Sept. 30

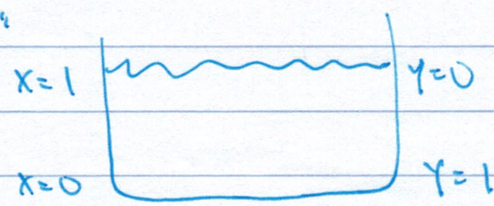
EEC 18

Boolean Algebra

variables take on 1 of 2 values

X	=	1	or	0	EEC 18
	=	ON	or	OFF	
	=	True	or	False	Philosophy
	=	1.0V	or	0V	circuits
	=	Full	or	Empty	

Ex: "Fullness" "Emptiness"



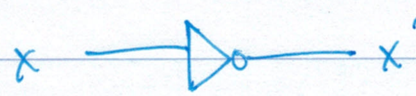
Basic Operations

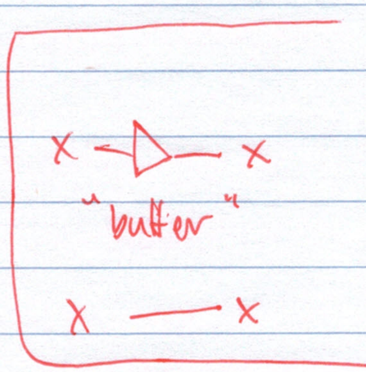
① Invert / NOT / complement

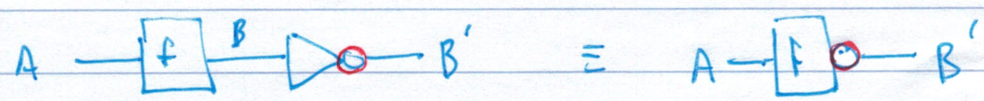
$\bar{X}$ or X' or $!X$ or $\sim X$

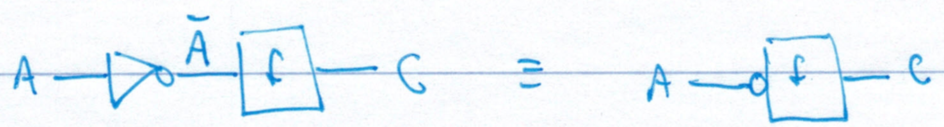
If $X = ON$, $\bar{X} = OFF$

$Y = 0$, $Y' = 1$









Boolean variables have a limited number of possible values, so we can enumerate all cases in a Truth Table (T.T.)

inputs — outputs

X	X'
0	1
1	0

* Number of rows in a T.T. $= 2^N$, N = number of inputs

② AND operator

Two or more inputs:

X AND Y AND Z

$X \cdot Y \cdot Z$

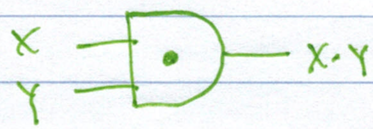
X & Y & Z

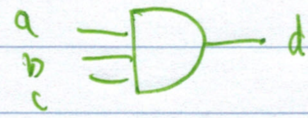
"Product" of X, Y, Z

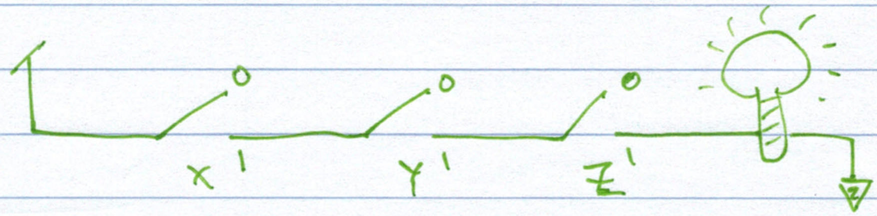
Def: output is 1, only when all inputs are 1

X	Y	$X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

X	Y	Z	X·Y·Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1







In "series"

③ OR operator

X OR Y OR Z

X + Y + Z

X | Y | Z

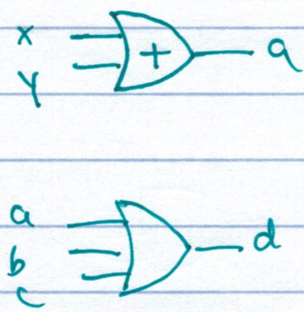
"Sum" of X, Y, Z

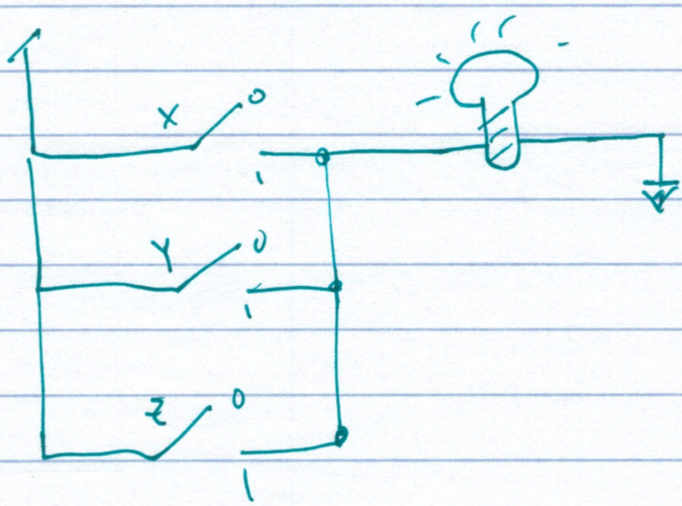
logical "sum", not addition

Output is 1, if any input is 1

X	Y	X + Y
0	0	0
0	1	1
1	0	1
1	1	1

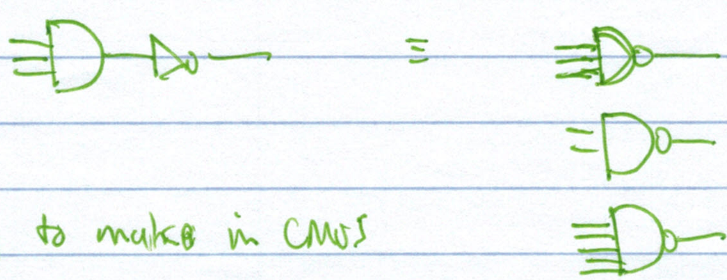
X	Y	Z	X|Y|Z
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1





"parallel"

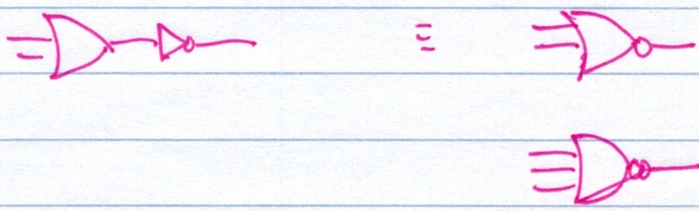
(2a) NAND - NOT AND



Easy to make in CMOS

X	Y	$\overline{X \cdot Y}$
0	0	1
0	1	1
1	0	1
1	1	0

(3a) NOR - NOT OR



X	Y	$\overline{X+Y}$
0	0	1
0	1	0
1	0	0
1	1	0

(4) XOR, Exclusive OR

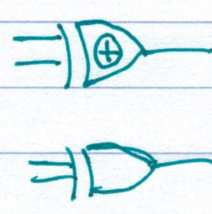
X XOR Y

$X \oplus Y$

X ^ Y

output is 1 if one or the other input is 1, but not both

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

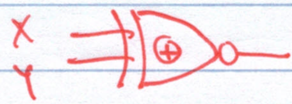


Ex: switches at ends of a hall

$$\begin{array}{r} X \\ +\ Y \\ \hline \end{array}$$

$\boxed{\text{sum}} = X \oplus Y$

4a) XNOR, ~~[illegible]~~

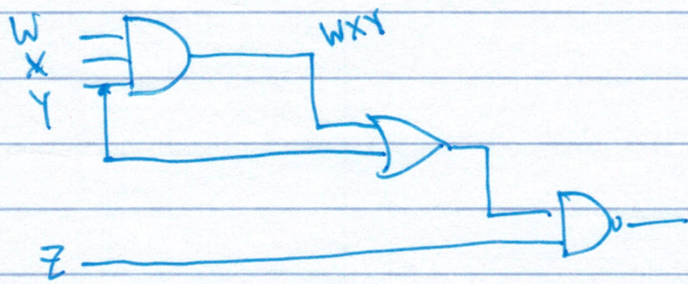


$\overline{X \oplus Y}$

X	Y	$\overline{X \oplus Y}$
0	0	1
0	1	0
1	0	0
1	1	1

XNOR or "Equivalence"

INV, AND, OR - functionally complete



$[(WXY + Y) \cdot Z]'$

5 literals

Literal Def: A variable or its complement in an expression

Theorems

Operations with 0, 1

$X + 0 = X$

$X + 1 = 1$ } set to 1

$X \cdot 0 = 0$ } clear to 0

$X \cdot 1 = X$

XXXX
OR 0001
XXX1 set

$$\begin{array}{lr} & XXXX \\ \text{AND} & \underline{0011} \\ & 00XX \end{array}$$ cleared

Idempotent Laws

x	y	x + y
0	0	0
0	1	1 unused
1	0	1 unused
1	1	1

OR : $x + x = x$

AND: $x \cdot x = x$

Involution law "double flip"

$$(x')' = \overline{\overline{x}} = x$$

Laws of Complementarity

$$x + x' = 1$$

$$x \cdot x' = 0$$

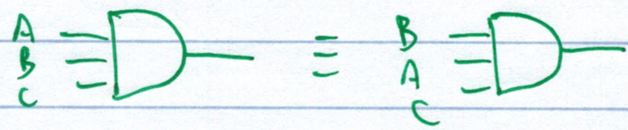
$$(x+y)\cdot(x+y) = x+y$$

$$(x' + y\cdot z')\cdot(x' + y\cdot z')' = 0$$

Commutative law

AND : $ABC = CBA = BAC \ldots$

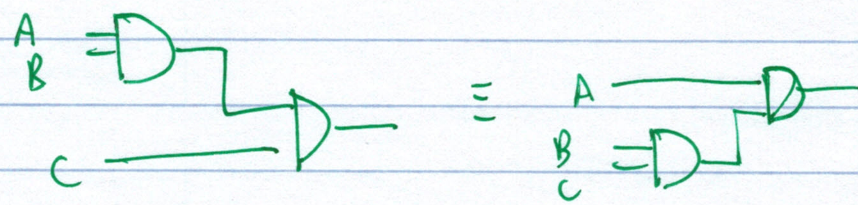
OR : $A+B+C = C+B+A = B+A+C \ldots$



Associative law

AND: $(AB)C = A(BC) = ABC$

OR: $(A+B)+C = A+(B+C) = A+B+C$



Distributive law

a) $A(B+C) = A \cdot B + A \cdot C$

b) $A + B \cdot C = (A+B) \cdot (A+C)$

Helpful Theorems, Roth p-49

Ex: $X + XY = X$

Ex: $DC(KL' + M)(G + H) + C = C$

X X

Y

00	0
10	0
01	0
11	1