On Modeling Random Topology Power Grids for Testing Decentralized Network Control Strategies

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Abstract: An electrical power grid is a critical infrastructure. Its reliable, robust, and efficient operation inevitably depends on underlying telecommunication networks. In order to design an efficient communication scheme and examine the efficiency of any networked control architecture, we need to characterize statistically its *information source*, namely the power grid itself. In this paper we studied both the topological and electrical characteristics of power grid networks based on a number of synthetic and real-world power systems. We made several interesting discoveries: the power grids are sparsely connected and the average nodal degree is very stable regardless of network size; the nodal degrees distribution has exponential tails, which can be approximated with a shifted Geometric distribution; the algebraic connectivity scales as a power function of network size with the power index lying between that of one-dimensional and two-dimensional lattice; the line impedance has a heavy-tailed distribution, which can be captured quite accurately by a Double Pareto LogNormal distribution. Based on the discoveries mentioned above, we propose an algorithm that generates random power grids featuring the same topology and electrical characteristics we found from the real data.

Keywords: Graph models for networks, Power grid topology.

1. INTRODUCTION

An electrical power grid is a critical infrastructure, whose reliable, robust, and efficient operation greatly affects national economics, politics, and people's everyday life. In the United States the bulk electric power system is operating ever closer to its reliability limits. It has been widely agreed that there are inseparable interdependencies between reliable, robust, and efficient operations of power grids and the efficient placement and operation of related telecommunication networks, as pointed out in the work of Heydt, Liu, Phadke, and Vital (2001).

1.1 Motivation of the work

One of our research questions is *what kind of communication network is needed to support the decentralized control of power grids*? To answer this question we need to do what communication designers have done in designing the large Public Switched Telephone Network (PSTN), the Internet and the cellular networks: understanding the nature of the source, and of the traffic it generates. One first step towards the goal of producing a statistical model for the data traffic is being able to generate a large number of random power grid test cases with realistic topologies, with scalable network size, and with realistic electrical parameter settings. The traditional practice of using a small number of historical test systems (e.g. IEEE model systems) is no longer sufficient for our research. Therefore it is essential to have a statistical model for power networks both as a simulation tool to generate such power grid test cases, as well as possibly an analytical tool to grasp *what class of communication network topologies* need to match the underlying power networks, so that the provision of the network control problem can be done efficiently.

1.2 Previous Work and Our Contributions

Several other researchers noticed the similar needs to generate scalable-size power grid test cases. Different power grid models were proposed based on observed statistical characteristics. For example, Parashar and Thorp (2004) used Ring-structured power grids to study the pattern and speed of contingency or disturbance propagation. Carreras, Lynch, Dobson, and Newman (2002) used a Tree-structured power grid model to study power grid robustness and to detect critical points and transitions in the transmission network. Both the models provided interesting perspectives to power grid characteristics.

The work from Watts and Strogatz (1998) in the context of their work on random graphs first proposed statistically modeling the power grid as a *small-world* network.

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What our work adds to the literature is a comprehensive study of all the topological as well as electrical features of the network, which clarifies and fills gaps left by previous work on the subject. We utilize our findings to provide a simulation platform that generates realistic power grids and that can aid in the evaluation of networked control protocols (see Section 5).

Our main findings are as follows: the nodal degrees follow a Poisson or shifted Geometric distribution, that is nearly invariant with respect to the network size; the algebraic connectivity has a very special scaling property, following $\lambda_2(L) \propto n^{-1.376} \sim n^{-1.088}$, where n is the network size; the distribution of line impedances Z_{pr} is heavy-tailed.

The rest of the paper is organized as follows: Section 2 discusses system model for power grid network; Section 3 and 4 present our study results on the topological and electrical characteristics of realistic power grids; Section 5 describes in details the model of *RT-nested-Smallworld*; and Section 6 concludes the paper.

2. SYSTEM MODEL

The power network dynamics are controlled by its network admittance matrix Y and by power generation distribution and load settings. The generation and load settings can take relatively independent probabilistic models, and have been discussed in our last paper [Wang, Thomas, and Scaglione (2008)]. Here in this work we mainly focus on the network admittance matrix, which is defined as:

$$Y = A^T \Lambda^{-1}(Z_{pr})A \quad (1)$$

It has two components: the line impedance Z_{pr} and the line admittance matrix A. If the network has N nodes, m links, its line admittance matrix A, with the size of $m \times N$, can be written as

$$A: \begin{cases} A(t,i) = \ \ 1 \\ A(t,j) = -1 \\ A(t,k) = \ \ 0, \text{ with } k \neq i, \text{or } j \end{cases} \quad (2)$$

if t-th link is from node i to node j

The Laplacian matrix L can be obtained as

$$L = A^T A \quad (3)$$

3. TOPOLOGICAL CHARACTERISTICS OF POWER GRIDS

A number of researchers have studied the statistical properties of the topology of an electrical grid, e.g. Watts and Strogatz (1998), Newman (2003), Whitney and Alderson (2006), *et al*. The metrics they studied include some basic ones, such as network size N, the total number of links m, average nodal degree $\langle k \rangle$, average shortest path length in hops $\langle l \rangle$, etc, and more complex ones, such as the ratio of nodes with larger nodal degrees than $\bar{k}$, $r\{k < \bar{k}\}$, and the Pearson coefficient ρ [introduced by Francis Galton in the 1880s, and named after Karl Pearson, see Rodgers and Nicewander (1988)]. Whitney and Alderson (2006) proposed to use the Pearson coefficient to distinguish technological networks and social networks. All the metrics mentioned above in fact can be deduced from the graph Laplacian.

3.1 Topological Characteristics

Table 1 below shows the parameter values resulting from the IEEE model systems, NYISO and WSCC systems based on selected metrics.

Table 1. Topological Characteristics of Real-world Power Networks

	(N,m)	$\langle l \rangle$	$\langle k \rangle$	ρ	$r(k_i > \bar{k})$
IEEE-30	(30, 41)	3.31	2.73	-0.0868	0.2333
IEEE-57	(57, 80)	4.95	2.80	0.1895	0.2281
IEEE-118	(118, 186)	6.31	3.15	-0.0518	0.1949
IEEE-300	(300, 411)	9.94	2.74	-0.2137	0.2467
NYISO	(2935,13136)	16.43	4.47	0.4593	0.1428
WSCC	(4941, 6954)	18.70	2.67	0.0035	0.2022

Two discoveries pertaining to the topology of the grid are most interesting:

(a) Power grids are sparsely connected. The average nodal degree is very stable, regardless of the network size.

(b) In Watts and Strogatz's work (1998) the authors hypothesized that power grids have the salient features of *small world graphs*. That is, while the vast majority of links are similar to that of a regular lattice, with limited near neighbor connectivity, a few links connect across the network. These bridging links significantly shorten the path length that connects every two nodes and critically increase the connectivity of the network. While the fundamental intuition is correct, we observe that small world graphs are only partially able to capture features of the power grid.

3.2 Nodal Degree Distribution

Sole, Rosas-Casals, Corominas-Murtra, and Valverde studied the robustness of European power grids under intentional attacks in 2007 and concluded that European power grids are sparsely connected with global average nodal degree of $\langle k \rangle = 2.8$ and the nodal degree distribution is exponential: the probability of having a node linked to k other nodes is $P(k) = \exp\left(-\frac{k}{\gamma}\right)/\gamma$, with the constant $\gamma = \langle k \rangle$. In this work we also examined the empirical distribution of nodal degrees $\underline{k} = diag(L)$ in the available real-world power grids. Fig. 1 shows the histogram probability density function (PDF) in log-scale for the nodal degrees of NYISO system. There obviously exists a straight line in the semi-log plot except the beginning part with small nodal degree (e.g. degree of 1 or 2), which implies underlying shifted Geometric distribution.

3.3 Wiring the Network

The *Small-world* model, proposed by Watts and Strogatz (1998), is generated starting from a regular ring lattice, then by a small probability, rewiring some local links to an arbitrary node chosen uniformly at random in the entire network. A tool to visually highlight small-world topologies is the Kirk graph, which was proposed by J. Kirk (2007): First, node numbers are assigned according to physical nodal adjacency, that is, physically closely located nodes are given close numbers; then all the nodes are sequentially and evenly spread around a circle and links between nodes are drawn as straight lines inside the circle. Fig. 2 below shows three representative network topologies, using Kirk graphs, of an *Erdös-Rényi* Rand-graph network [Erdős and Rényi (1959)], of a Watts-Strogatz *Small-world* network, and of a realistic power grid -- IEEE-57 network. The three networks have same network size and almost same total number of links.

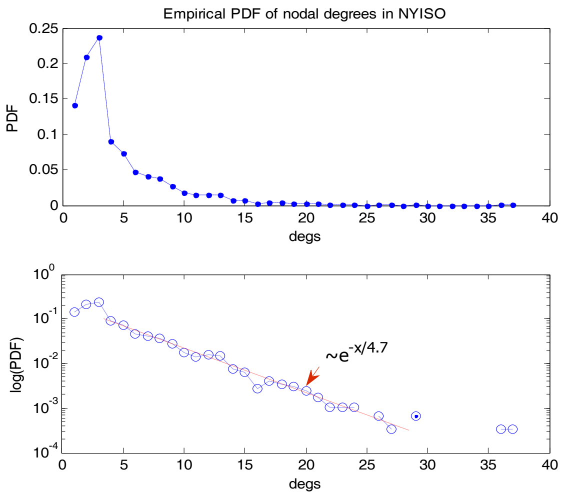


Fig. 1 Empirical PDF of Nodal Degrees in Real-world Power Grids (NYISO)

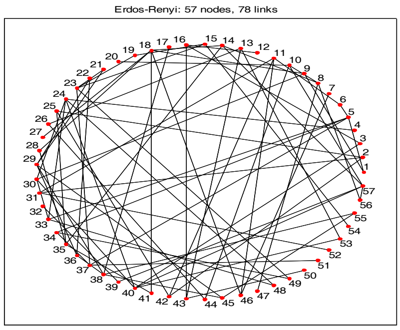
One can easily notice the topology difference between the *Erdös-Rényi* network and IEEE-57 network while observing as well a good degree of similarity between the Watts and Strogatz *small-world* model and the test power network.

The figure also shows that the rewiring of the test power grid is not independent, as in that of Watts and Strogatz *small-world* model; instead, long hauls appear over clusters of nodes. One property of the IEEE 57 network that is visually noticeable, and that differentiates it from the *small-world* graph, is the fact that the nodes that are rewired appear to be in clusters, rather than being chosen independently. This fact has an intuitive explanation: long hauls require having a right of way to deploy a long connection and it is highly likely that the long wires will reuse part of this space. These physical and economical constraints inevitably affect the structure of the topology.

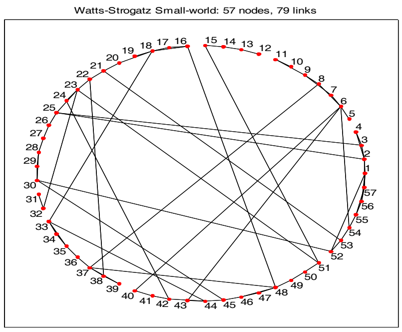
However, there is more than what meets the eyes. In our study we found that the *Small-world* model proposed by Watts and Strogatz has scaling property that cannot be validated by power grid topologies, precisely because the average nodal degree of a power network is almost invariant to the size of the network. Given a network size with its specified average nodal degree, the model cannot produce a connected power grid topologies for reasonably large network sizes using realistic power grid degree distributions. The main reason for the poor scaling lies in the fact that in order to produce a connected topology, the Watts and Strogatz *Small-world* model requires

$$\langle k \rangle \ll N \ll e^{\langle k \rangle} \quad (4)$$

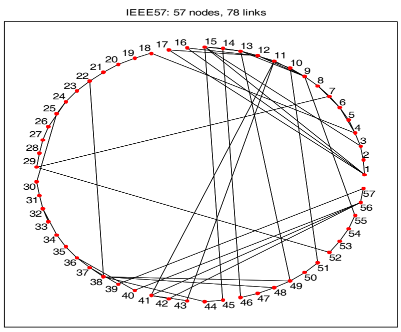
As we previously pointed out, power grids have very stable and low average nodal degree distribution with $\langle k \rangle = 2\sim 3 \; or \; 4\sim 5$, regardless of the network size. This limits the network size to be no greater than 30 or 300 in order to produce a connected topology. In the real world, large power grids are connected even if they are much sparser than what is required by Watts-Strogatz *Small-world* model to produce a connected topology.



(a) Erdős-Rényi Network



(b) Watts-Stogatz Small-world



(c) IEEE 57

Fig. 2. Topology Comparison Using Kirk Graph

3.4 Connectivity Scaling Property

In this paper our strategy for evaluating how well the models capture the topological features of the sample networks are based on two main characteristics: the nodal degree distribution $\underline{k} = diag(L)$ and the algebraic connectivity $\lambda_2(L)$, where L represents the Laplacian matrix of the network. The algebraic connectivity $\lambda_2(L)$ or Fiedler eigenvalue, introduced by Fiedler (1989), is the second smallest eigenvalue of the Laplacian and it reflects how well a network is connected and how fast information data can be shared across the network. If the algebraic connectivity $\lambda_2(L)$ is close to zero, the network is close to being disconnected. Otherwise, if $\varepsilon\lambda_2(L)$ tends to be 1, where ε is a normalized constant related with network size, the network tends to be fully connected.

Table 2 shows the algebraic connectivity of IEEE model systems and the NYISO system. Fig. 3 plots scaling curve of power grid versus network size and compares it with that of 1-Dimensional and 2-Dimensional lattices. For 1-D lattice, its connectivity scales as $\lambda_2(L) \propto n^{-2}$; for 2-D lattice , its connectivity grows as $\lambda_2(L) \propto n^{-1}$; interestingly, for power grids, its connectivity grows as $\lambda_2(L) \propto n^{-1.376} \sim n^{-1.088}$, lying between those of 1-D lattice and 2-D lattice.

Table 2. Algebraic Connectivity of Real-world Power Grids

	$\lambda_2(L)$
IEEE-30	0.21213
IEEE-57	0.09243
IEEE-118	0.028171
IEEE-300	0.0093895
NYISO-2935	0.0014238

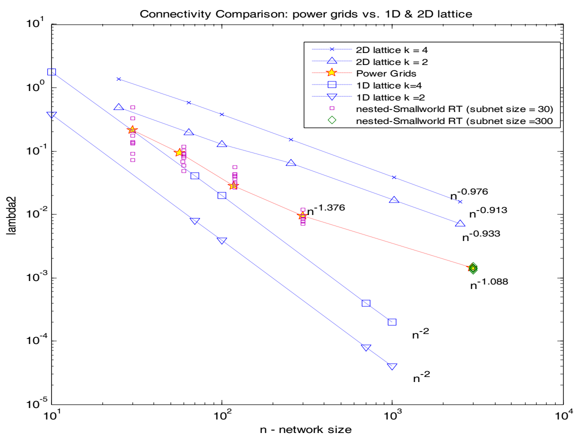


Fig. 3 Connectivity Scaling Curve versus Network Size

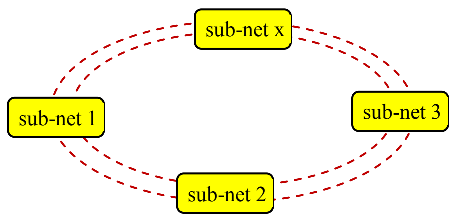


Fig. 4 Lattice-connections between Sub-networks

In trying to fit the random wiring of power network we postulated a new possible model, RT-*nested-Smallworld*. The model is resulted from nesting several "*small world*" sub-networks, whose size is likely to produce a connected topology with a realistic degree distribution, into a regular lattice again as shown in Fig. 4. This model is presented with more details in Section 5. The intuition guiding our modeling was that the network would have produced connected topologies with a connectivity that was intermediate between the 1 and 2 dimensional lattices. As shown in Fig. 3, there is an excellent match between the algebraic connectivity of this type of model and the real data.

4. ELECTRICAL CHARACTERISTICS

Another observation that guided our work and distinguishes our contribution from the referenced ones above is as follows: rather than focusing only on the network topology of power grid, we have made efforts aiming at reproducing accurately its electrical characteristics as well. One key element for a power grid network is its network admittance matrix Y , which can be expressed as a function of network connectivity matrix and line impedances. The topology and electrical characteristics of any power grid can be deduced from or closely related with them. Therefore our proposed power grid models include components which assign transmission line impedances according to specific distributions. We are not aware of other research effort to model the line impedances.

The data on line impedances of power grids we used are again from IEEE model systems and the NYISO system. The first clear observation from the empirical histogram probability density distribution (PDF) is that the distribution of the line impedances is heavy tailed.

In searching among the heavy tailed distributions for a fit, we estimated via the maximum likelihood (ML) criterion the parameters of the candidate distribution from the data and used the appropriately modified Kolmogorov-Smirnov test (K-S test) to check if the hypothesized distribution was a good fit. The candidate distribution functions include Gamma, Generalized Pareto (GP), Lognormal, Double-Pareto-Lognormal (DPLN), and two new distributions that we call Lognormal-clip, and DPLN-clip, which will be explained later.

The DPLN distribution was introduced by Reed and Jorgensen (2004), which proved to be very useful to model the size distributions of various phenomena, like incomes and earnings, human settlement sizes, etc. We applied the DPLN distribution to fit the line impedances of power grids because the latter implicitly relate to human settlement size in the network. Generally speaking, long-distance lines tend to interconnect large generation or load centers; long-distance lines usually exhibit large line impedance while large generation or load centers coincide with dense human settlements.

The Lognormal-clip and DPLN-clip are especially suited for fitting the NYISO data because the line impedances in this system appear to approximate a Lognormal or DPLN distribution very well except for having an *interrupted* tail, which is captured by the *clipping*. Therefore we assume the NYISO data is resulted from some original impedance data being "clipped" by an exponential cutoff tailing coefficient. That is, given the original impedance data Y following a specific distribution, $Y \sim f_Y(y)$, the clipped impedance data is

$$X = Z_{\max}(1 - e^{-Y/Z_{\max}}) \qquad (5)$$

with $Z_{\max}$ being the cutoff threshold.

We believe that the introduction of the clipping mechanism is reasonable. Since in real-world power grids, transmission lines are limited in length and correspondingly in the line impedance which is proportional to the length; due to the expensive right of way cost plus the high construction and maintenance cost.

Selected distribution functions are listed as below:

Double-Pareto-Lognormal (DPLN):

$$dPlN(x|\alpha,\beta,\mu,\sigma) = \frac{\alpha\beta}{\alpha+\beta}[A(\alpha,\mu,\sigma)x^{-\alpha-1}\Phi(\frac{\log x-\mu-\alpha\sigma^2}{\sigma}) + A(-\beta,\mu,\sigma)x^{\beta-1}\Phi^c(\frac{\log x-\mu+\beta\sigma^2}{\sigma})] \quad (6)$$

where $A(\theta,\mu,\sigma) = e^{(\theta\mu+\theta^2\sigma^2/2)}$

Lognormal-clip:

$$logn_{clip}(x|\mu,\sigma,Z_{\max}) = \frac{Z_{\max}}{Z_{\max}-x} logn(-Z_{\max}\log(1-x/Z_{\max})\,|\,\mu,\sigma) \quad (7)$$

DPLN-clip:

$$dPlN_{clip}(x|\alpha,\beta,\mu,\sigma,Z_{\max}) = \frac{Z_{\max}}{Z_{\max}-x} dPlN(-Z_{\max}\log(1-x/Z_{\max})\,|\,\alpha,\beta,\mu,\sigma) \quad (8)$$

With the ML parameter estimates from each candidate distribution function, K-S tests are run to pick the distribution which gives the best K-S test result. Table 3 shows the best-fitting distribution function with the corresponding ML parameter estimates. Fig. 5 compares the empirical PDFs and CDFs (in logarithm scales) with those from best-fitting distribution function for the system of NYSIO.

It also seems to agree with the topological properties of the graph, and it is reasonable to consider these heavy weight impedances as good candidates for line impedances of the links that are rewired in the small world islands, as well as the ones connecting difference islands in our *Nested- Smallworld* model.

Table 3. Distribution Fitting for the Line Impedances in Real-world Power Grids

System	Fitting Distribution	ML Parameter Estimates ($alph = 0.05$)
IEEE-30	$\Gamma(x\|a,b)$	$a = 2.14687, b = 0.10191$
IEEE-118	$\Gamma(x\|a,b)$	$a = 1.88734, b = 0.05856$
IEEE-57	$gp(x\|k,\sigma,\theta)$	$k = 0.33941$, $\sigma = 0.16963$, $\theta = 0.01814$
IEEE-300	$gp(x\|k,\sigma,\theta)$	$k = 0.45019$, $\sigma = 0.07486$, $\theta = 0.00046$
NYISO-2935	$logn_{clip}(x\|\mu,\sigma,Z_{max})$	$\mu = -2.37419$, $\sigma = 2.08285$, $Z_{max} = 1.9977$
	$dPlN_{clip}(x\|\alpha,\beta,\mu,\sigma,Z_{max})$	$\alpha = 44.25000$, $\beta = 44.30000$, $\mu = -2.37420$, $\sigma = 2.08260$, $Z_{max} = 1.9977$

5. GENERATION OF RANDOM TEST NETWORKS

In this section we present a new random topology power grid model, RT-*nested-Smallworld*. This model constructs a large scale power grid using a hierarchical way: first form connected sub-networks with size limited by the connectivity requirement indicated by equation (4); then connect the sub-networks through lattice connections; finally, generate the line impedances from some specific distribution and assign them to the links in the topology network. The hierarchy in the model is aroused from observation of real-world power grids: usually a large scale system consists of a number of smaller-size subsystem (e.g. control zones), which are interconnected by sparse and important tie lines.

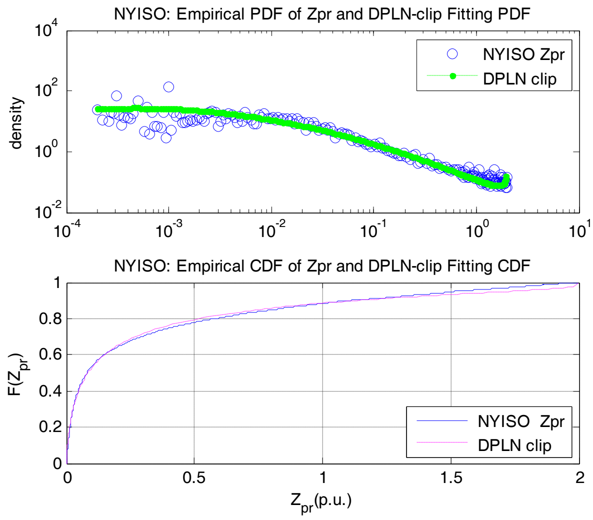


Fig. 5 Comparison between Empirical PDF/CDF Distributions and Fitting Distributions: NYISO

The model mainly contains three components: (a) *clusterSmallWorld* sub-network, (b) *Lattice*-connections, and (c) Generation and assignment of line impedances; which will be respectively described in details in following subsections.

5.1 clusterSmallWorld sub-Network

As mentioned in the previous section, power grid topology has "small-world" characteristics; it is sparsely connected with quite low and stable average nodal degree regardless of its network size. On the other side, in order for a *small-world* model to generate a connected topology, the network size has to be limited as in (4).

Therefore the first step of this new model is to select the size of sub-networks according to connectivity limitation. Then a topology is built up through a modified *small-world* model, called *clusterSmallWorld*. This model is different from Watts-Strogatz *SmallWorld* model in two aspects: the link selection and the link rewires. That is, instead of selecting links to connect most immediate $\langle k\rangle/2$ neighbors to form a regular lattice, our model selects a number k of links at random from local neighborhood, N_{d_0}, where k comes from a Poisson or shifted Geometric distribution. The local neighborhood N_{d_0} is defined as the group of close-by nodes with mutual node number difference less than the threshold d_0, that is, $N_{d_0}(i) = \{j;\ |j-i| < d_0\}$. For the link rewires, Watts-Strogatz *Small-world* model selects a small portion of the links to rewire to an arbitrary node chosen at random in the entire network to make it a *small-world* topology. Our *clusterSmallWorld* model uses a Markov chain with transition probabilities of (α,β), as shown in the below figure, to select and decide clusters of nodes and therefore links to be rewired. This mechanism is in order to capture the correlation among the rewired links.

After running the Markov transition as above for N times (i.e., for each node in the system according to the node number sequence), we get clusters of “0”s and “1”s. Then by a specific rewiring probability q_{rw}, the rewires are selected from the links originating from the group of 1-clusters and the local links get rewired to outside 1-clusters. The average cluster size for rewiring nodes is $K_{clst} = \frac{1}{\beta}$; and the steady-state probabilities are $p_0 = \frac{\beta}{\alpha+\beta}, p_1 = \frac{\alpha}{\alpha+\beta}$. Experiments are performed on available real-world power grid data to estimate the parameters.

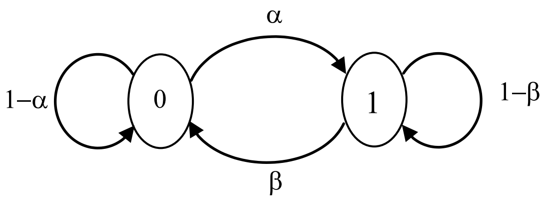


Fig. 6 Markov Chain for Selecting Cluster of Rewiring Nodes (0: not rewire; 1: rewire)

5.2 Lattice-connections

In this step lattice-connections are selected at random from neighboring sub-networks to form a whole large scale power grid network, as shown in Fig. 4. The number of lattice-connections between neighboring sub-networks is chosen to be an integer around $\langle k \rangle$.

5.3 Generation and Assignment of Line Impedances

In this part a number, m, of line impedances are generated from a specified heavy-tailed distribution, and then sorted by magnitude and group into local links, rewire links and lattice-connection links according to corresponding portions, as shown in Fig.7. Finally line impedances in each group are assigned at random to the corresponding group of links in the topology.

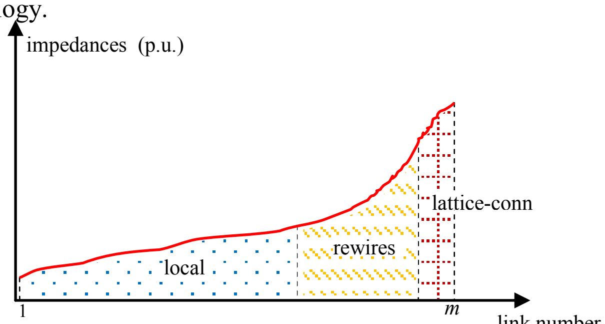


Fig. 7 Grouping Line Impedances

6. CONCLUSIONS

In this work we have studied both topological and electrical characteristics of electric power grids. Our findings conclude that power grids are sparsely connected with “small-world” characteristic and its associated “small-world” rewires usually happen with correlation in clusters; its nodal degrees follow a Poisson distribution or shifted Geometric distribution; its algebraic connectivity scales as a specific power function of network size; and its line impedances have a heavy-tailed distribution. A novel random topology power grid model, RT-*nested-Smallworld*, is proposed to capture the observed characteristics. This model is effective for generating a large number of power grid test cases with scalable network size, realistic topologies, and realistic electrical parameter settings. And it should become a useful tool for studying new networked control schemes in power grids under communication constraints.

REFERENCES

Albert, R., and Barabási, A. (2002), Statistical mechanics of complex networks, *Reviews of Modern Physics*, volume 74(1), 47-97.

Bezakova, I., Kalai, A., and Santhanam, R. (2006), Graph model selection using maximum likelihood, *Proceedings of the 23rd international conference on Machine learning*, Pittsburgh, Pennsylvania, 2006, 105 – 112.

Carreras, B. A., Lynch, V.E., Dobson, I., and Newman, D.E. (2002), Critical points and transitions in an electric power transmission model for cascading failure blackouts, *Chaos*, volume 12(4), 985-994.

Chen, J., Thorp, J., and Dobson, I. (2005), Cascading dynamics and mitigation assessment in power system disturbances via a hidden failure model, *Electrical Power and Energy Systems*, volume 133, 318-326.

Dijkstra, E. W. (1959). A note on two problems in connexion with graphs, Numerische Mathematik, volume 1, 269-271.

Erdős, P., Rényi, A. (1959), On random graphs. I., *Publicationes Mathematicae*, volume 6, 290–297.

Fiedler, M. (1989), Laplacian of graphs and algebraic connectivity, *Combinatorics and Graph Theory*, Banach Center Publications, Warsaw, volume 25, 57–70.

Heydt, G., Liu, C., Phadke, A. G., and Vital V. (2001), Solution for the crisis in electric power supply, *IEEE Computer Applications in Power*, volume 14(3), 22-30.

Kirk, E. J. (2007), Count loops in a graph, http://www.mathworks.com/matlabcentral/fileexchange/10722.

Newman M. (2003), The structure and function of complex networks, *SIAM Review,* volume (45), 167-256.

Parashar, M., Thorp, J. S. (2004), Continuum modeling of electromechanical dynamics in large-scale power systems”, *IEEE Trans. on Circuits and Systems*, volume 51(9), 1848-1858.

Power systems test case archive, (on line), http://www.ee.washington.edu/research/pstca/

Reed, W. J., and Jorgensen, M., (2004), The double pareto-lognormal distribution: a new parametric model for size distributions, *Communications in statistics, Theory and methods*, volume 33(8), 1733-1753.

Rodgers, J. L. and Nicewander, W. A. (1988), Thirteen ways to look at the correlation coefficient, *The American Statistician*, volume 42(1), 59–66.

Sole, R., Rosas-Casals, M., Corominas-Murtra, B., Valverde, S. (2007), Robustness of the European power grids under international attack, *International Journal of Bifurcations and Chaos*, volume 17(7), 2465-2475.

Wang, Z., Thomas R. J., Scaglione A., (2008), Generating random topology power grids, *Proc. 41st Annual Hawaii International Conference on System Sciences (HICSS-41)*, volume 41, Big Island, Jan 2008.

Watts D. J., Strogatz S. H. (1998), Collective dynamics of 'Small-World' networks, *Nature*, volume 393, 393-440.

Whitney D. E., Alderson D. (2006), Are technological and social networks really different, *6th International Conference on Complex Systems* (ICCS06), Boston, MA.