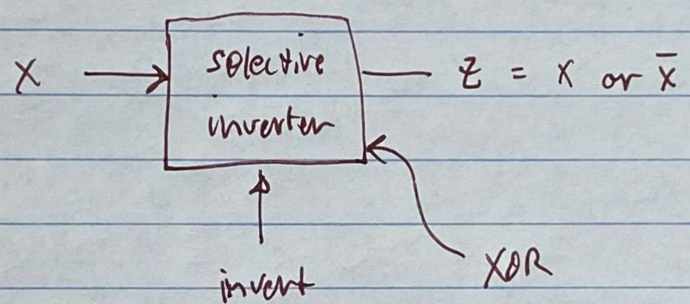
| X Y | $X \oplus Y$ | $X \equiv Y$ |
|---|---|---|
| 0 0 | 0 | 1 |
| 0 1 | 1 | 0 |
| 1 0 | 1 | 0 |
| 1 1 | 0 | 1 |

$X \oplus Y = XY' + X'Y$

XNOR / Equivalence

$(X \oplus 0)' = X'$

$(X \oplus 1)' = X$




| control | output |
|---|---|
| 0 | $X$ |
| 1 | $\overline{X}$ |

Ex: 3 or more inputs

$A \oplus B \oplus C = (A \oplus B) \oplus C$

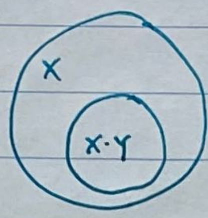
$= (AB' + A'B) \oplus C = (AB' + A'B) \cdot C' + (AB' + A'B)' \cdot C$

$= AB'C' + A'BC' + A'B'C + ABC$

$\underbrace{100 \quad 010 \quad 001 \quad 111}$ Odd # of inputs equal to one

$\underbrace{000}_{0} \quad \underbrace{110 \quad 101 \quad 011}_{2}$

Clearing up SoP, try $X + XY = X$




Consensus theorem

$$X\cdot Y + X'\cdot Z + \cancel{Y\cdot Z} = X\cdot Y + X'\cdot Z$$

Dual: $$(X+Y)(X'+Z)\cancel{(Y+Z)} = (X+Y)\cdot(X'+Z)$$

Ex: $AB + A'C + BCD$

$AB + A'C + BC + BCD$

consensus term

$AB + A'C + BC$ [Thm 10]

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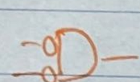
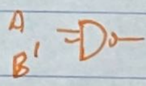
$[(AB' + A'B)']\cdot C$  = 

$[(A\cdot B')' \cdot (A'B)']\cdot C$ 

= 


$[(A'+B)\cdot(A+B')]\cdot C$

$X'$ $Z$ $X$ $Y$

$[AB + A'B']\cdot C$ [Thm 14] $ABC + A'B'C$ ✓